

WHEELER'S GRADED ARITHMETICS

INTERMEDIATE BOOK



SOMERVILLE AND GRAFF

CHICAGO
W.H. WHEELER & COMPANY



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WHEELER'S GRADED ARITHMETICS

BY

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INTERMEDIATE BOOK

CHICAGO

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PREFACE

THE real test of a textbook is its appeal to the experienced and practical teacher, for it is through her intuitive judgment, her keen understanding of children, and her sympathetic patience with their efforts, that the book must be adapted to the needs of each child. With this thought uppermost the advice and suggestions of the most successful teachers have been sought during the preparation of this series, and the constructive criticisms of these practical educators have been woven into the fabric of the books. Moreover, the authors have based their plan upon the courses in arithmetic as prescribed by the leading states and the larger cities, thus establishing an authoritative foundation upon which careful construction and intelligent criticism have produced a sound and practical series, adapted to all parts of the country. The general arrangement is distinctly topical rather than spiral, since the former has been found by experience to be more psychological than the scattered, fragmentary treatment which has frequently been attempted.

This second book of the series is intended to cover the work in arithmetic prescribed by the usual courses for the Fifth and Sixth Years. The principal feature of the Fifth Year's work is fractions, and that of the Sixth Year's work is decimals; but the usual treatment of Denominate Numbers, simple cases in Practical Measurements, and an elementary introduction to Percentage are added features in the last year's work. The character of this book changes naturally and materially from that of the Primary Book,

for the reason that the motivation of games and play does not dominate these years. However, the applications in this book continue in direct connection with the interests and activities of a more mature childhood; while offering at the same time a gradual provision for the more serious problems of simple home and business relations. This feature gives adaptability and elasticity, since it enables the skillful teacher to plan advantageously for those pupils who are to continue to the end of the course, and also for those who are compelled to leave school at the end of the sixth year.

The book gives abundant oral drills, carefully graded abstract practice so arranged as to permit of instant assignment at the blackboard, and a wealth of concrete applications that are interesting in subject matter, closely connected with the child's environment, and wholly free from obsolete and puzzling types. The concrete problems relate to saving, earning, thrift, games, playground sports, homes, farms, and business; and they visualize the everyday situations and experience in the ordinary walks of life in both the city and the country.

The authors believe that the series is teachable in its arrangement, sound in its methods, conservative in its treatment, and modern in its applications. As a carefully graded course it will serve as a constant guide for the young teacher of limited experience. On the other hand, its definite coincidence with the standard courses throughout the country should assure its welcome by the more experienced teachers who are desirous of producing genuine results.

The authors gratefully acknowledge their indebtedness to those whose criticisms and suggestions have been of material aid in the preparation of this series.

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FIFTH YEAR

REVIEW OF FUNDAMENTALS

NUMBER. NUMERATION. NOTATION

A **Unit** is any one thing.

For example, 1 cent, 1 dollar, one inch, one foot, one pound, are units.

A **Number** is a unit, or a collection of units.

A number answers the question "how many?"

Counting is the process of naming successive groups of units as we take them in regular order.

Number Words and Number Symbols convey the idea of groups. The symbols in almost universal use are the Arabic Numerals, and they are often called the Nine Digits.

One	Two	Three	Four	Five	Six	Seven	Eight	Nine
1	2	3	4	5	6	7	8	9

They originated in India about two thousand years ago, and were introduced into Europe in the thirteenth century.

The character 0, called Zero or Naught, or the Cipher, is used to represent the absence of value.

Numeration is the method of reading numbers.

Notation is the method of writing numbers.

Arithmetic is the science of numbers, and the art of computing by means of them.

THE DECIMAL SYSTEM OF NUMERATION

In the Decimal System

A group of ten units is called one ten.
 A group of ten tens is called one hundred.
 A group of ten hundreds is called one thousand.

The successive groups of tens in the hundred group are :

Two tens called twenty,	Six tens called sixty,
Three tens called thirty,	Seven tens called seventy,
Four tens called forty,	Eight tens called eighty,
Five tens called fifty,	Nine tens called ninety.

For continuous counting from ten to twenty we have :

one ten and one unit called eleven,
 one ten and two units called twelve,
 one ten and three units called thirteen ;

and, continuing, we have fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

For continuous counting from twenty to thirty we have :

two tens and one unit called twenty-one,
 two tens and two units called twenty-two,
 two tens and three units called twenty-three, etc.

Numbers from one hundred upwards follow naturally.
 Thus :

One hundred one,	Two hundred fifty-nine,
One hundred two,	Six hundred seventy-three,
One hundred three, etc.,	Nine hundred eighty-six.

THE DECIMAL SYSTEM OF NOTATION

In the Decimal System we write units at the extreme right of a number, with tens, hundreds, etc., in order, as we go towards the left. Thus :

Fifty-seven, meaning 5 tens and 7 units, is written 57,
 Sixty-eight, meaning 6 tens and 8 units, is written 68,
 Eighty, meaning 8 tens and 0 units, is written 80, etc.

Other illustrations of the notation of two-figure numbers are :

Eleven	11	Ten	10	Twenty-seven	27
Twelve	12	Twenty	20	Thirty-two	32
Thirteen	13	Thirty	30	Forty-five	45
Fourteen	14	Forty	40	Seventy-eight	78

For three-figure numbers we have :

One hundred twenty-six	126
Two hundred nineteen	219
Three hundred forty-two	342
Five hundred sixty	560
Seven hundred ninety-three	793
Eight hundred nine	809

Place Value. The value of a figure in a number depends upon its place in the number. Thus :

In the number 276 : 6 is in units' place and means 6 ones ; 7 is in tens' place and means 7 tens ; and 2 is in hundreds' place and means 2 hundreds.

Periods. For ease in reading a number that has more than three figures, we separate it into groups of three figures each called **Periods**.

Numbers from 1000 to 9999, which occur with great frequency in everyday life and in business, are not usually separated into periods.

The First Period includes numbers from 1 unit to 999 units.

The Second Period includes numbers from 1 thousand to 999 thousand.

The Third Period includes numbers from 1 million to 999 million, etc.

The table shows the complete notation through millions and including 1 billion.

BILLIONS	HUNDRED MILLIONS	TEN MILLIONS	MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	UNITS
1	5	2	0	4	7	3	6	9	2

To read a large number observe the following :

Beginning at the right, separate the number into periods of three figures each, and place a comma after each period.

In the illustration above this separation gives : 1,520,473,692.

Beginning at the left, read each group as if it stood alone, naming it according to the period in which it is found. Omit the name of the units' period.

The number is, " One billion, five hundred twenty million, four hundred seventy-three thousand, six hundred ninety-two."

In many numbers the left-hand period may have less than three figures.

ORAL PRACTICE

Name the hundreds, tens, and units in each of the following:

- | | | | | | | | | | |
|--------|-----|-----|-----|-----|--------|-----|-----|-----|-----|
| 1. 153 | 174 | 198 | 267 | 321 | 3. 201 | 344 | 561 | 700 | 819 |
| 2. 235 | 451 | 504 | 776 | 888 | 4. 399 | 507 | 705 | 750 | 999 |

Beginning at the left name each place in each number, then read the number:

- | | | | | | | |
|---------|-----------|------|-----------|-----------|-----------|--------|
| 5. 1356 | 2564 | 3115 | 4790 | 7. 11,516 | 12,314 | 15,465 |
| 6. 5617 | 7018 | 8111 | 9405 | 8. 17,651 | 34,800 | 35,566 |
| 9. | 125,446 | | 147,119 | | 157,452 | |
| 10. | 251,158 | | 461,109 | | 507,582 | |
| 11. | 1,465,489 | | 2,364,587 | | 3,765,408 | |
| 12. | 5,411,600 | | 7,500,400 | | 9,060,070 | |

BLACKBOARD PRACTICE

Write in figures, then read:

- Three hundred eleven thousand, nine hundred nine.
- Four hundred ninety-six thousand, three hundred ninety.
- Three million, four hundred forty-seven thousand, ninety-six.
- Twenty-six million, five hundred seventy-two thousand, one hundred two.
- One hundred two million, three hundred sixty thousand, ninety-one.
- Five hundred seventy-nine million, sixty-five thousand, one hundred fifty.

7. One billion, four hundred seventy million, twenty-five thousand, five hundred.

8. One billion, one hundred one million, one hundred one thousand, one hundred one.

UNITED STATES MONEY

United States Money is based on the decimal system.

10 mills = 1 cent

10 cents = 1 dime

10 dimes = 1 dollar

When dollars are written alone the dollar sign, \$, is written before the number. When cents are written alone the sign ¢ is written after the number.

Ten dollars is written \$10. Ten cents is written 10¢.

When dollars and cents are written together the **Decimal Point** is written between the number of dollars and the number of cents.

Fifteen dollars and twenty-five cents are written \$15.25.

Cents may be written as decimal parts of a dollar.

25¢ may be written \$.25.

ORAL PRACTICE

Read :

- | | |
|--|---------------------------------|
| 1. \$8, \$12, \$25, \$57 | 4. \$125, \$168, \$254, \$560 |
| 2. \$4.10, \$7.75, \$15.45 | 5. \$115.25, \$245.75, \$575.50 |
| 3. \$25.30, \$37.10, \$54.00 | 6. \$567.10, \$890.98, \$998.98 |
| 7. \$110.17, \$367.05, \$765.00, \$1157.64, \$2540.25 | |
| 8. \$701.01, \$1145.87, \$1150.15, \$35750.50, \$45107.20 | |
| 9. \$54000.00, \$135000.00, \$257650.85, \$1457327.00 | |
| 10. \$387520.75, \$1675000.00, \$3456125.50, \$87565110.00 | |

BLACKBOARD PRACTICE

Using the dollar sign and the decimal point, write :

1. Thirty-nine cents; sixty-two cents; eighty-seven cents; ninety cents.

2. One dollar and twenty-five cents; two dollars and seventy-five cents; four dollars and ten cents.

3. Twelve dollars and seventy-five cents; twenty-five dollars and twenty-five cents; forty-five dollars and five cents; eighty-three dollars and thirty-seven cents.

4. One hundred twenty-seven dollars; two hundred thirty-five dollars and fifty cents; five hundred dollars and seventy-five cents; nine hundred dollars and ninety-eight cents.

5. One thousand, two hundred fifty dollars; two thousand, seven hundred twenty-five dollars; five thousand, eight hundred ninety-seven dollars and forty-three cents.

6. Eleven thousand, five hundred seventy dollars; thirty-seven thousand, three hundred dollars; eighty-seven thousand, four hundred fifty dollars and ten cents.

7. Eighty-five thousand dollars; eight hundred fifty thousand dollars; eight million, five hundred thousand dollars; eighty-five million dollars.

THE ROMAN SYSTEM OF NOTATION

The Roman System of Notation uses seven capital letters for number symbols.

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

THE ROMAN TABLE			
I	X	C	M
II	XX	CC	MM
III	XXX	CCC	MMM
IV	XL	CD	$\overline{\text{IV}}$
V	L	D	$\overline{\text{V}}$
VI	LX	DC	$\overline{\text{VI}}$
VII	LXX	DCC	$\overline{\text{VII}}$
VIII	LXXX	DCCC	$\overline{\text{VIII}}$
IX	XC	CM	$\overline{\text{IX}}$

By the following laws the seven letters may be combined to represent other numbers :

(1) *The value of the letters I, X, C, and M, is repeated if either letter is repeated.*

III denotes 3. XX denotes 20. CC denotes 200, etc.

(2) *Addition of letter values is required when a letter of smaller value follows one of greater value.*

VI denotes 6. XVII denotes 17. LXXI denotes 71.

(3) *Subtraction of letter values is required when a letter of smaller value precedes one of greater value.*

IX denotes 9. XL denotes 40. XC denotes 90.

(4) *A letter between two letters, each of greater value, is regarded as combined with the last letter.*

XIV denotes 14. XIX denotes 19. MCM denotes 1900.

(5) *A bar over a letter multiplies its value by one thousand.*
 $\overline{\text{X}}$ denotes 10,000. $\overline{\text{IV}}$ denotes 4000. $\overline{\text{C}}$ denotes 100,000.

ORAL PRACTICE

Read :

- | | |
|--------------------|--------------------|
| 1. XLI, XLV, XLIX. | 6. LII, CXLI. |
| 2. XCI, CXI, CIII. | 7. DCIX, MDCC. |
| 3. DLXII, CCXXVI. | 8. CXLVIII, MDCCC. |
| 4. MMDC, CCXXV. | 9. MCMII, MDLX. |
| 5. MDCX, MDC. | 10. MCMV, MCMXIX. |

WRITTEN PRACTICE

Write in Roman numerals :

- | | |
|------------------------|-----------------------------|
| 1. 13, 21, 12, 17, 20. | 5. 110, 120, 125, 134, 155. |
| 2. 31, 39, 43, 48, 52. | 6. 213, 223, 247, 356, 453. |
| 3. 11, 24, 29, 36, 42. | 7. 753, 1215, 1492, 1682. |
| 4. 67, 71, 88, 91, 98. | 8. 1776, 1815, 1893, 1924. |
9. Write in their order all of the years from the year of your birth to the present year.
10. Write the chapter headings for a book containing thirty-four chapters.

REVIEW OF ADDITION

Addition is the process of finding the sum of two or more numbers.

The Sum of two or more numbers is the number that contains as many units as there are in all the numbers added.

The numbers to be added are called the **Addends**.

The Sign of Addition + is called the **Plus Sign**.

The Sign of Equality is the sign =, and is read "equals," or "is equal to."

Only like numbers can be added. The sum is the same regardless of the order in which given numbers are added.

REVIEW OF MENTAL ADDITION

DRILL TABLE

1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9
	2	2	2	2	2	2	2	2
	2	3	4	5	6	7	8	9
		3	3	3	3	3	3	3
		3	4	5	6	7	8	9
			4	4	4	4	4	4
			4	5	6	7	8	9
				5	5	5	5	5
				5	6	7	8	9
					6	6	6	6
					6	7	8	9
						7	7	7
						7	8	9
							8	8
							8	9
								9
								9

This table should be copied upon the blackboard and used as a daily drill.

Naming the Sum. In mental addition we carry in mind the successive totals: Thus: In counting by two's we do not say "2 plus 2 equals 4, 4 plus 2 equals 6, etc."

We think only of the totals, and we say, "2, 4, 6, 8, etc."

Using the same principle in adding $3+4+5+8+9$ we say aloud the totals thus: "3, 7, 12, 20, 29."

ORAL PRACTICE

Add the following groups from left to right. Test by adding from right to left :

1. $3+4+6+2$. 4. $4+7+6+4$. 7. $5+4+8+8$.

2. $8+7+4+6$. 5. $5+7+8+4$. 8. $6+9+9+7$.

3. $6+2+3+9$. 6. $5+8+6+9$. 9. $7+8+9+8$.

10. $4+3+7+8+7+8$. 14. $8+7+7+7+9+9$.

11. $6+8+9+3+2+7$. 15. $6+9+8+5+6+4$.

12. $1+5+8+6+9+4$. 16. $9+7+8+6+9+5$.

13. $9+8+7+6+5+4$. 17. $8+9+3+9+5+8$.

18. $5+8+7+3+1+9+5+8+7+5$.

19. $8+3+9+2+9+4+8+5+6+7$.

20. $1+2+3+4+5+6+7+8+9+5$.

ADDING BY GROUPS

In adding by groups we use the totals which we have learned from the drill table. After a little practice it is not difficult to give at sight all of the combinations of two figures. With practice we may learn to group many cases that include three figures.

In finding the sum in the illustration think :

Adding upwards, "8, 18, 27, 38, 45."

To test : Adding downwards, "7, 18, 27, 37, 45."

$$\begin{array}{r} 3 \\ 4 \end{array} \left. \vphantom{\begin{array}{r} 3 \\ 4 \end{array}} \right\} 7$$

$$\begin{array}{r} 5 \\ 6 \end{array} \left. \vphantom{\begin{array}{r} 5 \\ 6 \end{array}} \right\} 11$$

$$\begin{array}{r} 4 \\ 5 \end{array} \left. \vphantom{\begin{array}{r} 4 \\ 5 \end{array}} \right\} 9$$

$$\begin{array}{r} 8 \\ 2 \end{array} \left. \vphantom{\begin{array}{r} 8 \\ 2 \end{array}} \right\} 10$$

$$\begin{array}{r} 7 \\ 1 \end{array} \left. \vphantom{\begin{array}{r} 7 \\ 1 \end{array}} \right\} 8$$

45

ORAL PRACTICE

Add by grouping :

1. 4	2. 8	3. 3	4. 7	5. 6	6. 5	7. 8
6	7	9	5	2	2	2
7	4	8	9	8	3	6
3	9	3	2	4	6	5
5	1	6	6	9	7	9
8	6	7	8	5	4	1
2	5	4	4	9	9	9
7	3	5	2	6	7	8
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
8. 2	9. 8	10. 3	11. 1	12. 6	13. 6	14. 4
5	2	6	6	7	8	2
6	3	6	5	9	4	8
4	7	5	3	5	5	7
7	9	8	9	9	9	9
8	4	5	6	7	8	7
3	3	4	3	8	6	5
9	8	7	2	1	7	6
6	9	9	8	4	7	9
2	2	1	6	3	5	8
7	8	7	5	8	9	2
5	4	2	9	7	6	7
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

WRITTEN PRACTICE

Add, and test by adding in the opposite direction :

1. 18	2. 26	3. 38	4. 19	5. 34	6. 58
24	15	22	26	82	83
77	32	17	47	27	69
12	64	59	58	72	94
<u>38</u>	<u>56</u>	<u>85</u>	<u>92</u>	<u>97</u>	<u>76</u>

ADDITION

19

7.	27	8.	35	9.	13	10.	34	11.	34	12.	39
	52		28		67		46		25		77
	26		44		28		90		99		53
	40		59		47		23		72		96
	88		30		30		35		39		28
	63		53		74		61		86		82
	57		64		55		76		77		79
	<u>15</u>		<u>82</u>		<u>96</u>		<u>83</u>		<u>94</u>		<u>68</u>
13.	259	14.	398	15.	274	16.	808	17.	730	18.	189
	197		664		405		357		399		948
	118		759		650		634		507		807
	482		425		289		739		846		674
	<u>506</u>		<u>630</u>		<u>562</u>		<u>291</u>		<u>468</u>		<u>750</u>
19.	356	20.	478	21.	479	22.	869	23.	969	24.	788
	222		233		525		356		667		679
	107		609		368		978		858		501
	344		562		144		317		436		839
	638		237		200		109		640		438
	442		255		757		863		849		995
	510		814		656		717		974		780
	<u>104</u>		<u>575</u>		<u>333</u>		<u>407</u>		<u>707</u>		<u>997</u>
25.	142	26.	614	27.	2187	28.	3465	29.	905		
	12		3754		597		4785		5746		
	1623		42		4043		803		6993		
	853		2443		55		1006		847		
	200		503		842		7642		8072		
	85		354		5630		965		26		
	1164		30		6144		4400		866		
	<u>817</u>		<u>1819</u>		<u>421</u>		<u>6074</u>		<u>2099</u>		

30.	6858	31.	60	32.	72	33.	436
	12227		44823		66000		28009
	34		24089		3754		95
	8169		108		598		47880
	7070		67943		48826		2739
	53001		5390		77502		66964
	5946		87056		30060		55408
	41235		807		5989		95072
	733		56424		14445		2850
	<u>30600</u>		<u>8655</u>		<u>99027</u>		<u>70646</u>

34.	\$13.10	35.	\$74.49	36.	\$183.22	37.	\$580.98
	43.72		60.50		64.01		450.67
	86.43		88.87		257.80		19.49
	8.70		9.33		490.45		796.74
	40.88		96.78		500.00		845.80
	215.17		5.12		114.00		29.65
	109.12		10.00		12.50		110.15
	<u>38.55</u>		<u>142.70</u>		<u>16.19</u>		<u>13.74</u>

38.	\$ 4.95	39.	\$146.10	40.	\$.58	41.	\$ 58.47
	326.72		25.38		6.22		964.25
	73.60		.97		400.00		9.69
	500.23		2.54		758.36		295.22
	847.86		369.29		477.65		619.86
	196.00		620.00		563.10		886.58
	610.47		786.38		910.69		990.40
	68.54		448.63		659.21		1000.00
	31.15		15.10		5.84		18.45
	124.00		217.20		16.00		500.00
	<u>5.19</u>		<u>314.10</u>		<u>8.19</u>		<u>117.10</u>

42. \$1245.00	43. \$1900.00	44. \$4000.00	45. \$ 5400.00
560.00	54.00	4500.00	560.90
9.67	245.90	750.00	10000.00
118.56	1500.00	3456.00	37.50
23.87	65.10	9.70	5600.00
80.00	560.00	1500.00	675.10
654.11	1200.00	11.50	775.85
.87	89.00	768.19	1300.10
66.77	4350.00	5675.00	8.06
1000.00	11.98	.05	57.50
75.99	345.90	2010.01	100000.00
857.50	1.00	1.88	7500.00
65.10	1.45	100.00	2750.00
125.00	10.12	35.05	305.12
81.45	157.19	2100.00	7.96
35.20	8.07	9.17	18.70
<u>1120.15</u>	<u>19.64</u>	<u>31.40</u>	<u>124.12</u>

REVIEW OF SUBTRACTION

Subtraction is the process of finding the difference between two numbers.

The smaller of the two numbers is called the **Subtrahend**.

The larger of the two numbers is called the **Minuend**.

The **Difference** is the number that, added to the subtrahend, will give the minuend.

The difference may be defined as the number by which the greater number exceeds the smaller.

The Sign of Subtraction – is called the **Minus Sign**.

The Addition Method of Subtraction considers the number that, added to a given number, will give the larger number. Thus:

To find the difference between 13 and 8, we think

"8 and what number are 13?" We write the result, 5.

To find the difference between 9 and 16, we think

"9 and what number are 16?" We write the result, 7.

This method is commonly used in all business.

DRILL TABLES FOR SUBTRACTION

9	9	9	9	9	9	9	9
1	2	3	4	5	6	7	8
8	8	8	8	8	8	8	8
1	2	3	4	5	6	7	
	7	7	7	7	7	7	
	1	2	3	4	5	6	
		6	6	6	6	6	
		1	2	3	4	5	
			5	5	5	5	
			1	2	3	4	
				4	4	4	
				1	2	3	
					3	3	
					1	2	
						2	
						1	

- | | | |
|-------------------|--------------------|--------------------|
| 1. 3 and ? are 7. | 7. 3 and ? are 8. | 13. 2 and ? are 6. |
| 2. 4 and ? are 8. | 8. 5 and ? are 8. | 14. 3 and ? are 9. |
| 3. 4 and ? are 7. | 9. 4 and ? are 6. | 15. 6 and ? are 9. |
| 4. 2 and ? are 7. | 10. 5 and ? are 7. | 16. 6 and ? are 8. |
| 5. 3 and ? are 5. | 11. 2 and ? are 8. | 17. 5 and ? are 9. |
| 6. 4 and ? are 9. | 12. 3 and ? are 6. | 18. 7 and ? are 9. |

(When the minuend is a number greater than 9.)

				18				
				9				
			17		17			
			8		9			
		16		16		16		
		7		8		9		
	15		15		15		15	
	6		7		8		9	
	14		14		14		14	
	5		6		7		8	
	13		13		13		13	
	4		5		6		7	
	12		12		12		12	
	3		4		5		6	
	11		11		11		11	
	2		3		4		5	
10		10		10		10		10
1		2		3		4		5

ORAL PRACTICE

Give the difference between :

- | | | |
|--------------|---------------|---------------|
| 1. 8 and 13. | 9. 9 and 13. | 17. 7 and 12. |
| 2. 7 and 14. | 10. 6 and 14. | 18. 9 and 14. |
| 3. 6 and 13. | 11. 7 and 15. | 19. 9 and 16. |
| 4. 5 and 11. | 12. 9 and 15. | 20. 8 and 17. |
| 5. 5 and 12. | 13. 8 and 16. | 21. 7 and 16. |
| 6. 8 and 14. | 14. 5 and 13. | 22. 9 and 18. |
| 7. 9 and 17. | 15. 9 and 12. | 23. 8 and 12. |
| 8. 8 and 15. | 16. 7 and 13. | 24. 9 and 11. |

WRITTEN PRACTICE

Subtract, and test your work by showing that the sum of the subtrahend and the difference equals the minuend.

1. $\begin{array}{r} 110 \\ 20 \\ \hline \end{array}$	6. $\begin{array}{r} 232 \\ 121 \\ \hline \end{array}$	11. $\begin{array}{r} 1288 \\ 189 \\ \hline \end{array}$	16. $\begin{array}{r} 1675 \\ 1235 \\ \hline \end{array}$	21. $\begin{array}{r} 25486 \\ 2547 \\ \hline \end{array}$
2. $\begin{array}{r} 121 \\ 30 \\ \hline \end{array}$	7. $\begin{array}{r} 342 \\ 226 \\ \hline \end{array}$	12. $\begin{array}{r} 1373 \\ 246 \\ \hline \end{array}$	17. $\begin{array}{r} 3251 \\ 1167 \\ \hline \end{array}$	22. $\begin{array}{r} 33210 \\ 11745 \\ \hline \end{array}$
3. $\begin{array}{r} 130 \\ 26 \\ \hline \end{array}$	8. $\begin{array}{r} 546 \\ 358 \\ \hline \end{array}$	13. $\begin{array}{r} 2424 \\ 268 \\ \hline \end{array}$	18. $\begin{array}{r} 5609 \\ 2025 \\ \hline \end{array}$	23. $\begin{array}{r} 62504 \\ 28796 \\ \hline \end{array}$
4. $\begin{array}{r} 139 \\ 88 \\ \hline \end{array}$	9. $\begin{array}{r} 760 \\ 495 \\ \hline \end{array}$	14. $\begin{array}{r} 4635 \\ 676 \\ \hline \end{array}$	19. $\begin{array}{r} 7086 \\ 3747 \\ \hline \end{array}$	24. $\begin{array}{r} 83000 \\ 54975 \\ \hline \end{array}$
5. $\begin{array}{r} 124 \\ 48 \\ \hline \end{array}$	10. $\begin{array}{r} 809 \\ 547 \\ \hline \end{array}$	15. $\begin{array}{r} 7500 \\ 899 \\ \hline \end{array}$	20. $\begin{array}{r} 8540 \\ 4786 \\ \hline \end{array}$	25. $\begin{array}{r} 94867 \\ 9680 \\ \hline \end{array}$
26. $\begin{array}{r} \$5.00 \\ 1.75 \\ \hline \end{array}$	31. $\begin{array}{r} \$10.00 \\ 3.57 \\ \hline \end{array}$	36. $\begin{array}{r} \$25.50 \\ 12.75 \\ \hline \end{array}$	41. $\begin{array}{r} \$125.00 \\ 76.95 \\ \hline \end{array}$	
27. $\begin{array}{r} \$5.00 \\ 2.15 \\ \hline \end{array}$	32. $\begin{array}{r} \$10.00 \\ 4.49 \\ \hline \end{array}$	37. $\begin{array}{r} \$37.50 \\ 14.57 \\ \hline \end{array}$	42. $\begin{array}{r} \$375.45 \\ 196.53 \\ \hline \end{array}$	
28. $\begin{array}{r} \$5.00 \\ 3.15 \\ \hline \end{array}$	33. $\begin{array}{r} \$10.00 \\ 5.98 \\ \hline \end{array}$	38. $\begin{array}{r} \$45.75 \\ 32.11 \\ \hline \end{array}$	43. $\begin{array}{r} \$525.50 \\ 354.25 \\ \hline \end{array}$	
29. $\begin{array}{r} \$5.00 \\ 3.49 \\ \hline \end{array}$	34. $\begin{array}{r} \$10.00 \\ 7.75 \\ \hline \end{array}$	39. $\begin{array}{r} \$62.50 \\ 49.98 \\ \hline \end{array}$	44. $\begin{array}{r} \$875.25 \\ 590.00 \\ \hline \end{array}$	
30. $\begin{array}{r} \$5.00 \\ 4.35 \\ \hline \end{array}$	35. $\begin{array}{r} \$10.00 \\ 8.71 \\ \hline \end{array}$	40. $\begin{array}{r} \$75.25 \\ 63.56 \\ \hline \end{array}$	45. $\begin{array}{r} \$765.30 \\ 677.10 \\ \hline \end{array}$	

REVIEW OF MULTIPLICATION

Multiplication is the process of taking one number as many times as there are units in another number.

The Multiplicand is the number to be multiplied.

The Multiplier is the number by which we multiply.

The Product is the result of a multiplication.

The Sign of Multiplication $\times$ is read in two ways.

If the multiplier is written before the sign $\times$, the sign is read "times."

Thus: 4×9 lb. is read "four times nine pounds."

If the multiplier is written after the sign $\times$, the sign is read "multiplied by."

Thus: $9 \text{ lb.} \times 4$ is read "nine pounds multiplied by four."

The product always has the same name as the multiplicand.

A Concrete Number is a number to which a particular name is given.

12 feet, 15 pounds, and 7 gallons, are concrete numbers.

An Abstract Number is a number to which no particular name is given.

12, 15, and 7 are abstract numbers.

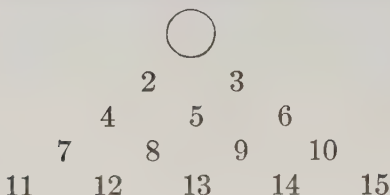
All number is really abstract. It is customary, however, to speak of a number that refers to a particular kind of object or measure as a concrete number.

In multiplication it is usually more convenient to use the smaller of two numbers for the multiplier. For example, instead of multiplying 14 by 354, we may multiply 354 by 14 with much less labor. Each process gives the same product. In the figure the product is 3 times 4 squares if we count the rows across the page, but it is 4 times 3 squares if the rows are counted up or down. In either case we have the same result, 12 squares.



Any two or more numbers may be multiplied as if all were abstract, but the name of the multiplicand must be given to the product when the multiplicand is concrete.

DRILL TABLE FOR MULTIPLICATION



		2		3	
		4		5	
				6	
	7		8		9
					10
11	12	13	14	15	

The pyramid above offers an opportunity for the review of all of the multiplication tables up to "fifteen times fifteen."

The pyramid should be copied upon the blackboard with any number from 2 to 15 inserted in the circle. As the leader points to any number outside of the circle, give instantly the product of that number by the number in the circle.

By changing the number in the circle any desired multiplication table is obtained instantly.

WRITTEN MULTIPLICATION WITH A MULTIPLIER OF TWO OR MORE FIGURES

Multiply 356 by 247.

$$\begin{array}{r}
 356 \\
 247 \\
 \hline
 2492 = 7 \times 356 \\
 14240 = 40 \times 356 \\
 71200 = 200 \times 356 \\
 \hline
 87932 = \text{Product.}
 \end{array}$$

Multiply \$541.15 by 142.

$$\begin{array}{r}
 \$541.15 \\
 142 \\
 \hline
 108230 = 2 \times \$541.15 \\
 216460 = 40 \times \$541.15 \\
 54115 = 100 \times \$541.15 \\
 \hline
 \$76843.30 = \text{Product.}
 \end{array}$$

In actual practice the zeros are not written in units' place in the second partial product, or in tens' place in the third partial product. Study each of the following and give aloud the steps taken in each :

$$\begin{array}{r} 248 \\ 65 \\ \hline 1240 \\ 1488 \\ \hline 16120 \end{array}$$

$$\begin{array}{r} 371 \\ 428 \\ \hline 2968 \\ 742 \\ \hline 1484 \\ \hline 158788 \end{array}$$

$$\begin{array}{r} 596 \\ 309 \\ \hline 5364 \\ 1788 \\ \hline 184164 \end{array}$$

To multiply by 10, 100, 1000, etc.

$$\begin{array}{r} 545 \\ 10 \\ \hline 5450 \end{array} \quad \begin{array}{r} 545 \\ 100 \\ \hline 54500 \end{array} \quad \begin{array}{r} 545 \\ 1000 \\ \hline 545000 \end{array} \quad \begin{array}{r} 545 \\ 10000 \\ \hline 5450000 \end{array} \quad \begin{array}{r} 545 \\ 100000 \\ \hline 54500000 \end{array}$$

To multiply a number by 10, annex one zero.

To multiply a number by 100, annex two zeros.

To multiply a number by 1000, annex three zeros.

In numbers like 240, 2400, etc., the figures 2 and 4 are called significant figures. To multiply by 20, 30, 40, etc., or by 200, 300, etc.

$$\begin{array}{r} 545 \\ 20 \\ \hline 10900 \end{array} \quad \begin{array}{r} 545 \\ 30 \\ \hline 16350 \end{array} \quad \begin{array}{r} 545 \\ 40 \\ \hline 21800 \end{array} \quad \begin{array}{r} 540 \\ 200 \\ \hline 108000 \end{array} \quad \begin{array}{r} 505 \\ 300 \\ \hline 151500 \end{array} \quad \begin{array}{r} 5400 \\ 2000 \\ \hline 10800000 \end{array}$$

From these products we have :

When the multiplicand, or the multiplier, or both, end in one or more zeros, multiply without regard to the zeros; and annex to the product as many zeros as there are at the end of the multiplicand and at the end of the multiplier.

A Power of a number is the result of taking the number two or more times in a product. Thus $2 \times 2 = 4$; $2 \times 2 \times 2 = 8$; $3 \times 3 \times 3 = 27$, etc. 4 is the *second power* of 2. 8 is the *third power* of 2. 27 is the *third power* of 3.

Testing Results in Multiplication. A product may be tested by interchanging the multiplicand and multiplier and multiplying again. The two results should be identical.

WRITTEN PRACTICE

Multiply and test :

- | | | |
|-----------------|-----------------|------------------|
| 1. 134 by 26. | 11. 365 by 234. | 21. 1506 by 132. |
| 2. 356 by 32. | 12. 463 by 178. | 22. 2137 by 245. |
| 3. 478 by 45. | 13. 587 by 365. | 23. 2243 by 421. |
| 4. 493 by 51. | 14. 623 by 489. | 24. 3965 by 478. |
| 5. 569 by 58. | 15. 742 by 596. | 25. 4648 by 629. |
| 6. 540 by 67. | 16. 869 by 753. | 26. 3779 by 845. |
| 7. 658 by 74. | 17. 925 by 581. | 27. 5672 by 784. |
| 8. 749 by 82. | 18. 987 by 638. | 28. 6324 by 697. |
| 9. 763 by 89. | 19. 899 by 425. | 29. 7895 by 867. |
| 10. 832 by 96. | 20. 927 by 786. | 30. 9674 by 958. |
| 31. 143 by 200. | 40. 909 by 782. | 49. 548 by 326. |
| 32. 234 by 310. | 41. 783 by 456. | 50. 540 by 762. |
| 33. 278 by 406. | 42. 870 by 645. | 51. 671 by 409. |
| 34. 327 by 600. | 43. 906 by 527. | 52. 748 by 650. |
| 35. 581 by 609. | 44. 968 by 754. | 53. 700 by 922. |
| 36. 469 by 780. | 45. 104 by 256. | 54. 846 by 608. |
| 37. 656 by 874. | 46. 345 by 160. | 55. 695 by 487. |
| 38. 538 by 796. | 47. 423 by 280. | 56. 899 by 670. |
| 39. 723 by 801. | 48. 527 by 307. | 57. 988 by 709. |

- | | |
|--------------------|-----------------------|
| 58. \$5.20 by 23. | 67. \$95.50 by 80. |
| 59. \$6.08 by 29. | 68. \$125.50 by 64. |
| 60. \$6.50 by 32. | 69. \$167.30 by 72. |
| 61. \$6.75 by 34. | 70. \$250.35 by 65. |
| 62. \$8.47 by 56. | 71. \$450.75 by 62. |
| 63. \$9.80 by 75. | 72. \$672.30 by 120. |
| 64. \$21.35 by 64. | 73. \$1257.20 by 135. |
| 65. \$49.63 by 82. | 74. \$2250.75 by 150. |
| 66. \$65.99 by 78. | 75. \$3750.75 by 246. |

REVIEW OF DIVISION

Division is the process of finding how many times one number contains another number.

The Dividend is the number to be divided.

The Divisor is the number by which we divide.

The Quotient is the result of a division.

The Sign of Division is the sign $\div$, and is read "divided by."

Division may be defined as the process of finding one of two numbers when their product and one of the numbers are known. In this definition of division the given product is the Dividend, the given number is the Divisor, and the required number is the Quotient.

ORAL PRACTICE

Give the quotient of :

- | | | | | |
|----------------------|-----------------------|------------------------|------------------------|------------------------|
| 1. $2\overline{)48}$ | 6. $4\overline{)56}$ | 11. $5\overline{)90}$ | 16. $7\overline{)154}$ | 21. $8\overline{)320}$ |
| 2. $2\overline{)62}$ | 7. $4\overline{)68}$ | 12. $6\overline{)102}$ | 17. $6\overline{)192}$ | 22. $9\overline{)288}$ |
| 3. $3\overline{)36}$ | 8. $5\overline{)75}$ | 13. $7\overline{)112}$ | 18. $8\overline{)216}$ | 23. $8\overline{)360}$ |
| 4. $2\overline{)88}$ | 9. $4\overline{)72}$ | 14. $8\overline{)112}$ | 19. $9\overline{)216}$ | 24. $9\overline{)360}$ |
| 5. $3\overline{)96}$ | 10. $5\overline{)85}$ | 15. $9\overline{)135}$ | 20. $7\overline{)280}$ | 25. $9\overline{)450}$ |

WRITTEN DIVISION WITH A DIVISOR OF TWO OR MORE FIGURES

Exact Division. Study each of the following and give aloud the steps in each.

$$\begin{array}{r} 254 \\ 36 \overline{)9144} \\ \underline{72} \\ 194 \\ \underline{180} \\ 144 \\ \underline{144} \\ 0 \end{array}$$

$$\begin{array}{r} 263 \\ 327 \overline{)86001} \\ \underline{654} \\ 2060 \\ \underline{1962} \\ 981 \\ \underline{981} \\ 0 \end{array}$$

$$\begin{array}{r} 504 \\ 84 \overline{)42336} \\ \underline{420} \\ 336 \\ \underline{336} \\ 0 \end{array}$$

Testing the Result of an Exact Division. An exact division may be tested by multiplying the divisor by the quotient. The product should equal the dividend.

In the first example above : $36 \times 254 = 9144$.

Inexact Division. The part of a dividend that, at the end of a division, is less than the divisor, is called the **Remainder**. Study the following :

$$\begin{array}{r} 38 \text{ Quotient} \\ 35 \overline{)1337} \\ \underline{105} \\ 287 \\ \underline{280} \\ 7 \text{ Remainder} \end{array}$$

$$\begin{array}{r} 1231 \text{ Quotient} \\ 42 \overline{)51703} \\ \underline{42} \\ 97 \\ \underline{84} \\ 130 \\ \underline{126} \\ 43 \\ \underline{42} \\ 1 \text{ Remainder} \end{array}$$

$$\begin{array}{r} 307 \text{ Quotient} \\ 125 \overline{)38477} \\ \underline{375} \\ 977 \\ \underline{875} \\ 102 \text{ Remainder} \end{array}$$

Testing the Result of an Inexact Division. To test a result in which a remainder occurs, multiply the divisor by

the quotient and add the remainder to the product. The sum obtained should equal the dividend.

In the first example above, $35 \times 38 = 1330$. $1330 + 7 = 1337$.

WRITTEN PRACTICE

Divide and test :

- | | | |
|-----------------|-------------------|--------------------|
| 1. 1625 by 25. | 11. 11622 by 198. | 21. 121518 by 133. |
| 2. 2090 by 55. | 12. 11286 by 209. | 22. 171814 by 271. |
| 3. 3186 by 27. | 13. 13195 by 290. | 23. 196803 by 374. |
| 4. 4368 by 78. | 14. 14335 by 305. | 24. 352256 by 344. |
| 5. 4876 by 23. | 15. 15652 by 333. | 25. 574287 by 609. |
| 6. 6104 by 56. | 16. 16884 by 402. | 26. 577536 by 564. |
| 7. 7020 by 45. | 17. 30150 by 455. | 27. 677039 by 671. |
| 8. 7276 by 68. | 18. 38286 by 709. | 28. 763904 by 746. |
| 9. 8640 by 72. | 19. 62284 by 724. | 29. 823296 by 804. |
| 10. 9890 by 46. | 20. 68100 by 908. | 30. 984064 by 961. |

Division by Powers of 10

By multiplication :

$$\begin{array}{r} 15 \\ 10 \\ \hline 150 \end{array}$$

$$\begin{array}{r} 15 \\ 100 \\ \hline 1500 \end{array}$$

$$\begin{array}{r} 15 \\ 1000 \\ \hline 15000 \end{array}$$

Therefore, by division :

$$150 \div 10 = 15.$$

Or, 150, with one zero cut off, is divided by 10.

$$1500 \div 100 = 15.$$

Or, 1500, with two zeros cut off, is divided by 100.

$$15000 \div 1000 = 15.$$

Or 15000, with three zeros cut off, is divided by 1000.

We may conclude, therefore :

To divide by 10, by 100, by 1000, etc., cut off from the right of the dividend as many places as there are zeros in the divisor. The resulting quotient is the number expressed by the figures at the left of the figures cut off. The remainder is the number expressed by the figures cut off.

Illustrations :

$$\begin{array}{rcl}
 & & 25 \\
 1. & 10 \overline{)250} & 1 \overline{)0}25 \overline{)0} \\
 & & 46 \frac{57}{100} \\
 3. & 100 \overline{)4657} & 1 \overline{)00}46 \overline{)57} \\
 \\
 2. & 100 \overline{)35125} & 4. & 1000 \overline{)37580} \\
 & 351 \frac{25}{100} & & 37 \frac{580}{1000} \\
 1 \overline{)00}351 \overline{)25} & & 1 \overline{)000}37 \overline{)580}
 \end{array}$$

WRITTEN PRACTICE

Divide :

- | | | |
|----------------|-------------------|---------------------|
| 1. 450 by 10. | 6. 12700 by 100. | 11. 98876 by 1000. |
| 2. 580 by 10. | 7. 45300 by 100. | 12. 14000 by 1000. |
| 3. 760 by 10. | 8. 76550 by 100. | 13. 15105 by 1000. |
| 4. 900 by 10. | 9. 91540 by 100. | 14. 25560 by 1000. |
| 5. 1250 by 10. | 10. 93725 by 100. | 15. 356126 by 1000. |

Division by Multiples of 10

Study each of the following :

$$\begin{array}{rcl}
 1. & 30 \overline{)150} & 2. & 40 \overline{)2560} & 3. & 60 \overline{)21600} & 4. & 600 \overline{)3600} \\
 & 5 & & 64 & & 360 & & 6 \\
 & 3 \overline{)0}15 \overline{)0} & & 4 \overline{)0}256 \overline{)0} & & 6 \overline{)0}2160 \overline{)0} & & 6 \overline{)00}36 \overline{)00} \\
 \\
 5. & 30 \overline{)253} & 6. & 40 \overline{)5625} & 7. & 50 \overline{)2001} & 8. & 700 \overline{)36200} \\
 & 8 \frac{13}{30} & & 140 \frac{25}{40} & & 40 \frac{1}{50} & & 51 \frac{500}{700} \\
 & 3 \overline{)0}25 \overline{)3} & & 4 \overline{)0}562 \overline{)5} & & 5 \overline{)0}200 \overline{)1} & & 7 \overline{)00}362 \overline{)00}
 \end{array}$$

WRITTEN PRACTICE

Divide :

- | | | |
|---------------|------------------|-------------------|
| 1. 360 by 30. | 7. 4500 by 50. | 13. 2805 by 40. |
| 2. 480 by 40. | 8. 4500 by 500. | 14. 3602 by 60. |
| 3. 640 by 40. | 9. 6400 by 80. | 15. 4501 by 90. |
| 4. 720 by 60. | 10. 6400 by 800. | 16. 5690 by 200. |
| 5. 780 by 60. | 11. 8100 by 90. | 17. 7846 by 300. |
| 6. 840 by 30. | 12. 81000 by 90. | 18. 13451 by 400. |

REVIEW OF FRACTIONS

How Fractions are Named. If a rectangle is divided into four equal parts, and these parts are shaded as in the figures below, the shaded portion of each is a **Fraction of the Whole Rectangle**.



In the first figure **one** part is shaded.

In the second figure **two** parts are shaded.

In the third figure **three** parts are shaded.

To indicate one of the four parts we write $\frac{1}{4}$.

To indicate two of the four parts we write $\frac{2}{4}$.

To indicate three of the four parts we write $\frac{3}{4}$.

$\frac{1}{4}$, called "One Fourth," $\frac{2}{4}$, called "Two Fourths," and $\frac{3}{4}$, called "Three Fourths," are **Fractions**.

The number over the line tells how many parts are taken.
This number is the **Numerator**.

The number under the line tells into how many equal parts the unit is divided.

This number is the **Denominator**.

Remember that a fraction tells

The number of parts taken
The number of equal parts into which a unit is divided.

Each of the rectangles below is of the same size. Write the fractions that are represented by the shaded portions of each rectangle.



CHANGING THE FORM OF A FRACTION

Divide a square into two equal parts as in the figure at the left. Each part is $\frac{1}{2}$ of the square.

How many halves are there in one unit?



Divide the same square into four equal parts as in the figure at the right. Each part is $\frac{1}{4}$ of the square.

How many fourths are there in one unit?

You have found, therefore, that

Two Halves = Four Fourths, or, $\frac{2}{2} = \frac{4}{4}$.

The four oblongs below are all of equal length and width.



Using the two oblongs at the left, we have $\frac{3}{4} = \frac{?}{8}$.

Using the two oblongs at the right, we have $\frac{5}{8} = \frac{?}{12}$.

At the left you see that $\frac{6}{8}$ are just as long as $\frac{3}{4}$. At the right you see that $\frac{10}{12}$ are just as long as $\frac{5}{6}$. Therefore,

The form of a fraction may be changed without changing its value:

In $\frac{1}{4}$ there are $\frac{2}{8}$. How many eighths are there in $\frac{3}{4}$?

In $\frac{1}{6}$ there are $\frac{2}{12}$. How many twelfths are there in $\frac{5}{6}$?

If both numerator and denominator of $\frac{3}{4}$ are multiplied by 2

$$\frac{2 \times 3}{2 \times 4} = \frac{6}{8}.$$

And, as we know that $\frac{6}{8}$ is another form of $\frac{3}{4}$, we conclude that:

Both the numerator and the denominator of a fraction may be multiplied by the same number without changing the value of the fraction.

$$\frac{2 \times 5}{2 \times 6} = \frac{?}{?}$$

$$\frac{2 \times 3}{2 \times 5} = \frac{?}{?}$$

$$\frac{3 \times 2}{3 \times 3} = \frac{?}{?}$$

$$\frac{4 \times 5}{4 \times 6} = \frac{?}{?}$$

Equivalent Fractions are fractions that have the same value.

To change a fraction to an equivalent fraction having a higher denominator:

Divide the required denominator by the given denominator.

Multiply both numerator and denominator by the quotient obtained.

Change $\frac{3}{5}$ to tenths.

$$10 \div 5 = 2.$$

$$\frac{2 \times 3}{2 \times 5} = \frac{6}{10} \text{ Result.}$$

Change $\frac{3}{5}$ to fifteenths.

$$15 \div 5 = 3.$$

$$\frac{3 \times 3}{3 \times 5} = \frac{9}{15} \text{ Result.}$$

To show that $\frac{2}{6}$, $\frac{4}{12}$, and $\frac{8}{24}$ are equivalent :

$$\frac{2}{6} = \frac{?}{24}$$

$$24 \div 6 = ?$$

$$\text{Then } \frac{4 \times 2}{4 \times 6} = \frac{?}{?}$$

$$\frac{4}{12} = \frac{?}{24}$$

$$24 \div 12 = ?$$

$$\text{Then } \frac{2 \times 4}{2 \times 12} = \frac{?}{?}$$

ORAL PRACTICE

Tell the number that must be used for a multiplier to change :

- | | |
|---------------------------|---------------------------|
| 1. Thirds to Sixths. | 6. Thirds to Fifteenths. |
| 2. Fourths to Eighths. | 7. Fifths to Tenths. |
| 3. Thirds to Twelfths. | 8. Thirds to Eighteenths. |
| 4. Fourths to Twelfths. | 9. Eighths to Fortieths. |
| 5. Fourths to Sixteenths. | 10. Fifths to Fiftieths. |

Change :

- | | | |
|-----------------------------|-----------------------------|-----------------------------|
| 11. $\frac{1}{2}$ to 8ths. | 16. $\frac{2}{3}$ to 12ths. | 21. $\frac{4}{5}$ to 20ths. |
| 12. $\frac{1}{3}$ to 9ths. | 17. $\frac{1}{5}$ to 10ths. | 22. $\frac{1}{4}$ to 24ths. |
| 13. $\frac{1}{2}$ to 10ths. | 18. $\frac{3}{8}$ to 16ths. | 23. $\frac{3}{8}$ to 24ths. |
| 14. $\frac{2}{3}$ to 6ths. | 19. $\frac{5}{8}$ to 16ths. | 24. $\frac{1}{6}$ to 24ths. |
| 15. $\frac{3}{4}$ to 12ths. | 20. $\frac{3}{4}$ to 16ths. | 25. $\frac{5}{8}$ to 24ths. |

How many

- | | |
|---------------------------|----------------------------|
| 26. 4ths are there in 1? | 33. 4ths are there in 2? |
| 27. 8ths are there in 1? | 34. 6ths are there in 3? |
| 28. 12ths are there in 1? | 35. 3ds are there in 10? |
| 29. 15ths are there in 1? | 36. 10ths are there in 2? |
| 30. 20ths are there in 1? | 37. 4ths are there in 20? |
| 31. 35ths are there in 1? | 38. 5ths are there in 11? |
| 32. 3ds are there in 2? | 39. 10ths are there in 10? |

REVIEW OF ADDITION OF FRACTIONS

We add inches to inches, pounds to pounds, and dollars to dollars because in each case the numbers are of the same kind. In the same way we add halves to halves, fourths to fourths, ninths to ninths, etc., because the fractions are of the same kind. Give the sums of each of the following:

$$1 \text{ dollar} + 3 \text{ dollars} = \text{How many dollars? } \frac{1}{5} + \frac{3}{5} = \frac{?}{5}$$

$$2 \text{ inches} + 4 \text{ inches} = \text{How many inches? } \frac{2}{7} + \frac{4}{7} = \frac{?}{7}$$

$$4 \text{ tenths} + 5 \text{ tenths} = \text{How many tenths? } \frac{4}{10} + \frac{5}{10} = \frac{?}{10}$$

$$1 \text{ ninth} + 2 \text{ ninths} + 4 \text{ ninths} = \text{How many ninths? } \frac{1}{9} + \frac{2}{9} + \frac{4}{9} = \frac{?}{9}$$

ORAL PRACTICE

Give the sum of:

$$1. \frac{1}{4} + \frac{1}{4}.$$

$$6. \frac{2}{5} + \frac{1}{5}.$$

$$11. \frac{1}{7} + \frac{3}{7} + \frac{2}{7}.$$

$$2. \frac{1}{5} + \frac{3}{5}.$$

$$7. \frac{3}{7} + \frac{2}{7}.$$

$$12. \frac{3}{10} + \frac{1}{10} + \frac{5}{10}.$$

$$3. \frac{1}{8} + \frac{3}{8}.$$

$$8. \frac{1}{9} + \frac{4}{9}.$$

$$13. \frac{4}{15} + \frac{3}{15} + \frac{2}{15}.$$

$$4. \frac{1}{6} + \frac{5}{6}.$$

$$9. \frac{3}{10} + \frac{5}{10}.$$

$$14. \frac{5}{12} + \frac{1}{12} + \frac{1}{12}.$$

$$5. \frac{1}{8} + \frac{5}{8}.$$

$$10. \frac{3}{10} + \frac{8}{10}.$$

$$15. \frac{3}{20} + \frac{7}{20} + \frac{1}{20}.$$

$$16. \frac{1}{8} + \frac{3}{8} + \frac{2}{8} + \frac{1}{8}.$$

$$19. \frac{1}{15} + \frac{2}{15} + \frac{5}{15} + \frac{3}{15}.$$

$$17. \frac{1}{10} + \frac{3}{10} + \frac{2}{10} + \frac{7}{10}.$$

$$20. \frac{8}{19} + \frac{1}{19} + \frac{6}{19} + \frac{3}{19}.$$

$$18. \frac{1}{12} + \frac{2}{12} + \frac{5}{12} + \frac{4}{12}.$$

$$21. \frac{2}{25} + \frac{1}{25} + \frac{8}{25} + \frac{1}{25}.$$

When the denominators of two or more given fractions are unlike, we may change the form of the given fractions so that the denominators will be of the same kind.

Illustration: Find the sum of $\frac{1}{4} + \frac{1}{2}$.

These fractions are not similar, but the fraction $\frac{1}{2}$ may be changed to 4ths, when both will be of the same kind.

Since $\frac{1}{2} = \frac{2}{4}$ we may write $\frac{2}{4}$ in place of $\frac{1}{2}$.

$$\text{Then } \frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}. \text{ Result.}$$

WRITTEN PRACTICE

Find the sum of :

1. $\frac{1}{2} + \frac{1}{6}$.

6. $\frac{1}{4} + \frac{1}{8}$.

11. $\frac{3}{4} + \frac{5}{8}$.

2. $\frac{1}{2} + \frac{1}{8}$.

7. $\frac{1}{4} + \frac{3}{8}$.

12. $\frac{4}{5} + \frac{3}{10}$.

3. $\frac{1}{2} + \frac{1}{10}$.

8. $\frac{1}{4} + \frac{7}{8}$.

13. $\frac{5}{8} + \frac{1}{6}$.

4. $\frac{1}{2} + \frac{1}{12}$.

9. $\frac{1}{4} + \frac{5}{12}$.

14. $\frac{3}{7} + \frac{5}{14}$.

5. $\frac{1}{2} + \frac{1}{14}$.

10. $\frac{1}{4} + \frac{7}{12}$.

15. $\frac{5}{9} + \frac{7}{18}$.

16. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.

19. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$.

17. $\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$.

20. $\frac{1}{3} + \frac{5}{6} + \frac{7}{12}$.

18. $\frac{1}{4} + \frac{1}{8} + \frac{1}{16}$.

21. $\frac{1}{4} + \frac{3}{8} + \frac{5}{16}$.

A whole number and a fraction that together represent one quantity are often written without the sign of addition but read as if it were included. Thus $3\frac{1}{2}$ means 3 units plus $\frac{1}{2}$ of 1 unit and is read "three *and* one half." Such a number is called a **Mixed Number**.

In adding mixed numbers it is customary to add the fractions first, and then the whole numbers. Thus :

$$4\frac{1}{2} + 5\frac{1}{4} = 4 + 5 + \frac{1}{2} + \frac{1}{4}. \quad \text{Then : } \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \text{ and } 4 + 5 = 9.$$

$$\text{Therefore, } 4\frac{1}{2} + 5\frac{1}{4} = 9 + \frac{3}{4} = 9\frac{3}{4}. \quad \text{Result.}$$

WRITTEN PRACTICE

Find the sum of :

1. $2\frac{1}{4} + 3\frac{1}{4}$.

5. $7\frac{1}{4} + 5\frac{1}{2}$.

9. $5\frac{2}{3} + 3\frac{1}{6} + 1\frac{1}{3}$.

2. $3\frac{1}{8} + 5\frac{1}{8}$.

6. $5\frac{1}{8} + 7\frac{1}{4}$.

10. $4\frac{1}{4} + 2\frac{1}{2} + 3\frac{3}{4}$.

3. $5\frac{1}{8} + 6\frac{3}{8}$.

7. $7\frac{1}{6} + 4\frac{1}{3}$.

11. $4\frac{1}{6} + 3\frac{1}{3} + 8\frac{2}{3}$.

4. $5\frac{3}{8} + 4\frac{1}{8}$.

8. $8\frac{1}{5} + 4\frac{1}{10}$.

12. $6\frac{1}{8} + 6\frac{1}{4} + 6\frac{1}{2}$.

13. $1\frac{1}{2} + 1\frac{1}{3} + 1\frac{1}{4} + 1\frac{1}{6}$.

19. $5\frac{1}{8} + 4\frac{5}{8} + 7\frac{3}{4} + 4\frac{3}{4}$.

14. $2\frac{1}{2} + 2\frac{1}{4} + 3\frac{1}{3} + 3\frac{1}{2}$.

20. $5\frac{3}{4} + 7\frac{5}{8} + 5\frac{4}{9} + 3\frac{5}{6}$.

15. $2\frac{1}{5} + 1\frac{1}{2} + 3\frac{1}{4} + 2\frac{1}{3}$.

21. $6\frac{7}{8} + 8\frac{5}{6} + 8\frac{1}{4} + 3\frac{5}{12}$.

16. $2\frac{2}{3} + 3\frac{3}{4} + 2\frac{4}{5} + 2\frac{1}{8}$.

22. $6\frac{8}{9} + 4\frac{5}{6} + 9\frac{7}{12} + 5\frac{5}{18}$.

17. $3\frac{1}{5} + 3\frac{3}{4} + 3\frac{5}{8} + 4\frac{1}{4}$.

23. $7\frac{7}{8} + 8\frac{8}{9} + 9\frac{5}{6} + 10\frac{7}{12}$.

18. $3\frac{3}{5} + 4\frac{5}{6} + 6\frac{7}{8} + 3\frac{3}{4}$.

24. $8\frac{3}{4} + 9\frac{5}{9} + 7\frac{7}{8} + 12\frac{1}{24}$.

REVIEW OF SUBTRACTION OF FRACTIONS

Fractions having denominators of the same kind may be subtracted.

Illustration :

1. Subtract $\frac{3}{5}$ from $\frac{7}{5}$.

The denominators are equal.

Therefore,
$$\frac{7}{5} - \frac{3}{5} = \frac{7-3}{5} = \frac{4}{5}. \quad \text{Result.}$$

2. Subtract $\frac{1}{2}$ from $\frac{7}{8}$.

The fractions are unlike, and must be made of the same kind.

$\frac{1}{2}$ = How many eighths? $8 \div 2 = 4$. Then $\frac{4 \times 1}{4 \times 2} = \frac{4}{8}$ or, $\frac{1}{2} = \frac{4}{8}$.

Then
$$\frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{4}{8} = \frac{7-4}{8} = \frac{3}{8}. \quad \text{Result.}$$

WRITTEN PRACTICE

From :

- | | | |
|--|---|--|
| 1. $\frac{1}{2}$ subtract $\frac{1}{8}$. | 6. $\frac{3}{4}$ subtract $\frac{1}{2}$. | 11. $\frac{4}{5}$ subtract $\frac{1}{10}$. |
| 2. $\frac{1}{2}$ subtract $\frac{1}{6}$. | 7. $\frac{5}{8}$ subtract $\frac{1}{4}$. | 12. $\frac{13}{16}$ subtract $\frac{3}{8}$. |
| 3. $\frac{1}{2}$ subtract $\frac{1}{16}$. | 8. $\frac{7}{8}$ subtract $\frac{1}{2}$. | 13. $\frac{14}{15}$ subtract $\frac{3}{5}$. |
| 4. $\frac{1}{4}$ subtract $\frac{1}{12}$. | 9. $\frac{5}{8}$ subtract $\frac{1}{16}$. | 14. $\frac{19}{20}$ subtract $\frac{3}{4}$. |
| 5. $\frac{1}{2}$ subtract $\frac{3}{16}$. | 10. $\frac{7}{8}$ subtract $\frac{1}{24}$. | 15. $\frac{23}{24}$ subtract $\frac{3}{8}$. |

In subtraction of mixed numbers it is customary to subtract the fraction from the fraction, then the whole number from the whole number.

Illustration :

1. From $8\frac{3}{4}$ subtract $1\frac{1}{2}$.

Subtracting the fractions: $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$.

Subtracting the whole numbers: $8 - 1 = 7$.

Therefore,
$$8\frac{3}{4} - 1\frac{1}{2} = 7\frac{1}{4}. \quad \text{Result.}$$

WRITTEN PRACTICE

Find the difference between :

- | | | |
|--|---|---|
| 1. $3\frac{1}{2}$ and $1\frac{1}{4}$. | 7. $5\frac{5}{6}$ and $4\frac{1}{3}$. | 13. $10\frac{7}{10}$ and $7\frac{1}{2}$. |
| 2. $4\frac{1}{2}$ and $2\frac{1}{8}$. | 8. $7\frac{3}{4}$ and $4\frac{1}{2}$. | 14. $12\frac{1}{3}$ and $10\frac{1}{5}$. |
| 3. $5\frac{1}{4}$ and $1\frac{1}{8}$. | 9. $8\frac{5}{8}$ and $2\frac{1}{4}$. | 15. $14\frac{1}{6}$ and $11\frac{5}{8}$. |
| 4. $7\frac{3}{4}$ and $5\frac{1}{2}$. | 10. $8\frac{7}{8}$ and $5\frac{1}{2}$. | 16. $20\frac{1}{2}$ and $13\frac{1}{6}$. |
| 5. $6\frac{3}{4}$ and $5\frac{1}{4}$. | 11. $9\frac{1}{3}$ and $4\frac{1}{6}$. | 17. $21\frac{3}{5}$ and $17\frac{3}{5}$. |
| 6. $8\frac{3}{4}$ and $3\frac{1}{8}$. | 12. $8\frac{5}{8}$ and $3\frac{1}{4}$. | 18. $30\frac{1}{4}$ and $11\frac{3}{8}$. |

EVERYDAY USES FOR ADDITION AND SUBTRACTION OF FRACTIONS

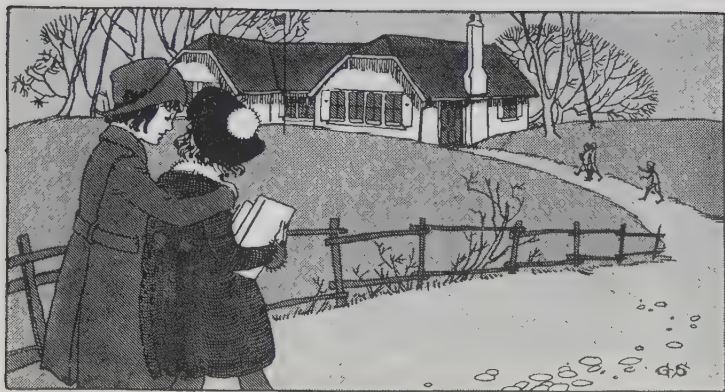
Find the sum or difference as indicated :

- | | |
|---|--|
| 1. $\frac{1}{2}$ yd. + $\frac{1}{4}$ yd. | 6. $\frac{2}{3}$ mile + $\frac{1}{6}$ mile. |
| 2. $\frac{3}{8}$ yd. - $\frac{1}{4}$ yd. | 7. $\frac{1}{8}$ mile + $\frac{1}{16}$ mile. |
| 3. $\frac{3}{8}$ yd. + $\frac{1}{2}$ yd. | 8. $\frac{2}{5}$ rod - $\frac{1}{10}$ rod. |
| 4. $\frac{5}{8}$ yd. - $\frac{1}{4}$ yd. | 9. $\frac{3}{8}$ ton - $\frac{1}{4}$ ton. |
| 5. $\frac{2}{3}$ yd. + $\frac{1}{6}$ yd. | 10. $\frac{5}{8}$ pound + $\frac{1}{2}$ pound. |
| 11. $1\frac{1}{4}$ ft. + $2\frac{1}{8}$ ft. + $3\frac{1}{2}$ ft. - 5 ft. | |
| 12. $4\frac{1}{2}$ gal. + 5 gal. + $11\frac{1}{4}$ gal. - $2\frac{1}{8}$ gal. | |
| 13. $10\frac{1}{2}$ ft. + $12\frac{1}{8}$ ft. + $8\frac{1}{4}$ ft. - $5\frac{1}{2}$ ft. | |

14. The cover of a box is made of two strips of wood. If one strip is $\frac{3}{8}$ ft. wide, and the other $\frac{3}{4}$ ft. wide, how wide is the cover of the box?

15. Tom worked $2\frac{1}{2}$ hours on Monday and $3\frac{1}{4}$ hours on Tuesday. How many hours did he work in the two days? How many more hours did he work on Tuesday than he did on Monday? *3/4 hr*

16. Marion used $3\frac{1}{4}$ yards of braid on one dress, and $5\frac{1}{8}$ yards on another dress. How many yards of braid did she use on both dresses? How many more yards did she use on the second dress than she did on the first?



17. Mary lives $\frac{3}{4}$ of a mile south of this schoolhouse and Dorothy lives $\frac{5}{8}$ of a mile north of it. How far is it from Mary's home to Dorothy's home?

18. Donald lives $1\frac{1}{4}$ miles east of this schoolhouse and David lives $\frac{3}{4}$ of a mile west of it. How much nearer the school does David live than Donald?

19. At the end of a term Marion weighed $67\frac{1}{2}$ pounds, but when the term began she weighed only $62\frac{3}{4}$ pounds. How much did she gain in weight during the term?

20. Tom spends $\frac{1}{4}$ of the day in school, $\frac{1}{12}$ of it at his meals, $\frac{1}{8}$ of it at play, and $\frac{1}{12}$ at evening study. How much of the day is left for sleep?

21. Jack's marks at the end of a term were $90\frac{1}{2}$ in Reading, 84 in Arithmetic, 90 in Geography, and $91\frac{1}{2}$ in Spelling. What average mark should be given him for the term?

22. In the sewing class at school Frances made an apron for which she used $3\frac{2}{3}$ yards of gingham, and Margaret made a dress of the same material using $5\frac{1}{4}$ yards. How many yards of gingham did they use?

23. Caroline preserved $4\frac{1}{3}$ dozen cans of strawberries, and her sister preserved $2\frac{1}{2}$ dozen cans. How many more cans did Caroline preserve than her sister? How many cans did both preserve? $6\frac{2}{3} / 6$

24. On Monday Jack's hens laid $2\frac{1}{6}$ dozen eggs, and on Tuesday they laid $2\frac{1}{2}$ dozen. On Wednesday they laid $1\frac{3}{4}$ dozen, and on Thursday 2 dozen. How many more eggs did they lay on the first two days than on the last two?

25. A train running between two cities is supposed to make the trip in exactly $3\frac{4}{5}$ hours, but on a certain day it was delayed $1\frac{1}{10}$ hours. How long did it take the train to make the trip on that day?

26. Jack and Edmund live on the same road outside of Philadelphia. From Jack's home to the City Hall is $12\frac{1}{2}$ miles, and from Edmund's home to the same point is $5\frac{1}{4}$ miles. What is the distance between their homes?

27. A grocer put $12\frac{1}{2}$ gallons of molasses into a cask containing $12\frac{3}{4}$ gallons. If he then sold $8\frac{1}{4}$ gallons from this cask, how many gallons of molasses were left in the cask?

28. Elizabeth and Eleanor weigh, respectively, $56\frac{1}{2}$ pounds and $47\frac{3}{4}$ pounds. Their father weighs $171\frac{1}{4}$ pounds. The sum of the weights of the two children is how much less than the father's weight? 69

29. A boy earned in March $\$4\frac{1}{2}$, in April $\$5\frac{3}{4}$, in May $\$4\frac{1}{4}$, and in June $\$6\frac{1}{2}$. He saved \$2 in March, \$4 in April, \$3 in May, and \$5 in June. If he spent the difference between his earnings and his savings, how much did he spend in that time?

FRACTIONS

Each oblong at the right is divided into equal parts.



In the first oblong two of five equal parts are shaded.



“Two of Five” equal parts are called
“Two Fifths.”



In the second oblong one of six equal parts is called
“One Sixth.”

In the third oblong three of seven equal parts are called
“Three Sevenths.”

In each of these fractions :

The number over the line tells “How many parts.”

The number under the line tells “What kind of parts”
and names them.

In the same way :

$\frac{2}{3}$ means a fraction containing two of the three equal parts of a unit.

$\frac{7}{8}$ means a fraction containing seven of the eight equal parts of a unit.

$\frac{5}{16}$ means a fraction containing five of the sixteen equal parts of a unit.

A Fraction is an expression for one or more of the equal parts into which a unit, a number, or a thing, is divided.

The Numerator indicates how many of the equal parts of the unit are taken.

The Denominator indicates the number of equal parts into which the unit is divided.

ORAL PRACTICE

Read :

- | | | | | |
|--------------------|--------------------|----------------------|-----------------------|-----------------------|
| 1. $\frac{2}{3}$. | 5. $\frac{7}{8}$. | 9. $\frac{8}{9}$. | 13. $\frac{15}{16}$. | 17. $\frac{13}{30}$. |
| 2. $\frac{2}{5}$. | 6. $\frac{1}{9}$. | 10. $\frac{9}{10}$. | 14. $\frac{16}{16}$. | 18. $\frac{15}{32}$. |
| 3. $\frac{4}{7}$. | 7. $\frac{5}{9}$. | 11. $\frac{9}{20}$. | 15. $\frac{17}{20}$. | 19. $\frac{21}{25}$. |
| 4. $\frac{5}{9}$. | 8. $\frac{7}{9}$. | 12. $\frac{9}{25}$. | 16. $\frac{11}{25}$. | 20. $\frac{23}{24}$. |

21. What is the meaning of $\frac{11}{12}$? $\frac{13}{17}$? $\frac{21}{25}$?22. What is the meaning of $\frac{13}{100}$? $\frac{15}{113}$? $\frac{25}{300}$? $\frac{31}{1000}$?

WRITTEN PRACTICE

Draw four straight lines on the blackboard. Divide the first line into fourths, the second line into fifths, the third line into sixths, and the fourth line into twelfths. Choose the line upon which you can show it, and point to :

- | | |
|--|--|
| 1. $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{4}$, $\frac{4}{4}$. | 4. $\frac{6}{12}$, $\frac{5}{12}$, $\frac{1}{12}$, $\frac{9}{12}$. |
| 2. $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{5}$, $\frac{4}{5}$. | 5. $\frac{4}{6}$, $\frac{11}{12}$, $\frac{4}{6}$, $\frac{7}{12}$. |
| 3. $\frac{1}{6}$, $\frac{3}{6}$, $\frac{5}{6}$, $\frac{2}{6}$. | 6. $\frac{3}{12}$, $\frac{6}{6}$, $\frac{8}{12}$, $\frac{5}{6}$. |

The Terms of a Fraction are its numerator and its denominator.

In the fraction $\frac{7}{10}$, the terms are 7 and 10.

A fraction whose numerator is less than its denominator is called a **Proper Fraction**.

A fraction whose numerator is greater than its denominator is called an **Improper Fraction**.

A fraction may be regarded as a division not yet performed.

Thus: $\frac{15}{3}$ means $15 \div 3$; $\frac{19}{6}$ means $19 \div 6$.

TO CHANGE A FRACTION TO LOWER TERMS

A fraction may be changed in form without changing its value.

In the illustration the two oblongs are of equal size.

The upper oblong is divided into 4 equal parts, 2 being shaded. Then the fraction of the oblong that is shaded is $\frac{2}{4}$.



The lower oblong is divided into 2 equal parts, 1 being shaded. And the fraction of the oblong that is shaded is $\frac{1}{2}$.



The shaded length in each oblong is the same, that is: $\frac{2}{4} = \frac{1}{2}$. The second fraction, $\frac{1}{2}$, is equivalent to the first fraction, $\frac{2}{4}$, but the second fraction is expressed in lower terms.

A fraction may be reduced to lower terms, *without changing its value*, by dividing both the numerator and the denominator by the same number.

Illustrations :

$$\frac{14}{16} = \frac{14 \div 2}{16 \div 2} = \frac{7}{8}$$

$$\frac{24}{30} = \frac{24 \div 2}{30 \div 2} = \frac{12}{15}$$

$$\frac{12}{21} = \frac{12 \div 3}{21 \div 3} = \frac{4}{7}$$

A fraction is reduced to its lowest terms when no further division of both the numerator and the denominator by the same number can be made.

Thus : $\frac{24}{36} = \frac{12}{18}$ (Dividing both numerator and denominator by 2.)

But $\frac{12}{18} = \frac{6}{9}$ (Dividing both numerator and denominator by 2.)

And $\frac{6}{9} = \frac{2}{3}$ (Dividing both numerator and denominator by 3.)

No further division is possible, for there is no number that will exactly divide both 2 and 3, excepting the number 1, which will make no change in them.

Therefore, $\frac{24}{36} = \frac{2}{3}$ when reduced to its lowest terms.

ORAL PRACTICE

Reduce to lowest terms :

- | | | | | |
|--------------------|---------------------|----------------------|----------------------|-----------------------|
| 1. $\frac{2}{4}$. | 5. $\frac{4}{6}$. | 9. $\frac{2}{10}$. | 13. $\frac{6}{10}$. | 17. $\frac{10}{12}$. |
| 2. $\frac{2}{8}$. | 6. $\frac{6}{8}$. | 10. $\frac{5}{10}$. | 14. $\frac{8}{12}$. | 18. $\frac{8}{16}$. |
| 3. $\frac{2}{8}$. | 7. $\frac{3}{12}$. | 11. $\frac{4}{10}$. | 15. $\frac{9}{12}$. | 19. $\frac{12}{16}$. |
| 4. $\frac{4}{8}$. | 8. $\frac{4}{12}$. | 12. $\frac{6}{12}$. | 16. $\frac{8}{10}$. | 20. $\frac{10}{20}$. |

WRITTEN PRACTICE

Reduce to lowest terms :

- | | | | | |
|----------------------|----------------------|-----------------------|-----------------------|-----------------------|
| 1. $\frac{2}{14}$. | 7. $\frac{6}{16}$. | 13. $\frac{12}{18}$. | 19. $\frac{8}{24}$. | 25. $\frac{16}{18}$. |
| 2. $\frac{3}{15}$. | 8. $\frac{9}{18}$. | 14. $\frac{9}{15}$. | 20. $\frac{15}{18}$. | 26. $\frac{20}{24}$. |
| 3. $\frac{8}{12}$. | 9. $\frac{5}{15}$. | 15. $\frac{12}{16}$. | 21. $\frac{14}{16}$. | 27. $\frac{16}{32}$. |
| 4. $\frac{8}{14}$. | 10. $\frac{8}{16}$. | 16. $\frac{11}{22}$. | 22. $\frac{7}{28}$. | 28. $\frac{18}{20}$. |
| 5. $\frac{4}{16}$. | 11. $\frac{6}{18}$. | 17. $\frac{12}{20}$. | 23. $\frac{18}{24}$. | 29. $\frac{22}{24}$. |
| 6. $\frac{10}{14}$. | 12. $\frac{4}{20}$. | 18. $\frac{8}{32}$. | 24. $\frac{12}{32}$. | 30. $\frac{24}{32}$. |

TO CHANGE A FRACTION TO HIGHER TERMS

In the illustration the oblongs are of equal length. Compare the part representing $\frac{1}{2}$ in the upper oblong with the part representing $\frac{6}{12}$ in the lower oblong. How do they compare?



Now the fraction $\frac{1}{2}$ becomes $\frac{6}{12}$ when both numerator and denominator are multiplied by 6.

In the same manner :

$\frac{3}{4}$ becomes $\frac{6}{8}$ when both of its terms are multiplied by 2.

$\frac{5}{7}$ becomes $\frac{15}{21}$ when both of its terms are multiplied by 3.

That is :

A fraction may be changed to higher terms, *without changing its value*, by multiplying both numerator and denominator by the same number.

Illustration :

Change $\frac{5}{8}$ to 24ths.

The required denominator is 24. The given denominator is 8. $24 \div 8 = 3$.

Then: $\frac{5}{8} = \frac{3 \times 5}{3 \times 8} = \frac{15}{24}$. The required fraction is fifteen twenty-fourths.

Therefore, to change a fraction to an equivalent fraction of higher terms :

Divide the required denominator by the given denominator.

Multiply both terms of the fraction by the quotient obtained.

ORAL PRACTICE

1. How many fourths are there in $\frac{1}{2}$? In 1? In $\frac{3}{2}$?
 2. How many quarters are there in one half dollar?
 3. How many quarter inches are there in one half inch?
 4. How many eighth inches are there in one half inch?
- In one quarter inch? In three quarter inches?
5. How many sixteenths of an inch are there in one half inch? In one quarter inch? In three quarter inches?

WRITTEN PRACTICE

Change :

- | | | |
|----------------------------|-----------------------------|------------------------------|
| 1. $\frac{3}{5}$ to 10ths. | 6. $\frac{3}{7}$ to 21sts. | 11. $\frac{3}{11}$ to 22ds. |
| 2. $\frac{4}{5}$ to 10ths. | 7. $\frac{5}{9}$ to 27ths. | 12. $\frac{5}{16}$ to 32ds. |
| 3. $\frac{3}{4}$ to 12ths. | 8. $\frac{6}{7}$ to 28ths. | 13. $\frac{7}{10}$ to 40ths. |
| 4. $\frac{5}{8}$ to 16ths. | 9. $\frac{7}{8}$ to 24ths. | 14. $\frac{5}{12}$ to 24ths. |
| 5. $\frac{3}{7}$ to 14ths. | 10. $\frac{5}{9}$ to 27ths. | 15. $\frac{3}{5}$ to 50ths. |

TO CHANGE A WHOLE NUMBER TO A FRACTIONAL FORM

How many halves are there in one unit? How many thirds in 1? How many fourths in 1? How many 5ths in 1? How many 10ths in 1? How many 25ths in 1?

ORAL PRACTICE

Give the number of :

- | | | |
|----------------|----------------|------------------|
| 1. 12ths in 1. | 5. 10ths in 5. | 9. 18ths in 5. |
| 2. 15ths in 1. | 6. 15ths in 6. | 10. 24ths in 10. |
| 3. 12ths in 2. | 7. 20ths in 5. | 11. 30ths in 10. |
| 4. 15ths in 2. | 8. 25ths in 4. | 12. 50ths in 10. |

TO CHANGE A MIXED NUMBER TO A FRACTION**ORAL PRACTICE**

- How many halves are there in 1? In $\frac{1}{2}$? In 1 and $\frac{1}{2}$?
- On your ruler count the number of half inches that answer the questions in the preceding example.
- How many fourths are there in 1? In $\frac{1}{2}$? In 1 and $\frac{1}{2}$?
- On your ruler count the number of quarter inches that answer the questions in the preceding example.
- How many eighths are there in 1? In $\frac{1}{2}$? In 1 and $\frac{1}{2}$?
- On your ruler count the number of eighth inches that answer the questions in the preceding example.
- Show on your ruler that $1\frac{1}{2}$ inches is the same as $\frac{3}{2}$ inches. Show that $1\frac{1}{2}$ inches is the same as $\frac{12}{8}$ inches.

A Mixed Number is a number composed of a whole number and a fraction.

$2\frac{1}{2}$, $3\frac{1}{4}$, and $10\frac{3}{4}$ are mixed numbers.

A mixed number is the sum of a whole number and a fraction, but in practice we do not write the plus sign between the two parts.

A mixed number may be changed to a fraction by :

(1) *Changing the whole number to a fractional form in which the denominator is the same as that of the fraction in the mixed number; and*

(2) *Adding the resulting fraction and the given fraction.*

Thus: $1\frac{1}{2}$ units = 1 unit + $\frac{1}{2}$ of 1 unit.

But 1 unit = $\frac{2}{2}$ units. Then, $\frac{2}{2}$ units + $\frac{1}{2}$ unit = $\frac{3}{2}$ units.

A briefer expression for the process may be given thus :

Multiply the whole number by the denominator of the fractional part.

Add the product to the numerator of the fractional part.

Write the result over the denominator.

Illustration :

Change $3\frac{2}{5}$ to a fraction.

$$3\frac{2}{5} = \frac{3 \times 5}{5} + \frac{2}{5} = \frac{15}{5} + \frac{2}{5} = \frac{15+2}{5} = \frac{17}{5}.$$

WRITTEN PRACTICE

Change to fractions :

- | | | | |
|----------------------|-----------------------|------------------------|------------------------|
| 1. $2\frac{1}{2}$. | 11. $5\frac{3}{5}$. | 21. $4\frac{4}{5}$. | 31. $2\frac{9}{16}$. |
| 2. $3\frac{1}{4}$. | 12. $7\frac{3}{10}$. | 22. $6\frac{9}{20}$. | 32. $4\frac{7}{24}$. |
| 3. $5\frac{2}{3}$. | 13. $6\frac{5}{8}$. | 23. $8\frac{7}{12}$. | 33. $3\frac{11}{18}$. |
| 4. $4\frac{3}{4}$. | 14. $8\frac{4}{5}$. | 24. $5\frac{8}{15}$. | 34. $5\frac{2}{24}$. |
| 5. $3\frac{5}{8}$. | 15. $7\frac{5}{12}$. | 25. $7\frac{11}{12}$. | 35. $6\frac{1}{16}$. |
| 6. $4\frac{3}{8}$. | 16. $5\frac{3}{11}$. | 26. $6\frac{9}{14}$. | 36. $7\frac{5}{32}$. |
| 7. $5\frac{2}{5}$. | 17. $6\frac{2}{3}$. | 27. $6\frac{3}{11}$. | 37. $8\frac{7}{25}$. |
| 8. $4\frac{7}{12}$. | 18. $7\frac{7}{8}$. | 28. $4\frac{7}{15}$. | 38. $5\frac{1}{20}$. |
| 9. $2\frac{1}{10}$. | 19. $8\frac{3}{16}$. | 29. $3\frac{5}{16}$. | 39. $6\frac{11}{32}$. |
| 10. $3\frac{1}{8}$. | 20. $4\frac{7}{10}$. | 30. $5\frac{1}{18}$. | 40. $8\frac{3}{32}$. |

**TO CHANGE AN IMPROPER FRACTION TO A WHOLE NUMBER
OR TO A MIXED NUMBER**

ORAL PRACTICE

1. How many quarters are there in one dollar? In one and one quarter dollars?

2. How many quarters are there in $\$ \frac{4}{4}$? In $\$ \frac{8}{4}$? In $\$ \frac{12}{4}$? In $\$ \frac{5}{4}$? In $\$ \frac{7}{4}$? In $\$ \frac{9}{4}$?

3. How many half inches are there in 1 inch? In 2 inches? In $1\frac{1}{2}$ inches? In $2\frac{1}{2}$ inches? In 3 inches? Use your ruler to prove your answers.

4. With your ruler point out the number of eighth inches in $1\frac{1}{8}$ inches. In 2 inches. In $2\frac{1}{4}$ inches. In $2\frac{1}{2}$ inches. In $3\frac{1}{4}$ inches. In $4\frac{3}{8}$ inches.

5. Change each of the following to inches :

$\frac{5}{4}$ inches, $\frac{7}{4}$ inches, $\frac{3}{2}$ inches, $\frac{5}{2}$ inches, $\frac{7}{2}$ inches. Prove each result by using your ruler.

If the numerator of the fraction $\frac{7}{3}$ is divided by the denominator, the Quotient = 2, and the Remainder = 1. Therefore, $\frac{7}{3} = 2\frac{1}{3}$.

The following examples show the usual process.

$$\frac{15}{4} = 15 \div 4 = 3 + \frac{3}{4} = 3\frac{3}{4}. \quad \text{Result.}$$

$$\frac{39}{7} = 39 \div 7 = 5 + \frac{4}{7} = 5\frac{4}{7}. \quad \text{Result.}$$

To change a fraction to a whole number or to a mixed number :

Divide the numerator of the fraction by the denominator.

If there is a remainder, write it over the denominator.

Add this fraction to the quotient.

WRITTEN PRACTICE

Change to whole numbers, or to mixed numbers :

1. $\frac{5}{2}$.	9. $\frac{13}{4}$.	17. $\frac{35}{15}$.	25. $\frac{41}{10}$.	33. $\frac{56}{18}$.
2. $\frac{8}{4}$.	10. $\frac{25}{12}$.	18. $\frac{27}{3}$.	26. $\frac{47}{5}$.	34. $\frac{65}{15}$.
3. $\frac{9}{3}$.	11. $\frac{18}{3}$.	19. $\frac{36}{9}$.	27. $\frac{52}{8}$.	35. $\frac{69}{10}$.
4. $\frac{13}{6}$.	12. $\frac{30}{20}$.	20. $\frac{38}{12}$.	28. $\frac{56}{12}$.	36. $\frac{38}{5}$.
5. $\frac{9}{2}$.	13. $\frac{18}{6}$.	21. $\frac{40}{18}$.	29. $\frac{48}{9}$.	37. $\frac{45}{8}$.
6. $\frac{19}{6}$.	14. $\frac{24}{4}$.	22. $\frac{46}{15}$.	30. $\frac{43}{14}$.	38. $\frac{72}{12}$.
7. $\frac{18}{8}$.	15. $\frac{25}{10}$.	23. $\frac{35}{6}$.	31. $\frac{60}{20}$.	39. $\frac{83}{9}$.
8. $\frac{12}{3}$.	16. $\frac{27}{5}$.	24. $\frac{37}{16}$.	32. $\frac{54}{18}$.	40. $\frac{88}{16}$.

DIVISORS, FACTORS, AND MULTIPLES

I. DIVISORS

An Exact Divisor of a number is a divisor that divides the number without a remainder.

Thus: 2, 3, and 5 are exact divisors of 30.

4, 7, and 12 are not exact divisors of 30.

When we say that one number is divisible by another, we mean exactly divisible, that is, *divisible without a remainder*.

An Even Number is a number that is divisible by 2.

An Odd Number is a number that is not divisible by 2.

2, 4, 6, 8, 10, 12, 14, 16, 24, 68, 104, etc., are even numbers.

1, 3, 5, 7, 11, 19, 31, 65, 93, etc., are odd numbers.

Divisibility of Numbers.

The digits of a number are the figures used to represent the number.

0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are the **Digits**.

In the number 347 the digits are 3, 4, and 7.

Divisibility by 2: A number is divisible by 2 if the units' figure is 0, 2, 4, 6, or 8.

Divisibility by 3 : A number is divisible by 3 if the sum of its digits is divisible by 3.

Divisibility by 5 : A number is divisible by 5 if the units' figure is 0 or 5.

Divisibility by 9 : A number is divisible by 9 if the sum of its digits is divisible by 9.

Divisibility by 10 : A number is divisible by 10 if the units' figure is 0.

279 is divisible by 3, because $2+7+9$, or 18, is divisible by 3.

6984 is divisible by 9, because $6+9+8+4$, or 27, is divisible by 9.

ORAL PRACTICE

From the following select those numbers divisible (1) by 2; (2) by 3; and (3) by 5.

1. 124 132 164 165 171 195 198 203 215 245
2. 154 162 177 186 193 211 225 227 254 270
3. 205 237 256 275 315 380 456 479 510 575
4. 358 448 479 567 650 718 805 945 990 996

Select the numbers that are divisible by 3:

5. 1140 1345 1416 1570 1605 1915 2118 2264 2510
6. 2520 2718 3677 4006 4517 4992 6501 7111 8961

Select the numbers that are divisible by 9:

7. 11359 16003 20016 25465 27883 31117 35546
8. 28881 30006 35005 39996 40059 57775 81253

Give two or more divisors for each of the following:

- | | | | |
|----------|-----------|------------|------------|
| 9. 1128 | 12. 25465 | 15. 311650 | 18. 475680 |
| 10. 2790 | 13. 47772 | 16. 458001 | 19. 688751 |
| 11. 5100 | 14. 58670 | 17. 768090 | 20. 995094 |

II. FACTORS

An Integer is a number that expresses whole units.

Thus: 12, 19, 25, etc., are integers or integral numbers.

The Factors of a number are the integral numbers which, multiplied, will produce the number.

Thus: 3 and 5 are the factors of 15.

2, 3, and 7 are the factors of 42.

A Prime Number is a number that has no integral factors except itself and 1.

The prime numbers from 1 to 30 are 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. You should memorize these prime numbers.

A Prime Factor is a factor that is a prime number.

Thus: The prime factors of 30 are 2, 3, and 5.

The prime factors of 75 are 3, 5, and 5.

Factoring is the process of separating a number into its integral factors.

In factoring any exact divisor may be taken for one factor and the quotient for the other factor.

In practical work we understand that the term "to factor" means to "find the prime factors."

The prime factors of a number are obtained by division.

Find the prime factors of 840.

Process:

The smallest possible divisor of 840 is 2.

$$\begin{array}{r} 2 \overline{)840} \end{array}$$

The division is repeated, using 2 as a divisor just as long as possible.

$$\begin{array}{r} 2 \overline{)420} \\ 2 \overline{)210} \end{array}$$

Each new result may be tested for divisibility.

$$\begin{array}{r} 3 \overline{)105} \end{array}$$

Finally, the last quotient, 7, is a prime number.

$$\begin{array}{r} 5 \overline{)35} \\ 7 \end{array}$$

Hence, 2, 2, 2, 3, 5, and 7 are the Prime Factors of 840.

WRITTEN PRACTICE

Find the prime factors of :

- | | | | | |
|-------|---------|---------|----------|----------|
| 1. 45 | 6. 120 | 11. 225 | 16. 1032 | 21. 2996 |
| 2. 72 | 7. 125 | 12. 432 | 17. 1176 | 22. 3549 |
| 3. 82 | 8. 156 | 13. 675 | 18. 1331 | 23. 5743 |
| 4. 90 | 9. 180 | 14. 848 | 19. 1728 | 24. 7624 |
| 5. 96 | 10. 196 | 15. 972 | 20. 2205 | 25. 9724 |

III. GREATEST COMMON DIVISOR

A Common Divisor of two or more numbers is a number that will exactly divide each of the numbers.

Thus: 4 is a divisor of 16 and 24.

The Greatest Common Divisor of two or more numbers is the largest number that will exactly divide each of the numbers.

Thus: 8 is the greatest *common* divisor of 16 and 24.

The abbreviation, "G. C. D.," is used for greatest common divisor.

ORAL PRACTICE

Give the greatest common divisor of :

- | | | |
|--------------|----------------|---------------------|
| 1. 6 and 8. | 6. 12 and 16. | 11. 12, 16, and 20. |
| 2. 8 and 12. | 7. 15 and 20. | 12. 16, 24, and 32. |
| 3. 8 and 14. | 8. 15 and 21. | 13. 15, 25, and 35. |
| 4. 9 and 12. | 9. 18 and 24. | 14. 18, 27, and 45. |
| 5. 9 and 15. | 10. 18 and 27. | 15. 20, 28, and 48. |

When the given numbers are too large for inspection, the greatest common divisor is found by factoring.

Find the greatest common divisor of 36, 72, and 90.

$$36 = 2 \times 2 \times 3 \times 3.$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3.$$

$$90 = 2 \times 3 \times 3 \times 5.$$

In each number we find the factors 2, 3, and 3. That is, 2, 3, and 3 are *common* to the numbers.

Therefore, the G. C. D. of 36, 72, and $90 = 2 \times 3 \times 3 = 18$. *Result.*

The G. C. D. of two or more numbers is the product of all the common prime factors of the numbers.

WRITTEN PRACTICE

Find the G. C. D. of :

- | | | |
|----------------|------------------|------------------------|
| 1. 18 and 42. | 6. 88 and 143. | 11. 42, 54, and 96. |
| 2. 36 and 63. | 7. 96 and 144. | 12. 48, 84, and 108. |
| 3. 48 and 72. | 8. 105 and 180. | 13. 64, 96, and 160. |
| 4. 64 and 96. | 9. 120 and 180. | 14. 98, 112, and 140. |
| 5. 80 and 128. | 10. 135 and 210. | 15. 100, 160, and 200. |

IV. LEAST COMMON MULTIPLE

The divisors of 24 are 2, 3, 4, 6, 8, and 12.

We say, therefore, that 24 is a **Multiple** of 2, 3, 4, 6, 8, and 12.

A **Common Multiple** of two or more given numbers is a number that is exactly divisible by each of the given numbers.

24 is a common multiple of 2, 3, 4, 6, 8, and 12.

The Least Common Multiple of two or more numbers is the least number that is exactly divisible by each of the given numbers.

Thus : 72 is a multiple of 2, 3, 4, 8, and 12.

But : 24 is the *least number* that will contain each, or, the *least common multiple* of 2, 3, 4, 8, and 12.

The abbreviation, "L.C.M.," is used for least common multiple.

ORAL PRACTICE

Give the least common multiple of :

- | | | |
|--------------|----------------|---------------------|
| 1. 4 and 6. | 6. 6 and 12. | 11. 4, 6, and 8. |
| 2. 6 and 8. | 7. 8 and 10. | 12. 6, 8, and 12. |
| 3. 6 and 9. | 8. 8 and 12. | 13. 6, 10, and 12. |
| 4. 6 and 10. | 9. 10 and 12. | 14. 10, 12, and 15. |
| 5. 6 and 15. | 10. 12 and 15. | 15. 12, 15, and 20. |

When the numbers are too large for inspection, the least common multiple is found by factoring.

Find the least common multiple of 20, 36, and 90.

$20 = 2 \times 2 \times 5$. Taking each factor the greatest number of times it
 $36 = 2 \times 2 \times 3 \times 3$. is found in any one of the given numbers, we find
 $90 = 2 \times 3 \times 3 \times 5$. two 2's, two 3's, and one 5.

Hence, the L. C. M. of 20, 36, and $90 = 2 \times 2 \times 3 \times 3 \times 5 = 180$.

The L. C. M. of two or more numbers is the product of all the different prime factors each used the greatest number of times it occurs in any one of the numbers.

WRITTEN PRACTICE

Find the L. C. M. of ·

- | | |
|---------------------|-------------------------|
| 1. 16 and 24. | 11. 10, 12, 15, and 20. |
| 2. 18 and 20. | 12. 12, 16, 20, and 24. |
| 3. 20 and 30. | 13. 18, 20, 36, and 54. |
| 4. 24 and 32. | 14. 15, 25, 40, and 75. |
| 5. 28 and 42. | 15. 25, 30, 35, and 40. |
| 6. 6, 10, and 15. | 16. 105, 120, and 135. |
| 7. 8, 12, and 16. | 17. 125, 200, and 275. |
| 8. 9, 12, and 18. | 18. 150, 160, and 240. |
| 9. 12, 16, and 24. | 19. 160, 200, and 400. |
| 10. 16, 18, and 24. | 20. 240, 480, and 600. |

21. 360, 480, and 720.
22. 20, 24, 25, 36, and 40.
23. 30, 36, 48, 60, and 72.
24. 48, 72, 84, 96, and 108.
25. 60, 90, 100, 120, and 200.

TO CHANGE TWO OR MORE FRACTIONS TO EQUIVALENT FRACTIONS HAVING THE LEAST COMMON DENOMINATOR

When the given fractions are in their lowest terms, the least common multiple of their denominators is their least common denominator. In practice the abbreviation, "L. C. D.," is used for the least common denominator.

Change $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$ to equivalent fractions having the least common denominator.

We find the L. C. M. of 3, 4, and 8 to be 24.

To change the fractions to equivalent fractions, each with the denominator 24, we proceed thus:

$$\left. \begin{array}{l} \frac{2}{3} = \text{How many 24ths? } 24 \div 3 = 8. \quad \frac{2}{3} = \frac{8 \times 2}{8 \times 3} = \frac{16}{24} \\ \frac{3}{4} = \text{How many 24ths? } 24 \div 4 = 6. \quad \frac{3}{4} = \frac{6 \times 3}{6 \times 4} = \frac{18}{24} \\ \frac{5}{8} = \text{How many 24ths? } 24 \div 8 = 3. \quad \frac{5}{8} = \frac{3 \times 5}{3 \times 8} = \frac{15}{24} \end{array} \right\} \text{Result.}$$

WRITTEN PRACTICE

Change to fractions having the least common denominator:

- | | | | |
|--------------------------------|----------------------------------|--|--|
| 1. $\frac{1}{2}, \frac{1}{3}.$ | 7. $\frac{3}{5}, \frac{2}{6}.$ | 13. $\frac{1}{2}, \frac{1}{4}, \frac{3}{8}.$ | 19. $\frac{3}{5}, \frac{3}{8}, \frac{5}{12}.$ |
| 2. $\frac{1}{4}, \frac{1}{6}.$ | 8. $\frac{4}{6}, \frac{3}{7}.$ | 14. $\frac{1}{3}, \frac{3}{4}, \frac{5}{8}.$ | 20. $\frac{2}{5}, \frac{5}{6}, \frac{5}{8}.$ |
| 3. $\frac{2}{3}, \frac{3}{4}.$ | 9. $\frac{5}{6}, \frac{5}{8}.$ | 15. $\frac{2}{3}, \frac{5}{6}, \frac{4}{9}.$ | 21. $\frac{1}{6}, \frac{4}{9}, \frac{5}{12}.$ |
| 4. $\frac{2}{5}, \frac{3}{8}.$ | 10. $\frac{3}{5}, \frac{4}{9}.$ | 16. $\frac{3}{4}, \frac{1}{8}, \frac{2}{3}.$ | 22. $\frac{3}{8}, \frac{4}{10}, \frac{8}{12}.$ |
| 5. $\frac{3}{4}, \frac{4}{5}.$ | 11. $\frac{5}{6}, \frac{7}{9}.$ | 17. $\frac{3}{4}, \frac{5}{6}, \frac{5}{8}.$ | 23. $\frac{2}{5}, \frac{3}{10}, \frac{8}{20}.$ |
| 6. $\frac{3}{4}, \frac{5}{6}.$ | 12. $\frac{3}{5}, \frac{3}{10}.$ | 18. $\frac{2}{5}, \frac{3}{10}, \frac{5}{12}.$ | 24. $\frac{5}{8}, \frac{7}{12}, \frac{9}{16}.$ |

ADDITION OF FRACTIONS

Find the sum of $\frac{1}{3}$, $\frac{4}{5}$, and $\frac{1}{2}$.

The L. C. D. of the denominators, 3, 5, and 2, is 30.

Then: $\frac{1}{3} = \frac{10}{30}$; $\frac{4}{5} = \frac{24}{30}$; and $\frac{1}{2} = \frac{15}{30}$.

Adding, $\frac{1}{3} + \frac{4}{5} + \frac{1}{2} = \frac{10}{30} + \frac{24}{30} + \frac{15}{30} = \frac{10+24+15}{30} = \frac{49}{30} = 1\frac{19}{30}$. *Result.*

To add two or more fractions:

Change the given fractions to equivalent fractions having a common denominator.

Add the numerators of the fractions obtained.

Write the sum over the common denominator.

If the sum is an improper fraction, change it to a mixed number or to a whole number.

WRITTEN PRACTICE

Find the sum of:

- | | | |
|--|-------------------------------------|--|
| 1. $\frac{3}{4} + \frac{2}{3}$. | 14. $\frac{4}{5} + \frac{5}{6}$. | 27. $\frac{2}{5} + \frac{5}{6} + \frac{3}{4}$. |
| 2. $\frac{3}{4} + \frac{5}{6}$. | 15. $\frac{4}{5} + \frac{7}{8}$. | 28. $\frac{4}{5} + \frac{1}{2} + \frac{3}{4}$. |
| 3. $\frac{3}{4} + \frac{3}{8}$. | 16. $\frac{5}{6} + \frac{8}{9}$. | 29. $\frac{5}{8} + \frac{1}{4} + \frac{3}{8}$. |
| 4. $\frac{3}{4} + \frac{4}{5}$. | 17. $\frac{5}{9} + \frac{3}{5}$. | 30. $\frac{3}{5} + \frac{1}{10} + \frac{4}{15}$. |
| 5. $\frac{5}{6} + \frac{3}{8}$. | 18. $\frac{7}{9} + \frac{3}{8}$. | 31. $\frac{5}{7} + \frac{3}{14} + \frac{1}{2}$. |
| 6. $\frac{1}{6} + \frac{5}{8}$. | 19. $\frac{5}{7} + \frac{3}{4}$. | 32. $\frac{8}{9} + \frac{1}{3} + \frac{5}{6}$. |
| 7. $\frac{7}{8} + \frac{5}{6}$. | 20. $\frac{4}{7} + \frac{5}{6}$. | 33. $\frac{6}{7} + \frac{1}{3} + \frac{1}{21}$. |
| 8. $\frac{5}{8} + \frac{7}{6}$. | 21. $\frac{6}{5} + \frac{3}{7}$. | 34. $\frac{5}{9} + \frac{2}{3} + \frac{1}{6}$. |
| 9. $\frac{3}{4} + \frac{7}{8}$. | 22. $\frac{7}{10} + \frac{3}{5}$. | 35. $\frac{8}{9} + \frac{3}{4} + \frac{11}{18}$. |
| 10. $\frac{5}{6} + \frac{1}{9}$. | 23. $\frac{7}{10} + \frac{1}{15}$. | 36. $\frac{3}{16} + \frac{1}{8} + \frac{3}{4}$. |
| 11. $\frac{5}{8} + \frac{5}{9}$. | 24. $\frac{3}{16} + \frac{5}{8}$. | 37. $\frac{8}{9} + \frac{2}{3} + \frac{5}{6}$. |
| 12. $\frac{7}{8} + \frac{4}{9}$. | 25. $\frac{3}{10} + \frac{2}{15}$. | 38. $\frac{7}{16} + \frac{5}{32} + \frac{1}{2}$. |
| 13. $\frac{8}{9} + \frac{7}{8}$. | 26. $\frac{5}{16} + \frac{7}{8}$. | 39. $\frac{4}{15} + \frac{5}{6} + \frac{7}{10}$. |
| 40. $\frac{1}{2} + \frac{3}{4} + \frac{5}{6} + \frac{2}{3}$. | | 43. $\frac{7}{8} + \frac{5}{4} + \frac{9}{10} + \frac{1}{5}$. |
| 41. $\frac{3}{4} + \frac{7}{8} + \frac{1}{2} + \frac{5}{16}$. | | 44. $\frac{2}{3} + \frac{7}{8} + \frac{8}{9} + \frac{5}{12}$. |
| 42. $\frac{2}{3} + \frac{3}{8} + \frac{1}{12} + \frac{5}{6}$. | | 45. $\frac{6}{7} + \frac{5}{14} + \frac{7}{2} + \frac{11}{21}$. |

When Mixed Numbers are added the fractions are usually added first, then the integers are added separately. These two sums are then added for the whole sum.

Find the sum of $2\frac{3}{4}$ and $5\frac{7}{8}$.

The sum of the fractions gives :

$$\frac{3}{4} = \frac{18}{24}, \text{ and } \frac{7}{8} = \frac{21}{24}. \quad \frac{18}{24} + \frac{21}{24} = \frac{39}{24} = 1\frac{15}{24} = 1\frac{5}{8}.$$

The sum of the integers is $2+5=7$.

Therefore, the whole sum is $1\frac{5}{8}+7$, or $8\frac{5}{8}$. *Result.*

WRITTEN PRACTICE

Find the sum of :

- | | | |
|---|---|---|
| 1. $2\frac{1}{2} + 3\frac{1}{4}$. | 9. $6\frac{1}{3} + 8\frac{3}{4}$. | 17. $5\frac{1}{8} + 3\frac{1}{4} + 6\frac{1}{2}$. |
| 2. $3\frac{1}{4} + 5\frac{1}{3}$. | 10. $8\frac{1}{3} + 3\frac{3}{4}$. | 18. $7\frac{3}{4} + 2\frac{2}{3} + 5\frac{1}{8}$. |
| 3. $3\frac{1}{3} + 4\frac{1}{2}$. | 11. $6\frac{5}{8} + 1\frac{3}{4}$. | 19. $2\frac{1}{3} + 3\frac{1}{4} + 5\frac{1}{2}$. |
| 4. $4\frac{1}{5} + 3\frac{1}{2}$. | 12. $7\frac{2}{3} + 9\frac{5}{6}$. | 20. $3\frac{1}{5} + 5\frac{1}{2} + 4\frac{1}{10}$. |
| 5. $5\frac{1}{8} + 3\frac{1}{2}$. | 13. $5\frac{3}{10} + 3\frac{1}{5}$. | 21. $8\frac{4}{5} + 6\frac{1}{10} + 7\frac{3}{10}$. |
| 6. $6\frac{3}{4} + 3\frac{7}{8}$. | 14. $8\frac{3}{4} + 7\frac{3}{5}$. | 22. $2\frac{1}{6} + 3\frac{1}{8} + 5\frac{7}{24}$. |
| 7. $4\frac{1}{3} + 3\frac{1}{5}$. | 15. $9\frac{2}{3} + 10\frac{5}{8}$. | 23. $6\frac{7}{16} + 5\frac{3}{2} + 8\frac{1}{2}$. |
| 8. $5\frac{1}{8} + 2\frac{1}{4}$. | 16. $4\frac{1}{8} + 9\frac{15}{16}$. | 24. $8\frac{3}{16} + 5\frac{3}{8} + 3\frac{3}{4}$. |
| 25. $\begin{array}{r} 32\frac{3}{4} \\ \underline{23\frac{1}{2}} \end{array}$ | 28. $\begin{array}{r} 35\frac{1}{8} \\ \underline{42\frac{3}{4}} \end{array}$ | 31. $\begin{array}{r} 40\frac{3}{8} \\ \underline{52\frac{1}{2}} \end{array}$ |
| 26. $\begin{array}{r} 24\frac{5}{6} \\ \underline{30\frac{2}{3}} \end{array}$ | 29. $\begin{array}{r} 41\frac{2}{3} \\ \underline{39\frac{8}{9}} \end{array}$ | 32. $\begin{array}{r} 43\frac{1}{3} \\ \underline{56\frac{3}{4}} \end{array}$ |
| 27. $\begin{array}{r} 32\frac{2}{3} \\ \underline{27\frac{1}{2}} \end{array}$ | 30. $\begin{array}{r} 40\frac{1}{6} \\ \underline{31\frac{3}{4}} \end{array}$ | 33. $\begin{array}{r} 49\frac{2}{3} \\ \underline{52\frac{4}{5}} \end{array}$ |
| | | 34. $\begin{array}{r} 56\frac{2}{5} \\ \underline{72\frac{1}{2}} \end{array}$ |
| | | 37. $\begin{array}{r} 73\frac{5}{8} \\ \underline{87\frac{7}{9}} \end{array}$ |
| | | 35. $\begin{array}{r} 63\frac{3}{5} \\ \underline{38\frac{2}{3}} \end{array}$ |
| | | 38. $\begin{array}{r} 86\frac{3}{10} \\ \underline{94\frac{5}{12}} \end{array}$ |
| | | 36. $\begin{array}{r} 72\frac{4}{5} \\ \underline{69\frac{3}{8}} \end{array}$ |
| | | 39. $\begin{array}{r} 93\frac{7}{12} \\ \underline{94\frac{5}{8}} \end{array}$ |

EVERYDAY USES FOR ADDITION OF FRACTIONS



1. Tom worked $2\frac{1}{3}$ hours mowing the lawn, and Clara worked $1\frac{1}{4}$ hours sweeping the walks. How many hours did both work?

2. In mowing their lawn Frank worked $1\frac{1}{4}$ hours before school, $\frac{1}{2}$ hour at the noon intermission, and $2\frac{1}{4}$ hours after school. How many hours did he work?

3. Mary worked two and one half hours in her flower bed, and one and one fourth hours in her war garden. How many hours did she work?

4. Marion swept a walk that was $12\frac{1}{2}$ yards long, another that was $24\frac{1}{3}$ yards long, and another that was $5\frac{1}{8}$ yards long. How many yards were there in the whole length of walks that she swept?

5. Jack and his father tried to see which could mow their lawn in the shortest time. Jack mowed it in $2\frac{3}{4}$ hours, and a week later his father mowed it in $3\frac{1}{4}$ hours. How much less time did it take Jack to mow the lawn than it did his father?

6. A piece of wood $\frac{3}{4}$ inch thick is glued to another piece $\frac{1}{2}$ inch thick. What is the thickness of the finished piece?

7. $4\frac{1}{2}$ yd. of cotton cloth were sold to a woman from one piece, and $15\frac{1}{4}$ yd. of cotton cloth from another piece. How many yards of cotton cloth were sold to her?

8. A young man spends $\frac{1}{5}$ of his salary for board, $\frac{1}{5}$ of it for clothing, and $\frac{1}{3}$ of it for other expenses. What part of his salary does he spend for these three items?

9. An automobile runs $24\frac{1}{2}$ miles the first hour, $27\frac{3}{4}$ miles the second hour, and $22\frac{3}{4}$ miles the third hour. How many miles does it run in the three hours?

10. A rug is $8\frac{1}{2}$ feet wide. At \$.20 a foot, find the cost of a fringe for both ends of the rug.

11. Jack trimmed $15\frac{1}{4}$ yd. of a hedge on Monday, and the remaining portion of it, $12\frac{3}{4}$ yd., on Tuesday. How many yards long is the hedge?

12. A boy travels two thirds of the distance between two towns by trolley, and one fifth by automobile, and walks the rest of it. What part of the distance between these towns does he walk?

13. The rough plaster coat on a wall is $\frac{3}{8}$ in. thick, and the finish coat is $\frac{3}{16}$ in. thick. What is the thickness of both coats?

14. Donald worked $3\frac{1}{4}$ hours on Monday, $4\frac{1}{3}$ hours on Tuesday, $3\frac{1}{2}$ hours on Wednesday, 4 hours on Thursday, and $4\frac{1}{2}$ hours on Friday. How many hours did he work?

15. Three timbers are placed end to end for the foundation of a partition. Their lengths being $12\frac{1}{2}$ ft., 11 ft., and $12\frac{3}{4}$ ft., find the length of the foundation.

16. The length of a sheet of cardboard is $12\frac{3}{4}$ inches, and the width is $7\frac{3}{8}$ inches. How many inches are there in the "perimeter," or distance around the edge of the sheet?

17. A farmer sold 4 lots from a field, the lots containing respectively $\frac{4}{5}$ A., $\frac{5}{8}$ A., $\frac{2}{3}$ A., and $\frac{3}{4}$ A. How many acres of land did he sell in the four lots?

18. A room is $18\frac{3}{4}$ ft. long and $11\frac{1}{4}$ ft. wide. At 5¢ a foot find the cost of a picture molding for the room; add 3¢ per yard for placing it on the wall, and find the total cost.

19. Four lots on a city street measure along the curb line $61\frac{1}{2}$ ft., $61\frac{1}{2}$ ft., $75\frac{1}{4}$ ft., and $82\frac{3}{4}$ ft., respectively. At \$.75 a foot for concrete curbing, find the cost for a curbing in front of these four lots.

SUBTRACTION OF FRACTIONS

In subtraction, as in addition, we must work with *similar* fractions. We make use, therefore, of the L. C. D., as we did in addition of fractions.

Subtract $\frac{2}{5}$ from $\frac{4}{5}$.

The L. C. D. of the given denominators is 15.

Then $\frac{2}{5} = \frac{10}{15}$, and $\frac{4}{5} = \frac{12}{15}$.

Subtracting, $\frac{12}{15} - \frac{10}{15} = \frac{12-10}{15} = \frac{2}{15}$. *Result.*

WRITTEN PRACTICE

Subtract the following:

1. $\frac{3}{4} - \frac{1}{5}$.

2. $\frac{2}{3} - \frac{1}{4}$.

3. $\frac{3}{4} - \frac{3}{8}$.

4. $\frac{5}{6} - \frac{1}{2}$.

5. $\frac{3}{4} - \frac{1}{8}$.

6. $\frac{5}{8} - \frac{1}{3}$.

7. $\frac{7}{8} - \frac{3}{4}$.

8. $\frac{7}{8} - \frac{1}{6}$.

9. $\frac{4}{5} - \frac{3}{10}$.

10. $\frac{5}{6} - \frac{3}{8}$.

11. $\frac{7}{8} - \frac{5}{6}$.

12. $\frac{9}{10} - \frac{3}{4}$.

13. $\frac{9}{10} - \frac{5}{8}$.

14. $\frac{9}{10} - \frac{4}{5}$.

15. $\frac{8}{15} - \frac{3}{10}$.

16. $\frac{5}{16} - \frac{1}{8}$.

17. $\frac{13}{16} - \frac{5}{8}$.

18. $\frac{15}{16} - \frac{7}{8}$.

19. $\frac{11}{15} - \frac{3}{10}$.

20. $\frac{5}{12} - \frac{3}{10}$.

21. $\frac{13}{15} - \frac{5}{6}$.

22. $\frac{17}{20} - \frac{3}{4}$.

23. $\frac{15}{16} - \frac{11}{12}$.

24. $\frac{11}{18} - \frac{5}{12}$.

TO SUBTRACT A FRACTION FROM A WHOLE NUMBER

Subtract $\frac{7}{9}$ from 4.

$$4 = \frac{4}{1} \quad \text{Subtracting } 4 - \frac{7}{9} = \frac{4}{1} - \frac{7}{9} = \frac{36}{9} - \frac{7}{9} = \frac{36-7}{9} = \frac{29}{9} = 3\frac{2}{9} \quad \text{Result.}$$

Another method frequently used is to consider that

$$4 - 1 = 3. \quad 1 = \frac{9}{9}. \quad \text{Subtracting } \frac{9}{9} - \frac{7}{9} = \frac{9-7}{9} = \frac{2}{9}$$

Adding the difference between the fractions to the whole number 3, we have $3\frac{2}{9}$. *Result.*

WRITTEN PRACTICE

Subtract the following:

- | | | | |
|------------------------|--------------------------|----------------------------|----------------------------|
| 1. $5 - \frac{3}{4}$. | 7. $8 - \frac{6}{7}$. | 13. $15 - \frac{6}{7}$. | 19. $25 - \frac{11}{16}$. |
| 2. $6 - \frac{2}{3}$. | 8. $9 - \frac{7}{8}$. | 14. $14 - \frac{4}{5}$. | 20. $40 - \frac{15}{16}$. |
| 3. $8 - \frac{3}{4}$. | 9. $10 - \frac{5}{8}$. | 15. $17 - \frac{8}{9}$. | 21. $52 - \frac{11}{16}$. |
| 4. $6 - \frac{5}{8}$. | 10. $10 - \frac{4}{5}$. | 16. $18 - \frac{7}{8}$. | 22. $60 - \frac{13}{15}$. |
| 5. $7 - \frac{3}{8}$. | 11. $12 - \frac{5}{9}$. | 17. $19 - \frac{11}{12}$. | 23. $72 - \frac{11}{16}$. |
| 6. $9 - \frac{5}{6}$. | 12. $11 - \frac{3}{8}$. | 18. $21 - \frac{11}{15}$. | 24. $85 - \frac{13}{25}$. |

TO SUBTRACT A MIXED NUMBER FROM A MIXED NUMBER

Subtract $3\frac{2}{3}$ from $5\frac{7}{8}$.

For the fractions: $\frac{7}{8} - \frac{2}{3} = \frac{21}{24} - \frac{16}{24} = \frac{21-16}{24} = \frac{5}{24}$

For the whole numbers: $5 - 3 = 2$.

Therefore, $2 + \frac{5}{24} = 2\frac{5}{24}$. *Result.*

If the fraction to be subtracted is larger than the fraction from which we subtract we may proceed as in the following:

Subtract $4\frac{1}{3}$ from $9\frac{2}{3}$.

$\frac{2}{3} = \frac{12}{15}$. $\frac{1}{3} = \frac{10}{15}$. We cannot subtract $\frac{12}{15}$ from $\frac{10}{15}$.

Consider $9\frac{2}{3}$ or $9\frac{10}{15} = 8 + 1\frac{10}{15}$. Then $1\frac{10}{15} = 2\frac{5}{15}$.

We have, therefore: $8\frac{25}{15} - 4\frac{12}{15}$. The difference = $4\frac{13}{15}$. *Result.*

WRITTEN PRACTICE

Subtract the following:

- | | | |
|-------------------------------------|--|--|
| 1. $3\frac{2}{3} - 2\frac{1}{2}$. | 6. $6\frac{1}{4} - 2\frac{1}{5}$. | 11. $5\frac{1}{3} - 2\frac{1}{2}$. |
| 2. $4\frac{3}{4} - 2\frac{1}{2}$. | 7. $7\frac{1}{3} - 3\frac{1}{4}$. | 12. $8\frac{2}{3} - 4\frac{3}{4}$. |
| 3. $4\frac{2}{3} - 3\frac{1}{4}$. | 8. $7\frac{2}{3} - 4\frac{1}{3}$. | 13. $10\frac{1}{4} - 7\frac{3}{5}$. |
| 4. $5\frac{1}{2} - 3\frac{1}{3}$. | 9. $8\frac{5}{8} - 6\frac{1}{3}$. | 14. $13\frac{3}{8} - 9\frac{2}{3}$. |
| 5. $5\frac{3}{8} - 3\frac{1}{4}$. | 10. $9\frac{6}{7} - 5\frac{1}{2}$. | 15. $12\frac{5}{12} - 9\frac{5}{8}$. |
| 16. $6\frac{5}{6} - 4\frac{1}{3}$. | 21. $8\frac{5}{6} - 6\frac{2}{3}$. | 26. $17\frac{5}{8} - 13\frac{7}{10}$. |
| 17. $7\frac{4}{5} - 5\frac{1}{2}$. | 22. $9\frac{9}{10} - 7\frac{5}{6}$. | 27. $20\frac{3}{10} - 15\frac{4}{5}$. |
| 18. $8\frac{5}{6} - 5\frac{1}{4}$. | 23. $10\frac{5}{12} - 8\frac{1}{8}$. | 28. $21\frac{5}{12} - 17\frac{3}{4}$. |
| 19. $8\frac{5}{8} - 6\frac{2}{3}$. | 24. $11\frac{7}{10} - 9\frac{1}{4}$. | 29. $34\frac{7}{16} - 23\frac{3}{4}$. |
| 20. $9\frac{6}{7} - 7\frac{1}{2}$. | 25. $12\frac{7}{16} - 10\frac{3}{8}$. | 30. $42\frac{9}{20} - 32\frac{3}{5}$. |

EVERYDAY USES FOR SUBTRACTION OF FRACTIONS

1. $\frac{1}{3}$ of a yard is cut from a piece of velvet $\frac{3}{4}$ of a yard long. What part of a yard is left?

2. Frank rode $12\frac{1}{2}$ miles on his bicycle in the forenoon, and $9\frac{1}{4}$ miles in the afternoon. How many more miles did he ride in the forenoon than he did in the afternoon?

3. If a boy earns \$12 $\frac{1}{2}$ in a week and spends \$5 $\frac{3}{4}$ of his earnings, how many dollars should he have left at the end of the week?

4. In payment for groceries costing \$2 $\frac{1}{4}$ Donald gave the grocer a \$5 bill. How much change should he receive?

5. In June Florence weighed 42 $\frac{3}{4}$ pounds, but in September she weighed 48 $\frac{1}{4}$ pounds. How many pounds did she gain in that time?

6. Mr. Emerson earns \$3 $\frac{3}{4}$ a day, and his boy earns \$1 $\frac{1}{4}$ a day. How much more does Mr. Emerson earn in a day than his boy earns?

7. In a jumping contest Franklin jumped $5\frac{1}{4}$ feet, and Ned jumped $6\frac{3}{4}$ feet. By how many feet did Ned beat Franklin in this contest?

8. The distance from the floor to the top of a window is $8\frac{3}{8}$ feet, and the distance from the floor to the bottom of this window is $2\frac{3}{4}$ feet. What is the distance between the bottom and the top of the window?

9. A board is $12\frac{3}{4}$ feet long, but the carpenter needs only $8\frac{3}{8}$ feet of it. How much must he cut off from it?

10. A Boy Scout's record for walking was $9\frac{3}{4}$ miles, but he increased this record to $12\frac{5}{8}$ miles. How many miles did he increase his record?

11. The weight of a basket and its contents is $23\frac{1}{4}$ pounds. If the basket weighs $4\frac{3}{4}$ pounds, what is the weight of the contents?

12. $\frac{3}{8}$ of a piece of cloth was sold on Monday, and $\frac{1}{3}$ of it was sold on Tuesday. By how much did the part sold on Monday exceed the part sold on Tuesday?

13. Gregory weighs $105\frac{1}{2}$ pounds and Jack weighs $101\frac{3}{4}$ pounds. Tom weighs $109\frac{1}{4}$ pounds and Charles 112 pounds. Show that the combined weight of the last two boys is 14 pounds more than the combined weight of the first two boys.

14. In a swimming contest Jack made the distance in $18\frac{1}{5}$ seconds. Tom made it in $17\frac{2}{5}$ seconds, and Edmund in $15\frac{4}{5}$ seconds. By how many seconds did Jack beat Tom? By how many seconds did Jack beat Edmund?

15. Three pieces of linen contain $14\frac{1}{2}$ yd., $12\frac{3}{4}$ yd., and $16\frac{1}{4}$ yd., respectively. From each of these three pieces $2\frac{1}{2}$ yd. are cut. How many yards of linen are left in the three pieces? (Can you suggest a short way to get the final result?)

COMBINING ADDITION AND SUBTRACTION OF FRACTIONS

Find the value of $\frac{2}{3} - \frac{2}{5} + \frac{3}{4} - \frac{1}{2}$.

The L. C. D. of the denominators is 60.

$$\begin{aligned} \text{Then: } \frac{2}{3} - \frac{2}{5} + \frac{3}{4} - \frac{1}{2} &= \frac{40}{60} - \frac{24}{60} + \frac{45}{60} - \frac{30}{60} \\ &= \frac{40 - 24 + 45 - 30}{60} = \frac{31}{60}. \quad \text{Result.} \end{aligned}$$

WRITTEN PRACTICE

Find the results in each of the following:

1. $\frac{2}{3} + \frac{3}{4} - \frac{1}{2}$.
2. $\frac{3}{4} - \frac{1}{3} + \frac{2}{3}$.
3. $\frac{5}{8} - \frac{1}{2} + \frac{2}{3}$.
4. $\frac{7}{8} - \frac{1}{4} - \frac{1}{2}$.
5. $\frac{3}{4} + \frac{1}{2} - \frac{5}{8}$.
6. $\frac{5}{16} - \frac{1}{8} + \frac{1}{4}$.
7. $\frac{7}{12} - \frac{1}{6} + \frac{2}{3}$.
8. $\frac{8}{9} - \frac{1}{3} + \frac{5}{6}$.
9. $\frac{15}{16} - \frac{1}{8} + \frac{3}{2}$.
10. $\frac{13}{32} + \frac{3}{8} - \frac{7}{16}$.
11. $\frac{13}{20} - \frac{2}{5} + \frac{3}{4}$.
12. $\frac{11}{24} + \frac{7}{12} - \frac{5}{8}$.
13. $\frac{2}{3} + \frac{1}{6} + \frac{5}{12} - \frac{3}{4}$.
14. $\frac{3}{4} - \frac{1}{2} + \frac{5}{8} - \frac{1}{4}$.
15. $\frac{1}{2} - \frac{1}{8} + \frac{5}{8} - \frac{2}{3}$.
16. $\frac{5}{6} + \frac{3}{4} - \frac{7}{8} - \frac{1}{12}$.
17. $\frac{3}{8} + \frac{5}{16} - \frac{1}{4} + \frac{1}{2}$.
18. $\frac{8}{9} + \frac{3}{2} + \frac{5}{6} - \frac{7}{3}$.
19. $\frac{5}{6} + 1 - \frac{2}{3} + \frac{1}{2}$.
20. $\frac{3}{8} - \frac{1}{4} + 5 - \frac{2}{3}$.
21. $\frac{3}{4} + 2 + \frac{1}{6} - \frac{5}{12}$.
22. $\frac{8}{9} - \frac{1}{6} + \frac{5}{3} + 1$.
23. $\frac{7}{8} + 3 - \frac{4}{3} + 2$.
24. $6 - \frac{2}{3} + \frac{5}{6} + \frac{3}{4}$.

EVERYDAY USES COMBINING ADDITION AND SUBTRACTION OF FRACTIONS

1. A barrel was $\frac{3}{8}$ full of sugar when $\frac{1}{3}$ of a barrel more sugar was poured into it. What fraction of a whole barrel of sugar did this barrel then contain?

2. $\frac{2}{3}$ of a farmer's land is planted to grains, and $\frac{3}{10}$ of it is in alfalfa. If the rest of his farm is in pasture, what fraction of the farm is pasture land?

3. A man spends $\frac{1}{4}$ of his income for board and room, $\frac{1}{5}$ of it for clothing, and $\frac{1}{3}$ for miscellaneous expenses. If he saves the balance, what fraction of his income does he save?

4. A man owned a field containing $12\frac{1}{2}$ acres of land and sold $5\frac{7}{8}$ acres of it. Later he bought an adjoining field, containing $4\frac{1}{4}$ acres. How many acres did he then have?

5. From a piece of timber $12\frac{3}{4}$ feet long a carpenter cut one piece $4\frac{1}{4}$ feet long and another piece $3\frac{5}{8}$ feet long. How much of the original piece was left?

6. A boy earned $\$3\frac{3}{4}$ caring for a neighbor's lawn, and $\$5\frac{1}{2}$ working Saturdays in a store. He spent $\$2\frac{1}{4}$ for a sweater and $\$1\frac{1}{5}$ for a cap. How many dollars should he have left from his earnings?

7. A merchant had two pieces of silk. From one piece containing $14\frac{5}{8}$ yards he sold $9\frac{3}{4}$ yards, and from the other piece that contained $11\frac{1}{4}$ yards he sold 8 yards. How many yards were left in both pieces after these sales?

8. A flagpole is 24 feet long. $16\frac{1}{3}$ feet of the pole are painted white, 5 feet of it are painted black, and the rest of the pole is in the ground. What is the length of the part in the ground?

9. Jack is $12\frac{1}{2}$ years old and Barbara is $10\frac{3}{4}$ years old. If their father is $38\frac{1}{4}$ years old, by how many years does the father's age exceed the sum of the years in both Jack's and Barbara's ages?

10. Fred, Donald, and Edmond together weigh 296 $\frac{1}{2}$ pounds. Fred weighs $112\frac{1}{2}$ pounds, and Donald weighs 89 $\frac{1}{4}$ pounds. How many pounds does Edmond weigh?

11. Harry spent $\frac{1}{5}$ of his money for a boat, $\frac{1}{4}$ of it for a golf club, $\frac{1}{3}$ of it for a tennis racket, and $\frac{1}{10}$ of it for a pair of Indian clubs. What fraction of his money had he left?

12. A company of four boys bought a canoe. Frank paid $\frac{1}{6}$ of the whole cost, Charles paid $\frac{1}{3}$ of it, Tom paid $\frac{1}{3}$ of it, and Lester paid the remaining fraction of it. For what fraction of the cost did Lester pay?

13. Jerry asked his father how his money was invested, and his father replied, " $\frac{1}{3}$ of it is in our farm, $\frac{2}{5}$ of it is in our grocery business, $\frac{1}{10}$ of it is loaned, and the remainder is in the savings bank." What fraction of his father's money is in the savings bank? $1/6$

CANCELLATION

Cancellation is the process of simplifying the work of multiplication and division by rejecting factors common to both the dividend and the divisor.

Illustrations:

1. Divide the product of $45 \times 56 \times 39$ by the product of $30 \times 52 \times 42$.

$$\begin{array}{ccccccc} & 3 & 4 & 3 & & & \\ 45 \times 56 \times 39 & = & \cancel{45} \times \cancel{56} \times \cancel{39} & = & 3 \times 4 \times 3 & = & \frac{3 \times 4 \times 3}{2 \times 4 \times 3} = \frac{3}{2}. \quad \text{Result.} \\ 30 \times 52 \times 42 & = & \cancel{30} \times \cancel{52} \times \cancel{42} & = & 2 \times 4 \times 3 & = & 2 \times 4 \times 3 \\ & 2 & 4 & 3 & & & \end{array}$$

The steps of the process are as follows:

(1) 45 and 30 have the common factor 15.

Dividing both 45 and 30 by 15 the quotients are 3 and 2 respectively.

(2) 56 and 42 have the common factor 14.

Dividing both 56 and 42 by 14 the quotients are 4 and 3 respectively.

(3) 39 and 52 have the common factor 13.

Dividing both 39 and 52 by 13 the quotients are 3 and 4 respectively.

(4) In like manner, 4 and 3 may be canceled with 4 and 3; and the uncanceled 3, with the uncanceled 2, give $\frac{3}{2}$. Result.

2. A city lot 450 feet long and 280 feet wide was laid out in building lots, each 140 feet deep and 50 feet wide. How many building lots were laid out?

$$\begin{array}{ccccccc} & 9 & 2 & & & & \\ 450 \times 280 & = & \cancel{450} \times \cancel{280} & = & 18, & \text{the number of lots.} & \text{Result.} \\ 140 \times 50 & = & \cancel{140} \times \cancel{50} & & & & \end{array}$$

WRITTEN PRACTICE

Using cancellation, divide :

$$1. \frac{12 \times 24}{6 \times 8}$$

$$5. \frac{18 \times 55}{22 \times 42}$$

$$9. \frac{57 \times 84}{28 \times 76}$$

$$2. \frac{12 \times 56}{7 \times 16}$$

$$6. \frac{54 \times 55}{45 \times 66}$$

$$10. \frac{105 \times 80}{128 \times 175}$$

$$3. \frac{18 \times 32}{16 \times 36}$$

$$7. \frac{36 \times 80}{48 \times 105}$$

$$11. \frac{125 \times 98}{245 \times 50}$$

$$4. \frac{28 \times 38}{19 \times 56}$$

$$8. \frac{72 \times 112}{56 \times 90}$$

$$12. \frac{215 \times 256}{172 \times 320}$$

$$13. 12 \times 15 \times 17 \times 21 \text{ by } 18 \times 34 \times 10 \times 28.$$

$$14. 22 \times 27 \times 35 \times 44 \text{ by } 36 \times 45 \times 55 \times 66.$$

$$15. 27 \times 34 \times 58 \times 74 \text{ by } 51 \times 37 \times 29 \times 18.$$

$$16. 54 \times 65 \times 81 \times 92 \text{ by } 91 \times 69 \times 36 \times 108.$$

$$17. 105 \times 117 \times 121 \times 128 \text{ by } 144 \times 143 \times 165 \times 158.$$

WRITTEN APPLICATIONS

1. If 15 bicycles cost \$525, find the cost of 12 such bicycles.
2. \$18.75 was paid for 15 chisels. At the same price apiece, how much will 19 such chisels cost?
3. A man worked 6 days each week for 3 weeks, and was paid with 54 bushels of potatoes worth \$1.75 per bushel. How much was he paid per day?
4. A woman exchanged 18 dozen eggs worth 50 cents a dozen for cloth worth 24 cents a yard. How many yards of the cloth should she receive?
5. A boy saved 15 cents a week for 52 weeks and spent all of the money for a set of storybooks costing \$.65 apiece. How many books are there in the set?
6. A grocer bought 30 pounds of butter at \$.56 per pound, and gave in payment 120 pounds of sugar. What was the price of the sugar per pound?

7. A farmer engaged a neighbor to work 18 days of 9 hours each at \$.20 per hour, and paid him with oats worth \$.75 per bushel. How many bushels of oats did it take to pay him for the work? *43 1/5*

8. 15 acres of a farm were sown to wheat, and the yield was 30 bushels to the acre. How many wagon-loads of 2500 pounds each are there in this crop? A bushel of the wheat weighs 60 pounds. *11 2/5*

MULTIPLICATION OF FRACTIONS

TO MULTIPLY A FRACTION BY A WHOLE NUMBER

ORAL PRACTICE

1. How many dollars are 3 times 1 dollar? 3 times 2 dollars? 3 times 5 dollars?

2. How many sevenths are 3 times 1 seventh? 3 times 2 sevenths? 3 times 5 sevenths?

$$3. \quad 3 \times \frac{1}{7} = \frac{?}{7} \quad 3 \times \frac{2}{7} = \frac{?}{7} \quad 3 \times \frac{5}{7} = \frac{?}{7}$$

4. If you multiply $3 \times \frac{1}{6}$ you get $\frac{3 \times 1}{6}$. Is the result in its simplest form?

5. If, instead of multiplying the numerator by 3, you had divided the denominator by 3, how would the result compare with that of Ex. 4?

6. Can you explain how cancellation simplifies the work in multiplying a fraction by a whole number?

1. Multiply $\frac{5}{7}$ by 4.

2. Multiply $\frac{5}{12}$ by 4.

$$4 \times \frac{5}{7} = \frac{4 \times 5}{7} = \frac{20}{7}. \text{ Result.}$$

$$4 \times \frac{5}{12} = \frac{4 \times 5}{12} = \frac{\cancel{4} \times 5}{\cancel{12}^3} = \frac{5}{3}. \text{ Result.}$$

To multiply a fraction by a whole number :

Multiply the numerator of the fraction by the whole number, or, Divide the denominator of the fraction by the whole number.

WRITTEN PRACTICE

Multiply, using cancellation wherever possible :

- | | | |
|--------------------------|-----------------------------|---------------------------------|
| 1. $\frac{7}{12}$ by 3. | 9. 7 by $\frac{13}{28}$. | 17. $\frac{13}{48}$ by 32. |
| 2. $\frac{7}{15}$ by 3. | 10. 8 by $\frac{11}{32}$. | 18. $15 \times \frac{11}{45}$. |
| 3. $\frac{5}{18}$ by 4. | 11. 9 by $\frac{13}{45}$. | 19. $\frac{31}{21}$ by 14. |
| 4. $\frac{11}{12}$ by 4. | 12. 12 by $\frac{15}{28}$. | 20. $25 \times \frac{19}{35}$. |
| 5. $\frac{11}{20}$ by 5. | 13. 15 by $\frac{11}{40}$. | 21. $\frac{17}{64}$ by 48. |
| 6. $\frac{13}{16}$ by 5. | 14. 12 by $\frac{3}{20}$. | 22. $26 \times \frac{15}{78}$. |
| 7. $\frac{11}{18}$ by 6. | 15. 15 by $\frac{17}{25}$. | 23. $\frac{21}{64}$ by 16. |
| 8. $\frac{12}{21}$ by 6. | 16. 16 by $\frac{9}{56}$. | 24. 38 by $\frac{5}{76}$. |

TO MULTIPLY A MIXED NUMBER BY A WHOLE NUMBER

ORAL PRACTICE

- How many dollars are there in 3 times $1\frac{1}{4}$ dollars? (3 times 1 dollar plus 3 times $\frac{1}{4}$ dollar equal what?)
- How many dollars are there in 4 times $1\frac{1}{5}$ dollars? In $4 \times 1\frac{1}{4}$ dollars? In 5 times $1\frac{1}{4}$ dollars?
- How many inches are there in 5 times $1\frac{1}{2}$ inches? In 6 times $1\frac{1}{4}$ inches? In $8 \times 1\frac{1}{2}$ inches? In $9 \times 1\frac{1}{3}$ inches?
- How much must you pay for 6 writing pads at 7 $\frac{1}{2}$ cents apiece? How much for 4 such pads? How much for 10 such pads?
- What is the cost of 10 tons of coal at $\$6\frac{1}{2}$ per ton? How much will 8 tons cost at the same price? 16 tons? 24 tons?

6. Explain why, in finding the product of a mixed number by a whole number, we must first find two products and then find their sum.

1. Multiply $2\frac{1}{2}$ by 5.

Since a mixed number is the sum of a whole number and a fraction, we want the product of 5 times 2, plus the product of $5 \times \frac{1}{2}$.

$$\begin{aligned}(5 \times 2\frac{1}{2}) &= (5 \times 2) + (5 \times \frac{1}{2}) \\ &= 10 + \frac{5}{2} = 10 + 2\frac{1}{2} = 12\frac{1}{2}. \text{ Result.}\end{aligned}$$

To multiply a mixed number by a whole number :

Multiply the whole number by the given multiplier.

Multiply the fraction by the given multiplier.

Add the products.

WRITTEN PRACTICE

Multiply :

- | | | |
|------------------------------|----------------------------|----------------------------------|
| 1. $3\frac{1}{2}$ by 4. | 9. $9\frac{3}{5}$ by 10. | 17. $16 \times 18\frac{3}{7}$. |
| 2. $3\frac{2}{5} \times 5$. | 10. $9\frac{5}{8}$ by 12. | 18. $20 \times 20\frac{1}{5}$. |
| 3. $4\frac{5}{6} \times 6$. | 11. $10\frac{1}{4}$ by 12. | 19. $18 \times 11\frac{5}{12}$. |
| 4. $5\frac{2}{3}$ by 7. | 12. $11\frac{2}{3}$ by 12. | 20. $21 \times 13\frac{2}{7}$. |
| 5. $6\frac{1}{4}$ by 8. | 13. $12\frac{3}{4}$ by 14. | 21. $20\frac{7}{8}$ by 18. |
| 6. $7\frac{2}{3}$ by 6. | 14. $11\frac{1}{3}$ by 15. | 22. 20 by $18\frac{7}{8}$. |
| 7. $8\frac{1}{4}$ by 8. | 15. $12\frac{2}{3}$ by 16. | 23. $25\frac{2}{5}$ by 20. |
| 8. $9\frac{1}{2}$ by 6. | 16. $14\frac{2}{3}$ by 15. | 24. 32 by $30\frac{5}{8}$. |

TO MULTIPLY A WHOLE NUMBER BY A FRACTION

ORAL PRACTICE

1. If you divide 6 by 2: $6 \div 2 = 3$. How does this result compare with $\frac{1}{2} \times 6$?

2. Finding "one half of 6" is the same as multiplying 6 by what fraction? What sign corresponds to the use of the word "of"?

3. What is $\frac{1}{2}$ of 10? $\frac{1}{2}$ of 16? $\frac{1}{2}$ of 20? $\frac{1}{3}$ of 12? $\frac{1}{3}$ of 18?
 $\frac{1}{4}$ of 20? $\frac{1}{5}$ of 30? $\frac{1}{6}$ of 42?

4. If the word "of" is equivalent to the expression "multiply by," or to the sign " $\times$," what do we mean when we say, "Find $\frac{2}{3}$ of 12"?

5. What is $\frac{2}{3}$ of 15? $\frac{2}{3}$ of 18? $\frac{3}{4}$ of 20? $\frac{2}{5}$ of 15? $\frac{3}{4}$ of 24?
 $\frac{3}{5}$ of 30? $\frac{4}{5}$ of 40?

1. Find $\frac{5}{8}$ of 56.

$$\frac{5}{8} \text{ of } 56 = \frac{5}{8} \times 56 = \frac{5 \times 56}{8} = \frac{5 \times \overset{7}{\cancel{8}8}}{8} = 35. \text{ Result.}$$

To multiply a whole number by a fraction :

Multiply the whole number by the numerator of the fraction, and write the product over the denominator.

Cancel, if possible, before actually multiplying.

WRITTEN PRACTICE

Find :

- | | | |
|--------------------------|----------------------------|----------------------------|
| 1. $\frac{2}{3}$ of 21. | 14. $\frac{6}{7}$ of 46. | 27. $\frac{10}{11}$ of 55. |
| 2. $\frac{3}{4}$ of 28. | 15. $\frac{7}{8}$ of 64. | 28. $\frac{12}{13}$ of 65. |
| 3. $\frac{3}{5}$ of 25. | 16. $\frac{8}{9}$ of 72. | 29. $\frac{14}{15}$ of 85. |
| 4. $\frac{3}{7}$ of 35. | 17. $\frac{8}{9}$ of 40. | 30. $\frac{17}{18}$ of 76. |
| 5. $\frac{4}{7}$ of 45. | 18. $\frac{9}{10}$ of 52. | 31. $\frac{12}{13}$ of 90. |
| 6. $\frac{5}{8}$ of 42. | 19. $\frac{7}{12}$ of 56. | 32. $\frac{23}{30}$ of 90. |
| 7. $\frac{7}{8}$ of 60. | 20. $\frac{11}{12}$ of 72. | 33. $\frac{15}{16}$ of 96. |
| 8. $\frac{8}{9}$ of 75. | 21. $\frac{11}{15}$ of 66. | 34. $\frac{20}{27}$ of 99. |
| 9. $\frac{4}{5}$ of 55. | 22. $\frac{10}{11}$ of 44. | 35. $\frac{21}{27}$ of 72. |
| 10. $\frac{6}{7}$ of 56. | 23. $\frac{11}{12}$ of 50. | 36. $\frac{25}{28}$ of 84. |
| 11. $\frac{7}{8}$ of 40. | 24. $\frac{10}{13}$ of 52. | 37. $\frac{34}{40}$ of 96. |
| 12. $\frac{6}{7}$ of 54. | 25. $\frac{11}{12}$ of 66. | 38. $\frac{23}{30}$ of 90. |
| 13. $\frac{7}{9}$ of 60. | 26. $\frac{10}{17}$ of 68. | 39. $\frac{24}{46}$ of 92. |

TO MULTIPLY A WHOLE NUMBER BY A MIXED NUMBER

ORAL PRACTICE

1. How many inches are there in 4×6 inches? How many inches are there in $\frac{1}{2}$ of 6 inches? How many inches are there in $4\frac{1}{2} \times 6$ inches?

2. How many dollars are there in $2\frac{1}{2}$ times 10 dollars? In $2\frac{1}{2}$ times 12 dollars? In $2\frac{1}{2} \times \$16$? In $2\frac{1}{2} \times \$20$?

3. How many inches are there in $3 \times \frac{1}{3}$ inches? In $2\frac{2}{3} \times 6$ inches? In $4\frac{1}{2} \times 6$ inches? In $3\frac{1}{2} \times 12$ inches? In $3\frac{1}{4}$ times 16 inches?

4. How much must I pay for $2\frac{1}{2}$ yards of braid at 10 cents a yard? For $3\frac{1}{2}$ yards at 10¢ a yard? For $4\frac{1}{2}$ yards at 16¢ a yard?

5. If I walk around a field which is $12\frac{1}{2}$ rods long on each of its four sides, how many rods do I walk? If I walked twice around this field, how many rods would I walk?

1. Multiply 15 by $12\frac{3}{4}$.

The product must include 15×12 and $15 \times \frac{3}{4}$

Then $15 \times 12\frac{3}{4} = (15 \times 12) + (15 \times \frac{3}{4})$.

But $(15 \times 12) = 180$, and $15 \times \frac{3}{4} = \frac{45}{4}$.

Hence, $15 \times 12\frac{3}{4} = 180 + \frac{45}{4}$
 $= 180 + 11\frac{1}{4}$
 $= 191\frac{1}{4}$. Result.

To multiply a whole number by a mixed number :

Multiply the given whole number by the whole number in the multiplier.

Multiply the given whole number by the fraction in the multiplier.

Add the products.

WRITTEN PRACTICE

Multiply :

- | | | |
|--------------------------------|---------------------------------|---------------------------------|
| 1. $2\frac{1}{2} \times 12.$ | 9. $4\frac{5}{8} \times 19.$ | 17. $9\frac{3}{20} \times 45.$ |
| 2. $2\frac{1}{4} \times 16.$ | 10. $4\frac{3}{10} \times 20.$ | 18. $10\frac{5}{8} \times 30.$ |
| 3. $3\frac{1}{2} \times 22.$ | 11. $4\frac{7}{12} \times 36.$ | 19. $10\frac{3}{22} \times 33.$ |
| 4. $4\frac{3}{8} \times 40.$ | 12. $5\frac{6}{17} \times 34.$ | 20. $12\frac{3}{16} \times 48.$ |
| 5. $5\frac{7}{8} \times 56.$ | 13. $8\frac{8}{21} \times 42.$ | 21. $13\frac{3}{10} \times 50.$ |
| 6. $3\frac{2}{3} \times 45.$ | 14. $10\frac{5}{24} \times 60.$ | 22. $13\frac{3}{8} \times 56.$ |
| 7. $4\frac{7}{8} \times 32.$ | 15. $11\frac{3}{25} \times 50.$ | 23. $14\frac{5}{12} \times 60.$ |
| 8. $5\frac{9}{10} \times 22.$ | 16. $12\frac{5}{12} \times 72.$ | 24. $15\frac{4}{15} \times 75.$ |
| 25. $3\frac{1}{2} \times 100.$ | 30. $5\frac{5}{8} \times 120.$ | |
| 26. $4\frac{1}{3} \times 120.$ | 31. $6\frac{5}{8} \times 126.$ | |
| 27. $4\frac{2}{3} \times 135.$ | 32. $7\frac{3}{7} \times 126.$ | |
| 28. $5\frac{2}{3} \times 105.$ | 33. $8\frac{6}{7} \times 133.$ | |
| 29. $6\frac{3}{4} \times 132.$ | 34. $9\frac{5}{8} \times 144.$ | |

TO MULTIPLY A FRACTION BY A FRACTION .

When we multiply a fraction by a fraction we find a fractional part of a given fraction.

The upper oblong is divided into two equal parts.



Each of these parts is one half of the oblong.



In the lower oblong each of these two parts is divided into four equal parts.

Each of these parts is one eighth of the oblong.

Each of the parts in the lower oblong equals one fourth of each part in one half of the upper oblong.

That is: $\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{8}$, or $\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$.

ORAL PRACTICE

1. 

The line above is divided into 4 equal parts.

How much is $\frac{1}{2}$ of $\frac{1}{2}$?2. 

Suppose the same line is divided into 8 equal parts.

How much is $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{3}{4}$?3. 

Suppose the same line is divided into 16 equal parts.

How much is $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{8}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{4}$?4. 

On the line above show

 $\frac{1}{2}$ of $\frac{3}{4}$. $\frac{1}{3}$ of $\frac{3}{4}$. $\frac{1}{5}$ of $\frac{5}{8}$. $\frac{1}{7}$ of $\frac{7}{8}$.

5. Using the line above question 4,

How much is $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{2}{3}$ of $\frac{3}{8}$? $\frac{5}{6}$ of $\frac{3}{4}$?

6. Draw a straight line 24 inches long on the blackboard, and divide it into inch lengths.

Point out $\frac{1}{12}$ of the line. $\frac{1}{6}$ of it. $\frac{1}{4}$ of it. $\frac{1}{3}$ of it.7. Using the line to prove to the class that you are right, find $\frac{1}{2}$ of $\frac{1}{12}$. $\frac{1}{4}$ of $\frac{1}{6}$. $\frac{1}{3}$ of $\frac{5}{8}$. $\frac{3}{4}$ of $\frac{1}{8}$. $\frac{1}{8}$ of $\frac{3}{4}$. $\frac{5}{6}$ of $\frac{3}{4}$. $\frac{3}{4}$ of $\frac{5}{8}$.

To multiply a fraction by a fraction :

*Multiply the numerators for the required numerator.**Multiply the denominators for the required denominator.**Cancel in the process if possible.*

WRITTEN PRACTICE

Multiply :

1. $\frac{2}{3} \times \frac{3}{4}$.

4. $\frac{2}{5} \times \frac{5}{6}$.

7. $\frac{3}{5} \times \frac{7}{12}$.

10. $\frac{5}{6} \times \frac{12}{5}$.

2. $\frac{2}{3} \times \frac{3}{8}$.

5. $\frac{3}{4} \times \frac{8}{9}$.

8. $\frac{4}{5} \times \frac{5}{14}$.

11. $\frac{5}{7} \times \frac{14}{15}$.

3. $\frac{2}{3} \times \frac{3}{5}$.

6. $\frac{3}{4} \times \frac{8}{15}$.

9. $\frac{4}{5} \times \frac{17}{16}$.

12. $\frac{6}{7} \times \frac{14}{15}$.

13. $\frac{3}{4} \times \frac{8}{9}$.

20. $\frac{5}{6} \times \frac{42}{55}$.

27. $\frac{7}{8} \times \frac{12}{25}$.

34. $\frac{19}{21} \times \frac{66}{95}$.

14. $\frac{3}{4} \times \frac{8}{15}$.

21. $\frac{6}{7} \times \frac{49}{54}$.

28. $\frac{8}{9} \times \frac{27}{40}$.

35. $\frac{26}{27} \times \frac{54}{65}$.

15. $\frac{3}{4} \times \frac{8}{21}$.

22. $\frac{6}{7} \times \frac{35}{66}$.

29. $\frac{9}{10} \times \frac{28}{33}$.

36. $\frac{23}{33} \times \frac{77}{92}$.

16. $\frac{3}{5} \times \frac{10}{21}$.

23. $\frac{7}{8} \times \frac{64}{63}$.

30. $\frac{12}{13} \times \frac{39}{48}$.

37. $\frac{8}{9} \times \frac{15}{22}$.

17. $\frac{4}{5} \times \frac{15}{28}$.

24. $\frac{7}{8} \times \frac{72}{49}$.

31. $\frac{14}{15} \times \frac{35}{36}$.

38. $\frac{9}{10} \times \frac{25}{27}$.

18. $\frac{4}{5} \times \frac{25}{28}$.

25. $\frac{4}{5} \times \frac{15}{16}$.

32. $\frac{16}{17} \times \frac{51}{80}$.

39. $\frac{13}{14} \times \frac{28}{39}$.

19. $\frac{5}{6} \times \frac{36}{35}$.

26. $\frac{5}{6} \times \frac{12}{25}$.

33. $\frac{13}{19} \times \frac{57}{90}$.

40. $\frac{15}{17} \times \frac{34}{50}$.

EVERYDAY USES FOR MULTIPLICATION OF FRACTIONS



1. An aviator flew from one city to another in $5\frac{1}{4}$ hours. At the same rate, how many hours should he require to fly $\frac{5}{8}$ of that distance? *2 $\frac{1}{2}$ hr*

2. Frank's baseball bat cost $\frac{3}{5}$ as much as his glove. If his glove cost \$2.75, how much did his bat cost? *1.65*

3. William's baseball team made 27 runs in a series of games, and Henry's team made $\frac{7}{9}$ as many runs. How many runs did Henry's team make? *21*

4. Jack made a "home run" in just 24 seconds, and Tom made one in three fourths of that time. How many seconds did Tom take to make his home run?

5. In April, 1916, an aviator flew to the height of 16,000 feet, and one month later another aviator flew $1\frac{1}{2}$ times as high. How high did the second aviator fly?

6. Each of 450 bags of flour contains $12\frac{1}{2}$ pounds. How many pounds of flour were required to fill these bags?

7. A house lot contains $\frac{4}{5}$ of an acre of land, and $\frac{3}{8}$ of this house lot is a garden. What part of an acre of land is there in the garden? $\frac{4}{5} \times \frac{3}{8} = \frac{3}{10}$

8. It takes a trolley car $\frac{3}{4}$ of an hour to run the distance between two towns. If Mr. Benton lives $\frac{3}{5}$ of the distance from one of these towns, how long should it take him to go to that town by this trolley line?

9. In covering a chair seat an upholsterer uses $\frac{7}{8}$ of a yard of tapestry, and for the back of the chair he uses $\frac{2}{3}$ as much as he does for the seat. How much tapestry does he use for both the back and the seat? $\frac{7}{8} + \frac{2}{3} \times \frac{7}{8} = \frac{17}{12}$

10. Margaret lives $\frac{3}{4}$ of a mile from the schoolhouse and Florence lives $\frac{1}{3}$ as far from the schoolhouse as Margaret. What fraction of a mile is the distance from Florence's home to the schoolhouse? $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

11. A contractor received $\$2\frac{1}{2}$ a yard for digging a ditch for a sewer. If the length of the ditch was $\frac{3}{4}$ of a mile, how much did he receive for the work? (How many yards are there in a mile?) $8\frac{3}{4} \times 1760 = \$15,120$

12. For each of the $5\frac{1}{2}$ working days in a week a laborer receives 24 cents an hour. If his working day is $8\frac{1}{2}$ hours long, how much does he receive per day? How much does he receive in a week of $5\frac{1}{2}$ working days?

13. $15\frac{3}{5}$ tons of coal were required to heat a schoolhouse during the month of December. If this coal cost the district $\$5\frac{1}{4}$ per ton delivered, how much was the expense of heating the building during the month?

14. A contractor employs 120 men at $\$24\frac{1}{2}$ a day, and 84 men at $\$31\frac{1}{4}$ a day. What is the total amount paid by the contractor to all of these men in a week of 6 working days?

15. A wholesale dealer in coal sold $410\frac{1}{2}$ tons of coal at $\$51\frac{1}{5}$ per ton, and $115\frac{3}{4}$ tons of a smaller size coal at $\$41\frac{1}{4}$ per ton. What was the total amount that the dealer received for all of the coal?

DIVISION OF FRACTIONS

TO DIVIDE A FRACTION BY A WHOLE NUMBER

ORAL PRACTICE

1. 8 inches divided by 2 equals how many inches? 8 dollars divided by 2 equals how many dollars? $\frac{8}{15}$ divided by 2 equals how many 15ths?

2. (a) $\frac{6}{7} \div 2 = \frac{?}{7}$. (c) $\frac{9}{10} \div 3 = \frac{?}{10}$. (e) $\frac{15}{16} \div 3 = \frac{?}{16}$.

(b) $\frac{8}{15} \div 4 = \frac{?}{15}$. (d) $\frac{15}{16} \div 5 = \frac{?}{16}$. (f) $\frac{21}{25} \div 7 = \frac{?}{25}$.

3. In example 2 you divided each of the fractions by a whole number. Which *term* of each fraction did you divide by the whole number?

4. Find on your ruler the point that marks $\frac{5}{8}$ of the first inch. How much is $\frac{5}{8} \div 2$? Instead of trying to tell how many 8ths result when $\frac{5}{8}$ are divided by 2, you read $\frac{5}{16}$. Therefore, you think, $\frac{5}{8} \div 2 = \frac{1}{2}$ of $\frac{5}{8} =$ What?

5. Give the following results:

(a) $\frac{3}{5} \div 2 = \frac{3}{10}$. (d) $\frac{5}{12} \div 3 = \frac{5}{?}$. (g) $\frac{5}{16} \div 4 = ?$

(b) $\frac{3}{10} \div 2 = \frac{3}{?}$. (e) $\frac{7}{10} \div 4 = \frac{7}{?}$. (h) $\frac{8}{15} \div 3 = ?$

(c) $\frac{3}{25} \div 2 = \frac{3}{?}$. (f) $\frac{9}{10} \div 2 = \frac{9}{?}$. (i) $\frac{7}{20} \div 4 = ?$

1. Divide $\frac{8}{15}$ by 2.

2. Divide $\frac{7}{15}$ by 3.

$$\frac{8}{15} \div 2 = \frac{8 \div 2}{15} = \frac{4}{15} \text{ Result.}$$

$$\frac{7}{15} \div 3 = \frac{1}{3} \text{ of } \frac{7}{15} = \frac{1 \times 7}{3 \times 15} = \frac{7}{45} \text{ Result.}$$

3. Divide $\frac{5}{18}$ by 10.

$$\frac{5}{18} \div 10 = \frac{1}{10} \text{ of } \frac{5}{18} = \frac{1 \times 5}{10 \times 18} = \frac{1 \times \cancel{5}}{\cancel{10} \times 18} = \frac{1}{36} \quad \text{Result.}$$

2

To divide a fraction by a whole number :

*Divide the numerator of the fraction by the whole number, or
Multiply the denominator of the fraction by the whole number.*

WRITTEN PRACTICE

Divide :

- | | | |
|------------------------------|------------------------------|------------------------------|
| 1. $\frac{5}{12} \div 5.$ | 11. $\frac{10}{11} \div 5.$ | 21. $\frac{32}{35} \div 16.$ |
| 2. $\frac{8}{15} \div 4.$ | 12. $\frac{15}{16} \div 5.$ | 22. $\frac{20}{27} \div 10.$ |
| 3. $\frac{9}{20} \div 3.$ | 13. $\frac{15}{17} \div 3.$ | 23. $\frac{25}{26} \div 15.$ |
| 4. $\frac{9}{10} \div 6.$ | 14. $\frac{16}{19} \div 4.$ | 24. $\frac{15}{28} \div 30.$ |
| 5. $\frac{9}{20} \div 12.$ | 15. $\frac{16}{27} \div 10.$ | 25. $\frac{16}{21} \div 12.$ |
| 6. $\frac{7}{15} \div 14.$ | 16. $\frac{20}{27} \div 12.$ | 26. $\frac{33}{14} \div 22.$ |
| 7. $\frac{8}{25} \div 12.$ | 17. $\frac{18}{25} \div 8.$ | 27. $\frac{39}{10} \div 13.$ |
| 8. $\frac{9}{16} \div 5.$ | 18. $\frac{24}{25} \div 10.$ | 28. $\frac{45}{49} \div 30.$ |
| 9. $\frac{11}{12} \div 4.$ | 19. $\frac{24}{25} \div 12.$ | 29. $\frac{35}{44} \div 28.$ |
| 10. $\frac{15}{16} \div 10.$ | 20. $\frac{25}{32} \div 15.$ | 30. $\frac{54}{65} \div 72.$ |

TO DIVIDE A FRACTION BY A FRACTION

ORAL PRACTICE

1. If you divide \$16 into portions of \$8 each, you have as many portions as $16 \div 8$, or 2. In the same way the fraction $\frac{16}{17} \div \frac{8}{7} = 2$. Similarly, $\frac{9}{10} \div \frac{3}{10} = 3$. $\frac{15}{16} \div \frac{5}{16} = 3$.

2. How would your final results compare if, instead of dividing as above in the case of $\frac{15}{16} \div \frac{5}{16}$, you had written $\frac{15}{16} \times \frac{16}{5}$, and then reduced the work by cancellation?

3. Would it change the results in a , b , and c , below, if, instead of dividing, you should invert each divisor and multiply?

$$(a) \frac{10}{21} \div \frac{5}{21} = \frac{10}{21} \times \frac{21}{5}.$$

$$(b) \frac{7}{18} \div \frac{5}{18} = \frac{7}{18} \times \frac{18}{5}.$$

$$(c) \frac{11}{32} \div \frac{11}{64} = \frac{11}{32} \times \frac{64}{11}.$$

4. You have found that in dividing a fraction by a fraction, you may invert the divisor and multiply the other fraction by it. Use this principle in each of the following:

$$(a) \frac{5}{8} \div \frac{5}{4} = ?$$

$$(c) \frac{7}{10} \div \frac{14}{15} = ?$$

$$(e) \frac{8}{9} \div \frac{2}{3} = ?$$

$$(b) \frac{8}{15} \div \frac{4}{5} = ?$$

$$(d) \frac{1}{20} \div \frac{1}{15} = ?$$

$$(f) \frac{5}{12} \div \frac{5}{4} = ?$$

5. Divide $\frac{3}{4}$ by $\frac{2}{5}$.

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{3 \times 5}{4 \times 2} = \frac{15}{8} = 1\frac{7}{8}. \quad \text{Result.}$$

6. Divide $\frac{8}{15}$ by $\frac{2}{3}$.

$$\frac{8}{15} \div \frac{2}{3} = \frac{8}{15} \times \frac{3}{2} = \frac{8 \times 3}{15 \times 2} = \frac{\overset{4}{\cancel{8}} \times \overset{4}{\cancel{3}}}{\cancel{15} \times \underset{5}{\cancel{2}}} = \frac{4}{5}. \quad \text{Result.}$$

7. Divide $2\frac{1}{5}$ by $5\frac{1}{4}$.

Changing to fractions: $2\frac{1}{5} = \frac{11}{5}$ and $5\frac{1}{4} = \frac{21}{4}$.

Inverting and multiplying

$$\frac{11}{5} \div \frac{21}{4} = \frac{11}{5} \times \frac{4}{21} = \frac{11 \times 4}{5 \times 21} = \frac{\overset{2}{\cancel{11}} \times \overset{4}{\cancel{4}}}{5 \times \underset{3}{\cancel{21}}} = \frac{8}{15}. \quad \text{Result.}$$

To divide a fraction by a fraction:

Invert the divisor.

Multiply as in multiplication of fractions.

WRITTEN PRACTICE

Find the quotient of:

1. $\frac{3}{4} \div \frac{3}{8}.$

3. $\frac{4}{5} \div \frac{2}{3}.$

5. $\frac{3}{4} \div \frac{5}{2}.$

2. $\frac{3}{5} \div \frac{2}{3}.$

4. $\frac{5}{8} \div \frac{3}{4}.$

6. $\frac{3}{8} \div \frac{1}{8}.$

- | | | |
|---------------------------------------|--|---|
| 7. $\frac{4}{7} \div \frac{2}{3}$. | 15. $\frac{5}{12} \div \frac{3}{8}$. | 23. $\frac{5}{12} \div \frac{3}{4}$. |
| 8. $\frac{5}{8} \div \frac{5}{6}$. | 16. $\frac{7}{12} \div \frac{1}{4}$. | 24. $\frac{8}{11} \div \frac{4}{5}$. |
| 9. $\frac{8}{9} \div \frac{2}{3}$. | 17. $\frac{5}{12} \div \frac{5}{8}$. | 25. $\frac{9}{16} \div \frac{3}{8}$. |
| 10. $\frac{7}{8} \div \frac{5}{6}$. | 18. $\frac{8}{15} \div \frac{4}{5}$. | 26. $\frac{11}{12} \div \frac{5}{6}$. |
| 11. $\frac{3}{8} \div \frac{1}{3}$. | 19. $\frac{9}{16} \div \frac{7}{8}$. | 27. $\frac{7}{24} \div \frac{5}{8}$. |
| 12. $\frac{5}{8} \div \frac{1}{2}$. | 20. $\frac{8}{15} \div \frac{3}{10}$. | 28. $\frac{9}{16} \div \frac{3}{4}$. |
| 13. $\frac{7}{9} \div \frac{2}{3}$. | 21. $\frac{3}{10} \div \frac{4}{5}$. | 29. $\frac{12}{25} \div \frac{9}{10}$. |
| 14. $\frac{9}{10} \div \frac{3}{5}$. | 22. $\frac{5}{12} \div \frac{2}{3}$. | 30. $\frac{14}{15} \div \frac{7}{12}$. |

Divide:

- | | | |
|---------------------------------------|---|---|
| 31. $2\frac{1}{2}$ by $\frac{5}{8}$. | 37. $4\frac{1}{2}$ by $\frac{3}{4}$. | 43. $3\frac{3}{4}$ by $3\frac{5}{8}$. |
| 32. $3\frac{1}{2}$ by $\frac{7}{8}$. | 38. $4\frac{1}{4}$ by $\frac{1}{4}$. | 44. $3\frac{1}{5}$ by $1\frac{1}{5}$. |
| 33. $2\frac{1}{4}$ by $\frac{6}{7}$. | 39. $5\frac{1}{3}$ by $\frac{4}{5}$. | 45. $5\frac{2}{3}$ by $3\frac{1}{3}$. |
| 34. $3\frac{1}{3}$ by $\frac{5}{8}$. | 40. $4\frac{2}{3}$ by $\frac{7}{9}$. | 46. $6\frac{3}{4}$ by $4\frac{1}{2}$. |
| 35. $3\frac{1}{2}$ by $\frac{2}{7}$. | 41. $6\frac{1}{4}$ by $\frac{5}{8}$. | 47. $7\frac{1}{3}$ by $1\frac{5}{6}$. |
| 36. $3\frac{1}{2}$ by $\frac{2}{3}$. | 42. $7\frac{1}{3}$ by $2\frac{1}{15}$. | 48. $8\frac{2}{5}$ by $2\frac{1}{10}$. |

TO DIVIDE A WHOLE NUMBER BY A FRACTION

1. Divide 12 by $\frac{3}{4}$.

As we have already learned to invert a fraction which is a divisor, and to use it as a multiplier, we invert the fraction $\frac{3}{4}$. Then:

$$12 \div \frac{3}{4} = 12 \times \frac{4}{3} = \frac{12 \times 4}{3} = 16. \quad \text{Result.}$$

2. Divide 15 by $\frac{4}{5}$.

$$15 \div \frac{4}{5} = 15 \times \frac{5}{4} = \frac{15 \times 5}{4} = 18\frac{3}{4}. \quad \text{Result.}$$

If the divisor is a mixed number, change it to a fraction.

3. Divide 12 by $2\frac{1}{4}$.

$$12 \div 2\frac{1}{4} = 12 \div \frac{9}{4} = 12 \times \frac{4}{9} = \frac{12 \times 4}{9} = \frac{16}{3} = 5\frac{1}{3}. \quad \text{Result.}$$

WRITTEN PRACTICE

Divide:

- | | | |
|---------------------------|-----------------------------|-----------------------------|
| 1. 4 by $\frac{2}{3}$. | 17. 64 by $\frac{7}{8}$. | 33. 12 by $6\frac{1}{2}$. |
| 2. 5 by $\frac{3}{4}$. | 18. 72 by $\frac{8}{9}$. | 34. 15 by $7\frac{1}{3}$. |
| 3. 6 by $\frac{3}{5}$. | 19. 84 by $\frac{7}{9}$. | 35. 17 by $5\frac{1}{8}$. |
| 4. 7 by $\frac{4}{5}$. | 20. 96 by $\frac{8}{9}$. | 36. 18 by $10\frac{1}{2}$. |
| 5. 8 by $\frac{7}{8}$. | 21. 84 by $\frac{7}{10}$. | 37. 20 by $3\frac{1}{3}$. |
| 6. 9 by $\frac{5}{9}$. | 22. 90 by $\frac{1}{16}$. | 38. 20 by $3\frac{3}{4}$. |
| 7. 10 by $\frac{6}{7}$. | 23. 96 by $\frac{1}{25}$. | 39. 20 by $6\frac{2}{3}$. |
| 8. 11 by $\frac{7}{9}$. | 24. 100 by $\frac{1}{16}$. | 40. 21 by $7\frac{1}{2}$. |
| 9. 12 by $\frac{8}{9}$. | 25. 4 by $3\frac{1}{2}$. | 41. 24 by $1\frac{5}{7}$. |
| 10. 15 by $\frac{5}{8}$. | 26. 4 by $3\frac{1}{4}$. | 42. 25 by $3\frac{4}{7}$. |
| 11. 16 by $\frac{4}{5}$. | 27. 5 by $4\frac{1}{3}$. | 43. 28 by $5\frac{3}{5}$. |
| 12. 18 by $\frac{4}{7}$. | 28. 5 by $4\frac{1}{2}$. | 44. 30 by $7\frac{1}{2}$. |
| 13. 24 by $\frac{5}{6}$. | 29. 6 by $5\frac{1}{3}$. | 45. 32 by $5\frac{1}{3}$. |
| 14. 28 by $\frac{7}{8}$. | 30. 6 by $7\frac{1}{4}$. | 46. 35 by $4\frac{2}{3}$. |
| 15. 32 by $\frac{6}{7}$. | 31. 7 by $7\frac{1}{2}$. | 47. 40 by $5\frac{5}{7}$. |
| 16. 45 by $\frac{5}{8}$. | 32. 8 by $7\frac{1}{3}$. | 48. 45 by $4\frac{3}{5}$. |

EVERYDAY USES FOR DIVISION OF FRACTIONS

1. A grocer sells sugar in bags containing $2\frac{1}{2}$ pounds each. How many bags must he sell to sell 150 pounds of sugar? 60

2. In $1\frac{1}{2}$ hours a steam shovel removed enough dirt to load 54 wagons. How many wagons could this shovel load with dirt in 1 hour? 81

3. If the loads of coal hauled in a truck average $2\frac{1}{2}$ tons, how many such loads will it take to haul 65 tons of coal?

4. A gallon of maple sirup weighs $11\frac{1}{2}$ pounds. How many gallons are there in a tank of this sirup that contains 840 pounds? $73\frac{1}{2}$

5. From $24\frac{1}{2}$ acres a man raised a crop of 2940 bushels of potatoes. What was the average yield of potatoes per acre? *120*

6. If flour is selling at $\$10\frac{1}{2}$ per barrel, how many barrels of flour can be bought for $\$78$? *7 $\frac{3}{4}$*

7. In going a distance of 297 miles an automobile averaged $24\frac{3}{4}$ miles per hour. How many hours did it take it to go this distance? *12*

8. $\$68\frac{3}{4}$ were paid for $13\frac{3}{4}$ tons of soft coal, and the freight on the coal was $\$4$. Give the cost of the coal per ton, including the freight. *5 $\frac{1}{4}$*

9. A field that contained $7\frac{1}{2}$ acres produced 174 bushels of grain. What was the average number of bushels produced per acre in this field? *23 $\frac{1}{2}$*

10. Fourteen and one half tons of hay cost a liveryman one hundred seventy-four dollars. How much did one ton of this hay cost him? *12*

11. A barrel holds $31\frac{1}{2}$ gallons, and a hogshead holds 63 gallons. How many barrels of molasses are there in 10 hogsheads of molasses? *20*

12. A merchant bought silk at $\$2\frac{3}{4}$ per yard, and the number of yards that he bought cost him a total of $\$316\frac{1}{4}$. How many yards of silk did he buy? *155*

13. The distance around the wheel of an automobile, called the circumference of the wheel, is $9\frac{1}{2}$ feet. How many times does this wheel turn around while running 1300 feet? *130 $\frac{2}{3}$*

14. A dealer in athletic supplies bought a lot of sweaters for $\$550$. If the cost of each sweater was $\$2\frac{1}{2}$, how many sweaters did he buy? *220*

FRACTIONAL RELATIONS

TO FIND WHAT FRACTIONAL PART ONE NUMBER IS OF
ANOTHER NUMBER

Into how many parts is this rectangle divided?

How many of the parts are

shaded?



What part of 9 is 4?

4 is what part of 15? Four of fifteen parts. $4 \times \frac{1}{15} = \frac{4}{15}$.

7 is what part of 12? Seven of twelve parts. $7 \times \frac{1}{12} = \frac{7}{12}$.

8 is what part of 20? Eight of twenty parts. $8 \times \frac{1}{20} =$

$$\frac{8}{20} = \frac{2}{5}.$$

ORAL PRACTICE

What part of

1. 4 is 2?

6. 10 is 2?

11. 18 is 3?

2. 8 is 4?

7. 12 is 3?

12. 20 is 5?

3. 6 is 3?

8. 12 is 4?

13. 24 is 6?

4. 9 is 3?

9. 15 is 5?

14. 24 is 8?

5. 8 is 2?

10. 18 is 6?

15. 25 is 5?

What part of

16. 3 feet is 2 feet?

24. 12 pounds is 6 pounds?

17. 5 feet is 3 feet?

25. 12 pounds is 9 pounds?

18. 8 feet is 5 feet?

26. 15 yards is 5 yards?

19. 10 feet is 7 feet?

27. 16 rods is 4 rods?

20. 15 feet is 8 feet?

28. 18 rods is 6 rods?

21. 4 pounds is 3 pounds?

29. 20 tons is 6 tons?

22. 5 pounds is 2 pounds?

30. 27 tons is 9 tons?

23. 8 pounds is 4 pounds?

31. 35 miles is 7 miles?

WRITTEN PRACTICE

What part of

- | | | |
|--------------|---------------|---------------|
| 1. 25 is 20? | 7. 42 is 14? | 13. 40 is 15? |
| 2. 32 is 16? | 8. 40 is 12? | 14. 54 is 39? |
| 3. 30 is 20? | 9. 45 is 30? | 15. 65 is 26? |
| 4. 32 is 20? | 10. 54 is 36? | 16. 75 is 35? |
| 5. 36 is 21? | 11. 56 is 24? | 17. 80 is 45? |
| 6. 39 is 13? | 12. 63 is 27? | 18. 90 is 54? |

19. By working an hour Marion solved 20 of the 32 problems in her lesson. What fractional part of the 32 problems had she not solved? $\frac{5}{8}$

20. From a crop of 135 bushels of fine potatoes a farmer saved 27 bushels for seed. What fractional part of this whole crop did he save? $\frac{27}{135} = \frac{1}{5}$

21. If a farmer sprayed 45 trees in one day, and his orchard contained 150 trees, what fractional part of the whole number of trees did he spray that day? $\frac{45}{150} = \frac{1}{3}$

22. A contractor finished paving 75 rods of a street that was 120 rods long. What fractional part of the whole street had he completed? What fractional part of the street remained to be finished? $\frac{75}{120} = \frac{5}{8}$

23. A car loaded with coal weighed 120,000 pounds. If the car weighed 20,000 pounds, what fractional part of the weight of the coal was the weight of the car? $\frac{1}{5}$

24. An automobile cost \$2400, and the owner paid \$100 more for wire wheels and \$50 for slip covers. What fraction of the cost of the automobile is the total cost of the wire wheels and slip covers? $\frac{1}{24}$

25. The distance from New York to Chicago is about 900 miles, and from New York to San Francisco about 3200 miles. What fraction of the distance from New York to San Francisco is the distance from New York to Chicago? $\frac{9}{32}$

TO FIND A NUMBER OF WHICH A GIVEN NUMBER IS A FRACTIONAL PART

Illustration :

Find the number of which 30 is $\frac{3}{4}$.

Since 30 is $\frac{3}{4}$ of some number,

Then $\frac{1}{4}$ of that number is $\frac{1}{3}$ of 30, or 10.

And $\frac{4}{4}$ of that number is 4 times 10, or 40.

That is, 40 is the number of which the given number, 30, is $\frac{3}{4}$.

WRITTEN PRACTICE

Find the number of which

- | | | |
|----------------------------|-----------------------------|-----------------------------|
| 1. 20 is $\frac{2}{3}$. | 11. 70 is $\frac{1}{10}$. | 21. 60 is $\frac{1}{2}$. |
| 2. 25 is $\frac{5}{7}$. | 12. 85 is $\frac{1}{20}$. | 22. 64 is $\frac{1}{25}$. |
| 3. 28 is $\frac{4}{5}$. | 13. 42 is $\frac{1}{15}$. | 23. 81 is $\frac{2}{9}$. |
| 4. 32 is $\frac{8}{9}$. | 14. 57 is $\frac{1}{10}$. | 24. 87 is $\frac{2}{10}$. |
| 5. 48 is $\frac{6}{7}$. | 15. 75 is $\frac{1}{25}$. | 25. 96 is $\frac{3}{8}$. |
| 6. 60 is $\frac{5}{9}$. | 16. 110 is $\frac{2}{27}$. | 26. 105 is $\frac{3}{48}$. |
| 7. 27 is $\frac{3}{8}$. | 17. 125 is $\frac{2}{5}$. | 27. 120 is $\frac{2}{3}$. |
| 8. 35 is $\frac{7}{11}$. | 18. 160 is $\frac{3}{5}$. | 28. 144 is $\frac{3}{5}$. |
| 9. 45 is $\frac{9}{13}$. | 19. 140 is $\frac{2}{31}$. | 29. 165 is $\frac{3}{50}$. |
| 10. 56 is $\frac{8}{15}$. | 20. 150 is $\frac{2}{27}$. | 30. 175 is $\frac{3}{48}$. |

USING FRACTIONAL RELATIONS IN PROBLEMS

Illustration :

A baseball team played 35 games and won 20 of them. What fractional part of the whole number of games played is the number of games won?

The question is : "What fractional part of 35 is 20?" 20 is $\frac{20}{35}$ of 35. That is, $\frac{20}{35}$, or $\frac{4}{7}$.

Therefore, the fractional part of games won is $\frac{4}{7}$. *Result.*

EVERYDAY USES FOR FRACTIONAL RELATIONS IN PROBLEMS

1. A baseball team played 24 games in a season and won 18 of them. What fractional part of the whole number of games played is the number of games won?

2. In a room of 45 pupils, 5 pupils failed to be promoted to the next higher grade. What fractional part of the whole number of pupils failed to be promoted?

3. A postman passes 54 doors on a city block and leaves mail at 36 of them. What fractional part of the whole number of doors is the number at which he left mail?

4. 18 of the 48 pupils in a schoolroom are boys. What fractional part of the whole number of pupils are girls?

5. A freight train is made up of 36 cars; 8 of them are flat cars and the others are box cars. What fractional part of the whole number of cars are the flat cars? What fractional part of the whole number of cars are the box cars?

6. In a town election 140 votes were cast, and the winning candidate received 84 of this number. What fractional part of the whole number of votes did he receive?

7. The cost of Mr. Merriman's dining room furniture was \$250, and the buffet alone cost \$75. What fractional part of the whole sum was paid for the buffet?

8. A man bought a second-hand automobile for \$400, and then bought four new tires at \$20 each. What fractional part of the whole cost was paid for the tires?

Illustration:

A baseball team won 12 games, which was $\frac{2}{3}$ of the number they played. How many games did they play?

Since 12 is $\frac{2}{3}$ of the number of games played,

Then $\frac{1}{3}$ of the number is $\frac{1}{3}$ of 12 games, or 4 games.

And $\frac{5}{3}$, or the number of games played, is 5×4 , or 20. *Result.*

9. A baseball team won 16 games, which was $\frac{2}{3}$ of the number of games they played. How many games did they play? *24*

10. John drove his father's automobile 45 miles, which was $\frac{3}{8}$ of the number of miles that his father drove it. How many miles did his father drive it?

11. At a Boy Scouts' contest 45 boys passed every test, and this number was $\frac{3}{7}$ of the number of boys taking part in this contest. How many boys took part in this contest?

12. A farmer sprayed 42 apple trees in a day, and this number was $\frac{7}{25}$ of the whole number of trees in his orchard. How many trees were there in the orchard? *150*

13. A man pays \$45 a month for house rent, and this amount is $\frac{3}{20}$ of his annual salary. What is his annual salary?

14. A young man saved $\frac{3}{11}$ of his salary in 1918. If the amount he saved was \$600, what was the amount of his salary that year? *\$2200*

15. A farmer paid \$800 down when he purchased a woodlot, and this sum was $\frac{5}{14}$ of the cost of the lot. How many dollars must he still pay in order to pay for it in full?

16. An ambitious young man earned \$1260 of the sum he paid for his expenses while at college. If this amount is $\frac{3}{8}$ of the whole expense of his college course, what did the course cost him? *\$2160*

17. A man promised to give his son when he should be twenty-five years old $2\frac{1}{2}$ times as much as the son should save from his own earnings up to that time. If the father gave him \$3600, how much had the young man saved?

18. A man left a clause in his will giving $\frac{5}{11}$ of his property to the college from which he was graduated, and the college received \$20,000. If the rest of his property went to his family, what amount should the family receive?

GENERAL REVIEW OF FRACTIONS

ORAL PRACTICE

Reduce to lowest terms :

1. $\frac{2}{6}$.

5. $\frac{6}{9}$.

9. $\frac{4}{12}$.

13. $\frac{10}{15}$.

2. $\frac{2}{8}$.

6. $\frac{6}{15}$.

10. $\frac{5}{20}$.

14. $\frac{20}{25}$.

3. $\frac{2}{10}$.

7. $\frac{9}{15}$.

11. $\frac{8}{18}$.

15. $\frac{12}{27}$.

4. $\frac{4}{14}$.

8. $\frac{9}{12}$.

12. $\frac{9}{24}$.

16. $\frac{14}{30}$.

Give the sum of :

17. $\frac{1}{4} + \frac{1}{2}$,

21. $\frac{1}{3} + \frac{2}{9}$.

25. $\frac{1}{2} + \frac{1}{3}$.

29. $\frac{1}{3} + \frac{1}{5}$.

18. $\frac{1}{6} + \frac{1}{3}$.

22. $\frac{1}{10} + \frac{2}{5}$.

26. $\frac{1}{3} + \frac{1}{4}$.

30. $\frac{1}{2} + \frac{1}{7}$.

19. $\frac{1}{8} + \frac{1}{4}$.

23. $\frac{3}{5} + \frac{1}{10}$.

27. $\frac{1}{4} + \frac{1}{5}$.

31. $\frac{1}{5} + \frac{1}{4}$.

20. $\frac{1}{10} + \frac{1}{5}$.

24. $\frac{3}{4} + \frac{1}{2}$.

28. $\frac{1}{2} + \frac{1}{5}$.

32. $\frac{1}{5} + \frac{1}{6}$.

Give the difference :

33. $\frac{3}{4} - \frac{1}{4}$.

37. $\frac{1}{2} - \frac{1}{4}$.

41. $\frac{3}{4} - \frac{3}{8}$.

45. $\frac{7}{8} - \frac{3}{4}$.

34. $\frac{5}{6} - \frac{1}{6}$.

38. $\frac{1}{3} - \frac{1}{6}$.

42. $\frac{2}{5} - \frac{1}{10}$.

46. $\frac{5}{9} - \frac{1}{3}$.

35. $\frac{3}{8} - \frac{1}{8}$.

39. $\frac{1}{2} - \frac{1}{8}$.

43. $\frac{3}{8} - \frac{1}{4}$.

47. $\frac{7}{15} - \frac{1}{3}$.

36. $\frac{7}{10} - \frac{3}{10}$.

40. $\frac{1}{2} - \frac{1}{10}$.

44. $\frac{5}{8} - \frac{1}{4}$.

48. $\frac{13}{18} - \frac{2}{3}$.

Give the product of :

49. $3 \times \frac{1}{8}$.

53. $\frac{2}{3} \times 5$.

57. $\frac{2}{3} \times \frac{1}{5}$.

61. $\frac{3}{4} \times \frac{5}{8}$.

50. $4 \times \frac{3}{7}$.

54. $\frac{3}{4} \times 16$.

58. $\frac{3}{4} \times \frac{5}{8}$.

62. $\frac{5}{8} \times \frac{4}{7}$.

51. $5 \times \frac{2}{7}$.

55. $\frac{5}{8} \times 12$.

59. $\frac{5}{8} \times \frac{3}{7}$.

63. $\frac{6}{7} \times \frac{7}{12}$.

52. $3 \times \frac{8}{9}$.

56. $\frac{3}{10} \times 15$.

60. $\frac{7}{9} \times \frac{2}{3}$.

64. $\frac{9}{10} \times \frac{5}{18}$.

Give the quotient of :

65. $\frac{5}{8} \div 5$.

69. $\frac{1}{3} \div 2$.

73. $\frac{1}{2} \div \frac{1}{4}$.

77. $\frac{2}{3} \div \frac{1}{3}$.

66. $\frac{6}{7} \div 2$.

70. $\frac{1}{4} \div 3$.

74. $\frac{3}{4} \div \frac{1}{4}$.

78. $\frac{3}{5} \div \frac{1}{10}$.

67. $\frac{3}{10} \div 3$.

71. $\frac{1}{5} \div 4$.

75. $\frac{1}{2} \div \frac{1}{8}$.

79. $\frac{4}{5} \div \frac{7}{20}$.

68. $\frac{15}{16} \div 3$.

72. $\frac{1}{6} \div 5$.

76. $\frac{3}{4} \div \frac{1}{8}$.

80. $\frac{5}{8} \div \frac{5}{2}$.

WRITTEN PRACTICE

Reduce to lowest terms:

- | | | | |
|----------------------|----------------------|-----------------------|-------------------------|
| 1. $\frac{12}{21}$. | 5. $\frac{16}{24}$. | 9. $\frac{30}{63}$. | 13. $\frac{110}{125}$. |
| 2. $\frac{15}{27}$. | 6. $\frac{28}{42}$. | 10. $\frac{44}{77}$. | 14. $\frac{128}{144}$. |
| 3. $\frac{20}{28}$. | 7. $\frac{36}{54}$. | 11. $\frac{51}{66}$. | 15. $\frac{156}{288}$. |
| 4. $\frac{32}{36}$. | 8. $\frac{54}{72}$. | 12. $\frac{94}{96}$. | 16. $\frac{168}{300}$. |

Change to improper fractions:

- | | | | |
|----------------------|-----------------------|-----------------------|------------------------|
| 17. $3\frac{2}{5}$. | 21. $7\frac{2}{3}$. | 25. $15\frac{7}{8}$. | 29. $15\frac{3}{16}$. |
| 18. $5\frac{4}{5}$. | 22. $9\frac{3}{4}$. | 26. $21\frac{8}{9}$. | 30. $16\frac{5}{12}$. |
| 19. $6\frac{3}{7}$. | 23. $10\frac{4}{5}$. | 27. $32\frac{8}{9}$. | 31. $18\frac{7}{15}$. |
| 20. $8\frac{5}{8}$. | 24. $13\frac{5}{6}$. | 28. $41\frac{7}{8}$. | 32. $20\frac{8}{21}$. |

Change to whole numbers or to mixed numbers:

- | | | | |
|----------------------|-----------------------|------------------------|------------------------|
| 33. $\frac{23}{5}$. | 37. $\frac{41}{12}$. | 41. $\frac{94}{15}$. | 45. $\frac{213}{25}$. |
| 34. $\frac{24}{7}$. | 38. $\frac{53}{15}$. | 42. $\frac{110}{13}$. | 46. $\frac{245}{36}$. |
| 35. $\frac{35}{8}$. | 39. $\frac{69}{17}$. | 43. $\frac{145}{18}$. | 47. $\frac{277}{10}$. |
| 36. $\frac{42}{9}$. | 40. $\frac{70}{23}$. | 44. $\frac{178}{25}$. | 48. $\frac{312}{65}$. |

Find the sum of:

- | | | |
|--|---|--|
| 49. $\frac{2}{3} + \frac{3}{4}$. | 53. $\frac{1}{4} + \frac{1}{3} + \frac{1}{2}$. | 57. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$. |
| 50. $\frac{3}{8} + \frac{5}{6}$. | 54. $\frac{2}{3} + \frac{3}{4} + \frac{5}{8}$. | 58. $\frac{2}{3} + \frac{3}{4} + \frac{1}{2} + \frac{5}{8}$. |
| 51. $\frac{5}{9} + \frac{5}{6}$. | 55. $\frac{4}{5} + \frac{3}{10} + \frac{7}{15}$. | 59. $\frac{5}{6} + \frac{7}{9} + \frac{1}{16} + \frac{6}{8}$. |
| 52. $\frac{7}{12} + \frac{3}{10}$. | 56. $\frac{5}{9} + \frac{1}{6} + \frac{7}{12}$. | 60. $\frac{3}{4} + \frac{7}{8} + \frac{5}{16} + \frac{2}{3}$. |
| 61. $\frac{2}{3} + 1 + \frac{3}{4}$. | 65. $\frac{2}{3} + 4\frac{1}{2} + \frac{3}{5} + 2$. | |
| 62. $\frac{3}{5} + 2 + \frac{5}{7}$. | 66. $\frac{5}{8} + 9 + \frac{2}{3} + 3\frac{5}{8}$. | |
| 63. $\frac{4}{9} + \frac{3}{4} + 10$. | 67. $5\frac{3}{4} + \frac{5}{8} + \frac{7}{12} + 3$. | |
| 64. $8 + \frac{5}{16} + \frac{3}{8}$. | 68. $8 + 7\frac{1}{3} + \frac{5}{9} + \frac{5}{12}$. | |

Find the difference:

- | | | | |
|-------------------------------------|---------------------------|--------------------------------------|--------------------------------------|
| 69. $\frac{7}{8} - \frac{3}{5}$. | 73. $8 - \frac{5}{6}$. | 77. $4\frac{7}{8} - \frac{3}{4}$. | 81. $3\frac{3}{4} - 2\frac{1}{3}$. |
| 70. $\frac{7}{8} - \frac{2}{3}$. | 74. $12 - \frac{3}{8}$. | 78. $8\frac{5}{6} - \frac{3}{8}$. | 82. $4\frac{5}{8} - 3\frac{5}{6}$. |
| 71. $\frac{8}{9} - \frac{3}{4}$. | 75. $21 - \frac{5}{9}$. | 79. $12\frac{8}{9} - \frac{3}{4}$. | 83. $10\frac{5}{6} - 4\frac{3}{4}$. |
| 72. $\frac{13}{15} - \frac{5}{6}$. | 76. $32 - \frac{7}{10}$. | 80. $15\frac{3}{10} - \frac{5}{6}$. | 84. $15\frac{3}{8} - 8\frac{4}{5}$. |

Multiply:

85. $\frac{5}{12}$ by 5. = $2\frac{1}{4}$ ✓
 86. $\frac{8}{21}$ by 3.
 87. $\frac{11}{48}$ by 8.
 88. $\frac{25}{64}$ by 24.
 89. $5\frac{3}{4}$ by 8.
 90. $8\frac{5}{8}$ by 9.
 91. $10\frac{2}{3}$ by 15.
 92. $15\frac{5}{8}$ by 12.
 93. 15 by $\frac{8}{9}$.
 94. 24 by $\frac{5}{16}$.
 95. 36 by $\frac{7}{12}$.
 96. 45 by $\frac{11}{30}$.
 97. 24 by $3\frac{7}{8}$.
 98. 30 by $5\frac{3}{8}$.
 99. 42 by $7\frac{5}{7}$.
 100. 54 by $12\frac{4}{9}$. = 67
 101. $\frac{8}{15}$ by $\frac{7}{12}$.
 102. $\frac{9}{10}$ by $\frac{5}{8}$.
 103. $\frac{8}{21}$ by $\frac{7}{16}$.
 104. $\frac{14}{25}$ by $\frac{5}{16}$.
 105. $\frac{17}{36}$ by $\frac{7}{34}$.
 106. $\frac{63}{50}$ by $\frac{25}{21}$.
 107. $\frac{2}{3}$ by $\frac{4}{5}$ by $\frac{9}{8}$.
 108. $\frac{3}{4}$ by $\frac{17}{27}$ by $\frac{9}{32}$.
 109. $\frac{6}{11}$ by $\frac{13}{21}$ by $\frac{7}{10}$.
 110. $\frac{9}{20}$ by $\frac{17}{17}$ by $\frac{5}{18}$.
 111. $\frac{12}{25}$ by $\frac{5}{36}$ by $\frac{15}{17}$.
 112. $\frac{11}{34}$ by $\frac{40}{33}$ by $\frac{51}{50}$.
 113. $\frac{3}{4}$ by $2\frac{1}{4}$ by $\frac{8}{15}$.
 114. $\frac{7}{8}$ by $3\frac{1}{4}$ by $\frac{16}{21}$.
 115. $\frac{9}{11}$ by $6\frac{2}{3}$ by $\frac{25}{45}$.
 116. $\frac{15}{28}$ by $3\frac{5}{9}$ by $\frac{21}{20}$.
 117. $\frac{27}{25}$ by $\frac{5}{18}$ by $13\frac{1}{3}$.
 118. $\frac{17}{18}$ by $14\frac{9}{7}$ by $\frac{15}{11}$.

Divide:

119. $\frac{5}{8}$ by 2.
 120. $\frac{5}{16}$ by 2.
 121. $\frac{2}{15}$ by 4.
 122. $\frac{1}{53}$ by 10.
 123. $\frac{7}{18}$ by 2.
 124. $\frac{4}{15}$ by 8.
 125. $6\frac{1}{4}$ by 5.
 126. $8\frac{1}{3}$ by 5.
 127. $9\frac{3}{5}$ by 6.
 128. $12\frac{1}{4}$ by 7.
 129. $13\frac{1}{3}$ by 10.
 130. $14\frac{2}{3}$ by 11.
 131. $\frac{5}{8}$ by $\frac{2}{3}$.
 132. $\frac{7}{8}$ by $\frac{5}{6}$.
 133. $\frac{9}{10}$ by $\frac{3}{5}$.
 134. $\frac{11}{16}$ by $\frac{3}{4}$.
 135. $\frac{11}{13}$ by $\frac{1}{9}$.
 136. $\frac{21}{25}$ by $\frac{7}{10}$.
 137. $3\frac{1}{2}$ by $2\frac{1}{3}$.
 138. $5\frac{1}{3}$ by $3\frac{2}{3}$.
 139. $7\frac{1}{2}$ by $3\frac{1}{3}$.
 140. $8\frac{1}{4}$ by $6\frac{2}{3}$.
 141. $16\frac{2}{9}$ by $10\frac{1}{9}$.
 142. $21\frac{1}{3}$ by $20\frac{1}{3}$.
 143. $24\frac{1}{2}$ by $16\frac{1}{3}$.
 144. $35\frac{2}{3}$ by $4\frac{1}{24}$.
 145. $40\frac{1}{2}$ by $6\frac{3}{4}$.
 146. $42\frac{2}{3}$ by $3\frac{5}{9}$.
 147. $48\frac{2}{5}$ by $4\frac{2}{5}$.
 148. $50\frac{1}{4}$ by $10\frac{1}{20}$.
 149. $51\frac{2}{3}$ by $5\frac{1}{6}$.
 150. $54\frac{2}{3}$ by $11\frac{5}{7}$.
 151. $60\frac{1}{4}$ by $3\frac{2}{25}$.
 152. $64\frac{1}{3}$ by $6\frac{7}{15}$.

EVERYDAY USES FOR FRACTIONS IN THE HOME



I. THE HOME GARDEN

1. Mary raised vegetables for which she received \$12.60. If she received $\frac{1}{4}$ of this sum for tomatoes, $\frac{5}{12}$ of it for sweet corn, and $\frac{1}{3}$ of it for string beans, how much did she receive for each of these crops?

2. Mary raised 240 tomatoes, and sold three tenths of them to a grocer. If the family ate the rest, find the number that the family consumed, and the number that she sold to the grocer.

3. Henry has a garden whose area is 1200 square feet. If two thirds of it is planted to potatoes, and one half of the remainder is planted to beets, find how many square feet of the garden is planted to each crop.

4. William raised 150 ears of corn which he sold for 22 cents per dozen, and $3\frac{1}{3}$ bushels of beans which he sold for \$4.50 per bushel. How much did he receive for both crops?

17.75

5. Tom's father raised 10 bushels of potatoes (60 pounds to the bushel), and stored them through the winter in dry sand. If they lost $\frac{1}{10}$ of their weight while in storage, how many pounds did they shrink? What part of a bushel of potatoes is this loss? *20 lbs. 1/10 bushel*

II. DAILY EXPENSES FOR FOOD

Much of the food used in a home must be purchased daily and in small quantities because of its perishable nature. In most homes arithmetic is used every day in connection with the purchase of food.

WRITTEN PRACTICE

1. A family used $2\frac{1}{2}$ quarts of milk daily during the month of March. At 11 cents a quart, what was their expense for milk in that month? *\$8.52 1/2*

2. A woman found that her expense for meat in the month of May was \$15.65. Find, to the nearest fraction of a cent, her daily expense for meat. *31/100*

3. A man pays \$5 a week for his meals. How much is this for each meal, if three meals are served to him daily?

4. Five persons sat down to a dinner. The meat for the dinner cost \$.90, the vegetables \$.65, the bread \$.12, the butter \$.20, and the dessert \$.58. What was the cost of the dinner per person? *55/100*

5. On the last day of June, 1918, a housekeeper found that the meat used by her family for the six months of that year had cost \$130.32. Find the actual number of days in that time and the average daily cost of the meat used by her family. Find, also, how much the meat bill should be in this family, at the same rate, for the months of September, October, and November.

130.32 - 30 days = 181 days. 781 7/10 130.32.

III. MONTHLY EXPENSES FOR RENT, GAS, ELECTRICITY, ETC.

In most cases the rent for a house or apartment is payable on the first of each month. The gas and the electric companies usually send bills on the first of each month, or at regular periods of three months. Telephone bills are sent out monthly.

WRITTEN PRACTICE

1. If a man rents a house for \$400 a year, how much is this per month? $33.3\overline{3}$

2. A family rented a house for \$350 a year. If the family remained in this house $3\frac{1}{2}$ years, what is the total amount of rent that should have been paid? \$1225

3. In three winter months last year a family used $12\frac{3}{4}$ thousand cubic feet of gas. If the gas cost \$1.10 per thousand cubic feet, find the amount the family spent for gas in that time. \$14.025

4. Mr. French paid \$32 last year for the use of his telephone. What was the average cost of the telephone per month expressed in dollars and a fraction of a dollar? $2\frac{2}{3}$

5. A family paid \$300 rent, \$31 for gas, and $\$22\frac{3}{4}$ for a telephone in one year. Find the average monthly expense of this family for these three items in dollars and a fraction of a dollar.

6. A family spent \$360 for rent and \$128 for gas and electricity in one year. If their annual income was \$2000, what fraction of it was spent for rent, and what fraction for gas and electricity?

7. A housekeeper spent \$50 each month for rent, and her average expenses for gas and electricity for the same period were \$3.50 and \$2.50 respectively. If her annual income was \$1400, what fraction of that sum did she spend for each of the three items in each month?

IV. ANNUAL EXPENSES FOR COAL, INSURANCE, TAXES, ETC.

Every person who owns a house or other property pays a part of the cost of running the government of the town and the state in which he lives. This sum is called a **Tax**, and is paid each year.

To protect himself from loss in case of fire a man may buy **Insurance** on his home. This sum is usually paid annually in advance. Many families also buy at one time sufficient coal or wood to last for a year. So taxes, insurance, and fuel are generally considered as "annual expenses."

WRITTEN PRACTICE

1. Mr. Johnson's taxes for the year 1917 were \$21. What was the average amount of his tax per month?

2. A man buys insurance to protect himself from loss on his house and also on his furniture. One policy costs him $\$15\frac{1}{4}$ a year, and the other $\$8\frac{1}{2}$ a year. What is the average monthly cost for both?

3. A man finds that his coal bill for a year is \$90. If he runs his heater exactly $8\frac{1}{2}$ months, what is the average monthly cost of heating his house? What is the average daily cost if he heats his house 258 days?

4. Mr. Emerson paid in 1918 as follows: Taxes, $\$31\frac{1}{2}$; insurance, $\$18\frac{3}{4}$; and coal, \$101. Find the average monthly expense for all three items. Find, also, the average daily expense in dollars and cents.

5. A man rented one of his houses for \$450 a year. His taxes during the year on the house were $\$23\frac{3}{4}$, and his water tax was $\$18\frac{1}{2}$; and he paid for repairs \$17. Subtract the sum of all these expenses from the amount of rent he received from the tenant, and tell, to the nearest cent, just what monthly income he got from his house.

V. THE BUDGET PLAN FOR MEETING LIVING EXPENSES, AND FOR SAVING A FIXED SUM EACH YEAR

Many families plan their living expenses so that a certain fixed sum is paid each year for food, another sum for clothing, and other sums for other expenses that are sure to occur.

Such a plan is widely encouraged by the thrift societies and by the banks, for the plan always provides for a fixed amount of saving each year. When a family uses such a system they are said to use a "Budget."

Illustration :

A family has an income of \$1500. How much can be spent for food if the allowance for that expense is $\frac{3}{10}$ of this income? How much can be spent for rent if the allowance for rent is $\frac{1}{5}$ of this income? How much should be saved if the allowance for saving is $\frac{1}{10}$ of this income?

$$\left. \begin{array}{l} \text{For the food : } \frac{3}{10} \text{ of } \$1500 = \frac{3 \times \$1500}{10} = \$450. \\ \text{For the rent : } \frac{1}{5} \text{ of } \$1500 = \frac{1 \times \$1500}{5} = \$300. \\ \text{For the saving : } \frac{1}{10} \text{ of } \$1500 = \frac{1 \times \$1500}{10} = \$150. \end{array} \right\} \text{Result.}$$

WRITTEN PRACTICE

1. If a family has an annual income of \$1200 and spends $\frac{3}{10}$ of it for food, what is the family's annual expense for food?
 $\$360.00$

2. A man earns \$1800 and spends $\frac{1}{5}$ of it for rent, $\frac{1}{8}$ of it for clothing, and $\frac{3}{10}$ of it for food. How much does he spend for all of these three items?

3. A man spends $\frac{1}{4}$ of his income for miscellaneous expenses. If these miscellaneous expenses amount to \$625, how much is his income?

4. A young man received a salary of \$1400 and managed to save $\frac{3}{10}$ of it. Another young man received a salary of \$1800 and saved $\frac{1}{4}$ of it. How much does the saving of the last young man exceed that of the first?

5. A family spends $\frac{1}{5}$ of its income for rent; $\frac{3}{10}$ for food; $\frac{1}{6}$ for clothing; $\frac{1}{4}$ for miscellaneous expenses, and saves the balance. If the income is \$3000 a year, how much is saved?

6. If a family spends $\frac{1}{6}$ of its income for rent, and $\frac{2}{3}$ of it for food, what fraction of their income do they spend for both rent and food?

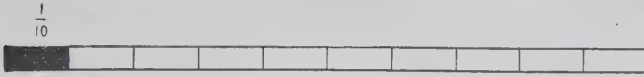
7. If $\frac{5}{6}$ of the income of a family is spent for rent and food, and if $\frac{1}{2}$ of the remainder of it is spent for clothing, what fraction of the income is spent for clothing?

8. A family has an income of \$1800. How much should this family save if three fifths of their income is spent for food, one fourth as much for clothing as for food, and two thirds as much for miscellaneous expenses as for clothing?

9. A man earns \$2500 annually, and divides his expenses so that $\frac{1}{6}$ of his earnings are spent for rent, $\frac{1}{4}$ of them for clothing, $\frac{1}{3}$ of them for food, and $\frac{1}{16}$ of them for fuel and light. If he saves $\frac{3}{5}$ of the remainder, how much does he save?

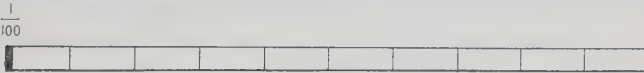
10. In 1917 a family's income was \$2400, of which they spent \$480 for rent, \$1620 for all other expenses, and saved the remainder. If, in 1918, their income was \$2800, and if they spent the same fractional parts on rent and other expenses as in the previous year, find how much they saved.

DECIMALS



The whole length in the first figure is divided into ten equal parts.

The shaded part is **One Tenth** or $\frac{1}{10}$ of the whole length.



The whole length in the second figure is divided into tenths, but the first tenth is again divided into ten equal parts. Each small division, therefore, is **One Tenth of One Tenth** of the whole line. And $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$.

The shaded part, therefore, is **One Hundredth**, or $\frac{1}{100}$ of the whole length.

Decimal Fractions are fractions whose denominators are 10, 100, 1000, etc.

By Using a **Decimal Point**, and omitting the denominator, such fractions are written thus:

$$.1 = \frac{1}{10}$$

$$.01 = \frac{1}{100}$$

$$.001 = \frac{1}{1000}, \text{ etc.}$$

In this form decimal fractions are called **Decimals**.

You have used the decimal point many times in writing United States Money.

Thus: \$2.75 means 2 dollars plus $\frac{75}{100}$ of one dollar.

Dimes and cents are decimals of the dollar.

One dime equals what decimal of one dollar?

One cent equals what decimal of one dollar?

One cent equals what decimal of one dime?

READING DECIMALS

Whole Numbers occupy places at the left of the decimal point. Decimals occupy places at the right of the decimal point.

- .3 is read three tenths.
- .35 is read thirty-five hundredths.
- .356 is read three hundred fifty-six thousandths.
- .0475 is read four hundred seventy-five ten thousandths.

When a whole number and a decimal occur in the same number, the word "and" is read between the whole number and the decimal.

- 2.56 is read two and fifty-six hundredths.
- 15.7 is read fifteen and seven tenths.
- 125.018 is read one hundred twenty-five and eighteen thousandths.
- 65.0075 is read sixty-five and seventy-five ten thousandths.

The number in the table at the right is read :

Two hundred forty-five and seven thousand six hundred forty-one hundred thousandths.

HUNDREDS	TENS	UNITS	DECIMAL POINT	TENTHS	HUNDREDTHS	THOUSANDTHS	TEN THOUSANDTHS	HUNDRED THOUSANDTHS
2	4	5	.	0	7	6	4	1

ORAL PRACTICE

Read :

- | | | | | | | | |
|--------|-----------|--------|---------|-----------|-------|-------|-------|
| 1. .3 | .15 | .25 | .125 | 5. .27 | .105 | .319 | .1275 |
| 2. .7 | .07 | .007 | .375 | 6. .117 | .076 | .706 | .0706 |
| 3. .9 | .91 | .091 | .901 | 7. .217 | .0217 | .2017 | .2107 |
| 4. .12 | .012 | .202 | .991 | 8. .0009 | .0099 | .0999 | .9001 |
| | 9. 12.15 | 37.125 | 45.015 | 75.005 | | | |
| | 10. 32.75 | 192.15 | 345.018 | 1235.0025 | | | |

WRITING DECIMALS

In writing decimals the last figure in the number called for must be written in the place that names the decimal. Thus :

Forty-five hundredths	is written	.45
Forty-five thousandths	is written	.045
Forty-five hundred thousandths	is written	.00045
Twenty-five and six thousandths	is written	25.006

WRITTEN PRACTICE

Write in figures :

1. Fourteen hundredths ; twenty-nine hundredths ; sixty-nine hundredths ; one hundred twenty-two thousandths.

2. Sixty-one hundredths ; seventy-nine thousandths ; sixteen ten thousandths ; two hundred one ten thousandths.

3. Two hundred eleven thousandths ; two hundred eleven ten thousandths ; two thousand and eleven ten thousandths.

4. Eleven hundred thirty-seven ten thousandths ; eleven hundred and thirty-seven ten thousandths ; three thousand seven hundred eleven ten thousandths.

5. 59 thousandths. 9. 265 thousandths.

6. 92 thousandths. 10. 712 ten thousandths.

7. 116 ten thousandths. 11. 819 ten thousandths.

8. 215 ten thousandths. 12. 8002 hundred thousandths.

13. 25 and 37 hundredths.

14. 85 and 125 ten thousandths.

15. 175 and 175 ten thousandths.

16. 1201 and 301 ten thousandths.

17. Three thousand five hundred seventy and one hundred fifteen thousandths.

18. One thousand seventy-nine and ten thousand three hundred seven hundred thousandths.

Similar Decimals are decimals that have the same number of decimal places.

.25 and .64 are similar decimals. .156, .005, and .027 are similar decimals.

Annexing a zero, or annexing zeros, to a decimal does not change its value.

$$\frac{5}{10} = \frac{50}{100} = \frac{500}{1000}. \text{ Therefore, } .5 = .50 = .500, \text{ etc.}$$

Dissimilar Decimals may be changed to similar decimals by annexing zeros. Thus:

.25 and .175 are dissimilar. But .250 and .175 are similar.

WRITTEN PRACTICE

Change to similar decimals:

1. .25 and .125.
2. .154 and .75.
3. .08 and .1125.
4. .7 and .12675.
5. .7, .07, .007, and .0007.
6. .135, 1.5, .15, and 15.0005.
7. .001, .01, .0001, and .1.
8. .0035, .035, .5, and .00175.
9. .125, .2, .15, .0125, and .525.
10. .5, .15, .015, .25, and .0005.

CHANGING DECIMALS TO FRACTIONS

$$.35 = \frac{35}{100} = \frac{7}{20}. \text{ (Reduced to lowest terms.)}$$

Therefore, to change a decimal to a fraction:

Write the figures of the decimal for the numerator.

Write the denominator of the decimal for the denominator.

Reduce the resulting fraction to its lowest terms.

WRITTEN PRACTICE

Change to common fractions, and reduce each to its lowest terms :

1. .4.	6. .35.	11. .125.	16. .045.	21. .0375.
2. .8.	7. .55.	12. .245.	17. .008.	22. .0075.
3. .12.	8. .75.	13. .485.	18. .084.	23. .0008.
4. .41.	9. .83.	14. .504.	19. .096.	24. .0042.
5. .50.	10. .90.	15. .775.	20. .099.	25. .0095.

ADDITION OF DECIMALS

In adding decimals, units of the same kind and decimals of the same kind are written in vertical columns. You have already added one kind of decimals in adding United States Money.

Give aloud the steps taken in each of the illustrations.

\$125.50	10.15	.0019	.37
1.75	4.72	.72	1.414
<u>13.00</u>	<u>1.08</u>	<u>1.5</u>	<u>12.1127</u>
\$140.25 Sum.	15.95 Sum.	2.2219 Sum.	13.8967 Sum.

WRITTEN PRACTICE

Copy, add, and test :

1. \$4.50	2. 5.14	3. 12.451	4. 125.115
1.70	1.16	16.025	110.125
3.15	2.55	31.175	120.075
<u>1.15</u>	<u>3.16</u>	<u>40.005</u>	<u>100.685</u>
5. 31.50	6. 15.75	7. 21.001	8. 110.011
.75	.15	3.110	1.512
1.25	2.18	.009	13.008
<u>12.50</u>	<u>11.20</u>	<u>5.546</u>	<u>.675</u>

9. 39.	10. .75	11. 1.	12. 91.01
.75	5.	3.091	6.
1.50	.65	11.08	9.8
45.	.19	.029	.725
1.25	10.5	4.	1.15
<u>.07</u>	<u>1.05</u>	<u>.271</u>	<u>.005</u>
13. 65.17	14. 7.125	15. 215.7	16. .115
1.96	.751	119.	.075
21.75	1.565	456.25	.325
37.11	.887	7.125	.007
9.08	.75	91.6	.725
89.67	3.25	110.55	.805
<u>19.77</u>	<u>1.115</u>	<u>56.375</u>	<u>.048</u>

Write in vertical columns and add :

17. $2.14 + 3.151 + 8.02 + 6.413 + 51.7$.

18. $27.3 + 61.14 + .005 + 51.142 + 31.09$.

19. $.009 + .0001 + 54. + .186 + 114.77 + .0021$.

20. $71.19 + .4116 + 100.1 + 800.102 + 333.03 + 41.009$.

SUBTRACTION OF DECIMALS

In subtracting decimals, units of the same kind and decimals of the same kind are written in vertical columns. You have subtracted decimals of one type in subtracting United States Money.

Give aloud the steps taken in each of the illustrations :

\$56.75	12.7	14.161	9.15	100.005
<u>\$19.91</u>	<u>5.3</u>	<u>3.12</u>	<u>3.0125</u>	<u>8.19</u>
\$36.84	7.4	11.041	6.1375	91.815

WRITTEN PRACTICE

Copy, subtract, and test :

$$\begin{array}{r} 1. \ \$15.51 \\ \underline{6.32} \end{array}$$

$$\begin{array}{r} 2. \ \$18.54 \\ \underline{9.67} \end{array}$$

$$\begin{array}{r} 3. \ 35.172 \\ \underline{10.024} \end{array}$$

$$\begin{array}{r} 4. \ 51.109 \\ \underline{37.112} \end{array}$$

$$\begin{array}{r} 5. \ 83.07 \\ \underline{41.115} \end{array}$$

$$\begin{array}{r} 6. \ 116.9 \\ \underline{7.112} \end{array}$$

$$\begin{array}{r} 7. \ 71.009 \\ \underline{9.3} \end{array}$$

$$\begin{array}{r} 8. \ 85.1 \\ \underline{1.1715} \end{array}$$

$$\begin{array}{r} 9. \ 100. \\ \underline{41.125} \end{array}$$

$$\begin{array}{r} 10. \ 200. \\ \underline{32.375} \end{array}$$

$$\begin{array}{r} 11. \ 1000. \\ \underline{62.625} \end{array}$$

$$\begin{array}{r} 12. \ 2000. \\ \underline{18.875} \end{array}$$

$$13. \ 8.12 \text{ from } 15.$$

$$22. \ 12.19 \text{ from } 32.$$

$$14. \ .716 \text{ from } 42.$$

$$23. \ 31.42 \text{ from } 500.$$

$$15. \ .019 \text{ from } 54.1.$$

$$24. \ 41.09 \text{ from } 43.$$

$$16. \ .032 \text{ from } 16.$$

$$25. \ 117.19 \text{ from } 118.$$

$$17. \ 16.04 \text{ from } 79.01.$$

$$26. \ 131.001 \text{ from } 1135.$$

$$18. \ 31.5 \text{ from } 70.14.$$

$$27. \ 271.93 \text{ from } 1275.$$

$$19. \ 12.18 \text{ from } 30.26.$$

$$28. \ 107.15 \text{ from } 1000.$$

$$20. \ 35.07 \text{ from } 37.04.$$

$$29. \ 210.7 \text{ from } 257.5.$$

$$21. \ 38.19 \text{ from } 54.6.$$

$$30. \ 31.52 \text{ from } 96.04.$$

EVERYDAY USES FOR ADDITION AND SUBTRACTION OF DECIMALS

1. A railroad time-table gives the distance from Washington to Baltimore as 40 miles, from Baltimore to Philadelphia as 95.2 miles, and from Philadelphia to New York as 91.7 miles. Find the distance from Washington to New York over this railroad.

2. A land surveyor runs four lines on the boundary of a property. The first line is 1375.25 feet; the second line, 807.65 feet; the third line, 611.09 feet; and the fourth line 711.15 feet. What is the total length of the four lines that he measured?

3. In four consecutive years a cow produced butter fat as follows: In 1915, 218.5 pounds; in 1916, 256.75 pounds; in 1917, 261.25 pounds; and in 1918, 251 pounds. What was the total amount of butter fat produced by the cow in all four years?

4. Find the distance from New York to St. Louis over the Pennsylvania Railroad system, if the several divisions give the distance as follows: New York to Pittsburgh, 440.5 miles; Pittsburgh to Columbus, 190.9 miles; Columbus to Indianapolis, 187.8 miles; Indianapolis to Terre Haute, 72 miles; and Terre Haute to St. Louis, 169.1 miles.

5. The width of four lots measured along a city street is 354.75 feet. Three of the lots measure 61.5 feet, 110 feet, and 91.75 feet, respectively. Find the length of the fourth lot.

6. Altoona is between Harrisburg and Pittsburgh. From Harrisburg to Pittsburgh the distance by rail is 244.9 miles, and from Pittsburgh to Altoona it is 113.8 miles. What is the distance from Altoona to Harrisburg?

7. A customer gave a merchant a twenty-dollar bill in paying for shoes costing \$5.45, a hat costing \$3.75, a tie costing \$.75, a shirt costing \$1.50, and stockings costing \$1.15. How much change should the customer receive?

MULTIPLICATION OF DECIMALS

I. MULTIPLYING A DECIMAL BY A WHOLE NUMBER

Study the following additions and multiplications.

.5	Three times five tenths is the same as adding .5+.5+.5.	.5
.5	$3 \times .5 = 1.5$.	
.5	The multiplicand has only one decimal place.	<u>3</u>
1.5	Therefore, the product has only one decimal place.	1.5

.05 Three times five hundredths is the same as adding .05+.05+.05.

.05 $3 \times .05 = .15$. .05

.05 The multiplicand has two decimal places. 3

.15 Therefore, the Product has two decimal places. .15

Apply the same reasoning to $3 \times .005 = .015$. $3 \times .0005 = ?$
 $4 \times .07 = ?$ $4 \times .017 = ?$

In multiplying a decimal by a whole number the product has as many decimal places as there are decimal places in the multiplicand.

WRITTEN PRACTICE

Find the product of :

1. 5×1.3 .

7. 12×25.17 .

13. 25×1.125 .

2. 6×5.4 .

8. 12×26.15 .

14. 24×2.376 .

3. 7×6.1 .

9. 12×32.24 .

15. 31×8004 .

4. 8×9.3 .

10. 14×45.26 .

16. 32×18.124 .

5. 8×12.5 .

11. 16×56.71 .

17. 43×35.125 .

6. 9×20.5 .

12. 21×75.15 .

18. 52×61.117 .

II. MULTIPLYING A DECIMAL BY A DECIMAL

$$.1 \times .1 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = .01.$$

$$.01 \times .01 = \frac{1}{100} \times \frac{1}{100} = \frac{1}{10000} = .0001.$$

$$.1 \times .01 = \frac{1}{10} \times \frac{1}{100} = \frac{1}{1000} = .001.$$

In each multiplication tell how the number of decimal places in the product can be found by observing both the multiplicand and multiplier. You have found that in multiplying a decimal by a decimal you multiply as if they were whole numbers, and then point off as many decimal places in the product as there are decimal places in both the multiplicand and the multiplier.

Give aloud the steps taken in each of the illustrations :

$$\begin{array}{r} 1.4 \\ .8 \\ \hline 1.12 \end{array}$$

$$\begin{array}{r} 2.35 \\ 11.5 \\ \hline 1175 \\ 235 \\ \hline 235 \\ \hline 27.025 \end{array}$$

$$\begin{array}{r} 12.75 \\ .25 \\ \hline 6375 \\ 2550 \\ \hline 3.1875 \end{array}$$

$$\begin{array}{r} 1.375 \\ 1.13 \\ \hline 4125 \\ 1375 \\ \hline 1375 \\ \hline 1.55375 \end{array}$$

WRITTEN PRACTICE

Multiply :

- | | | |
|--------------------|--------------------|--------------------|
| 1. 2.1 by 3.2. | 11. 18.75 by 11.7. | 21. .105 by .5. |
| 2. 3.4 by 4.1. | 12. 25.42 by 12.4. | 22. .0105 by 5. |
| 3. 5.4 by 6.3. | 13. 4.15 by 2.13. | 23. 10.5 by .005. |
| 4. 8.2 by 5.4. | 14. 6.91 by 3.14. | 24. .5 by .1005. |
| 5. 10.1 by 12.4. | 15. 9.23 by 8.17. | 25. 25.01 by .012. |
| 6. 13.5 by 16.3. | 16. 8.114 by 12.6. | 26. 3.125 by .215. |
| 7. 3.21 by 2.3. | 17. 7.023 by 11.9. | 27. 15.27 by .125. |
| 8. 5.16 by 7.8. | 18. .1175 by 10.7. | 28. 312.5 by 1.15. |
| 9. 7.35 by 9.3. | 19. 10.5 by .05. | 29. .0152 by 2.12. |
| 10. 12.15 by 10.5. | 20. 1.05 by .05. | 30. .0212 by 1.52. |

EVERYDAY USES FOR ADDITION, SUBTRACTION AND MULTIPLICATION OF DECIMALS

I. Keeping a Cash Account. You are familiar with forms similar to the one given on the next page. Books ruled for this type of account may be found at many stores.

This account began on May 1st, with a balance of \$2.15 left over from April. It closed May 31st, and the \$3.60 on hand at that date is carried over to the account for June and begins the account for that month.

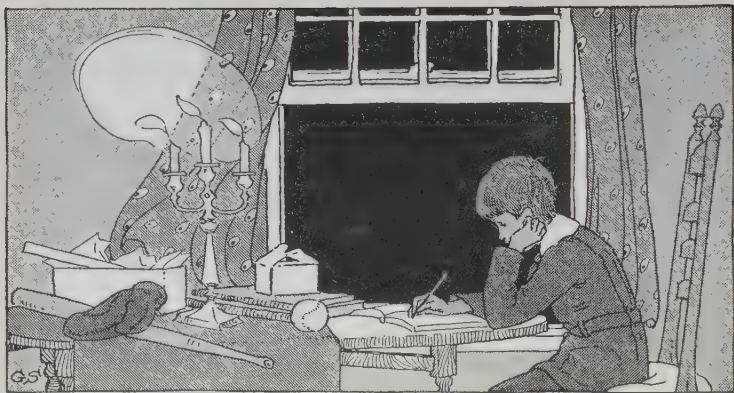
<i>Cash Received, May '18</i>				<i>Cash Paid, May '18</i>			
May 1	Cash on hand	2	15	May 5	Pencils		20
" 8	Mowing lawn		75	" 9	Moving pictures		10
" 15	Selling papers	1	60	" 10	Railroad fare		65
" 19	Mowing lawn		75	" "	Lunch		20
				" 20	Baseball		50
				" 31	On hand	3	60
			5 25				5 25
June 1	Cash on hand	3	60				

WRITTEN PRACTICE

Upon the pages of a small book made for the purpose, or upon paper carefully ruled and spaced, make out cash accounts with the following items:

1. September 1, 1918, On hand, \$4.15. Receipts: Sept. 3d, for errand, \$.20; Sept. 12th, mowing lawn, \$.60; Sept. 14th, gift, \$1.00; Sept. 20th, washing auto, \$.75. Payments: Sept. 15th, football suit, \$3.85; Sept. 20th, carfare, \$.45. Find the balance on hand Sept. 30th.

2. June 1, 1918, On hand, \$.75. Receipts: June 1, cleaning yard, \$.50; June 2, trimming hedge, \$1.25; June 8, mowing lawn, \$.75; June 18, picking berries, \$1.00; June 25, caddying, \$1.75. Payments, June 9, baseball glove, \$2.25; June 19, expenses at game, \$.93; June 27, ball, \$.50, and bat, \$.75. Balance the account on June 30, and show how the amount of cash on hand should be entered at the beginning of the account for July.



3. Jack is making out his cash account for the month of April. He is making the following entries :

April 1, 1918, On hand, \$2.51. April 4, earned \$1.25 cleaning auto; April 5, earned \$.75 trimming hedge; April 6, paid \$1.25 for a baseball glove; April 7, paid \$1 for a baseball bat; April 12, earned \$1.35 mowing lawns; April 20, earned \$1.25 selling magazines; April 29, paid \$2.75 for a tennis racket. Balance his account on April 30.

II. Buying Food. Prices of most foods change frequently, many of them from day to day. Make up two or three problems similar to those that follow, using the prices now prevailing in your locality.

WRITTEN PRACTICE

Find the cost of :

- | | |
|------------------------------|-----------------------------|
| 1. 5 bu. potatoes at \$1.25. | 6. 25 lb. flour at 8¢. |
| 2. 4 doz. eggs at \$.48. | 7. 100 lb. meal at \$.02½. |
| 3. 5 lb. sugar at \$.09. | 8. 5 doz. oranges at \$.75. |
| 4. 10 lb. ham at \$.42. | 9. 5 doz. candles at \$.23. |
| 5. 2 bu. beets at \$2.25. | 10. 2 doz. lemons at \$.30. |

III. Saving through Buying in Quantities. Many careful housekeepers watch for "bargain days," when certain goods are sold at reduced prices. Then by purchasing in quantities sufficient to last for some time, they save considerable sums in the course of a year.

WRITTEN PRACTICE

1. Mrs. Emerson bought five pounds of cocoa for 19 cents a pound, buying it in bulk. In 1-pound cans a similar grade of cocoa would have cost 30 cents per pound. How much did she save by buying in bulk?

2. At a special sale of laundry soap a woman bought a box of 100 cakes for \$2.45. If this box of soap had been bought a few cakes at a time and at 5 cents a cake, how much would it have cost her? How much did she save by buying the box of soap at the special sale?

3. How much would a woman save in a year if, instead of buying a half bushel of potatoes every week, at an average price of \$.95, she should buy two lots of potatoes of 13 bushels each, at a cost of \$1.65 per bushel?

4. If tub butter is 5 cents a pound cheaper than print butter, and if a family uses an average of $3\frac{1}{2}$ pounds of butter per week, find the saving this family can make in a year by using only tub butter.

5. A department store advertised 5 pounds of coffee for \$.95, and 10 pounds of rice for \$.45. If the regular price of the coffee was 25 cents per pound, and the regular price of the rice 8 cents per pound, how much would a purchaser save by buying both the coffee and the rice at the quantity price?

IV. Using Market Quotations on Food Prices. In the morning papers of a Middle West city early in 1919, the following food prices were published :

QUOTATIONS

Print butter . 65¢ per pound.	Bananas . . . 30¢ per dozen.
Cheese 36¢ per pound.	Lemons . . . 40¢ per dozen.
Eggs 55¢ per dozen.	Oranges . . . 50¢ per dozen.
Potatoes . . . 35¢ per peck.	Navy beans . 20¢ per pound.
Grapefruit . . 10¢ each.	Cauliflower . 25¢ each.

WRITTEN PRACTICE

Using the prices in the table, find the cost of :

- | | |
|-------------------------------|---------------------------------|
| 1. 3 lb. butter. | 7. 2 heads cauliflower. |
| 2. 5 doz. eggs. | 8. $5\frac{1}{2}$ doz. eggs. |
| 3. 4 doz. bananas. | 9. 3 doz. oranges. |
| 4. $2\frac{1}{2}$ lb. cheese. | 10. $3\frac{1}{2}$ lb. butter. |
| 5. 4 lb. beans. | 11. $3\frac{1}{3}$ doz. lemons. |
| 6. 2 pk. potatoes. | 12. 8 grapefruit. |

Find the cost of :

- 3 doz. eggs, 5 lb. butter, $1\frac{1}{2}$ pk. potatoes, and 5 lb. beans.
- 5 doz. oranges, 3 doz. lemons, and $4\frac{1}{2}$ lb. cheese.
- 1 bu. potatoes, 10 doz. eggs, 15 lb. butter, and 10 lb. beans.
- 4 doz. eggs, 5 lb. beans, 3 pk. potatoes, 2 doz. oranges, and 10 grapefruit.
- 5 doz. lemons, 6 doz. bananas, 10 doz. oranges, 18 grapefruit, and 5 cauliflower.
- 5 lb. print butter, 4 lb. cheese, 1 pk. potatoes, 8 lb. navy beans, and 8 doz. eggs.

V. Making out Bills. A bill is a written statement of the purchases made by one person or firm from another person or firm.

W·H·WHEELER & COMPANY**PUBLISHERS of SCHOOL BOOKS****CHICAGO****June 20, 1919****SOLD TO. James & Law Company****Clarksburg, West Virginia**

100 Primers	\$.50	\$50.00
150 First Readers	.50	75.00
150 Primary Arithmetics	.60	90.00
		\$215.00

In the specimen bill above we find :

1. The place of purchase.
2. The date of purchase.
3. The name of the purchaser.
4. The name and the business of the merchant.
5. The amount, kind and unit price of each article sold.
6. The amount of each sale.
7. The total amount of all the sales.

WRITTEN PRACTICE

Make some carefully ruled bill forms, and fill out one for each of the following. Consider yourself the purchaser, and use the name of some local merchant who sells the particular kind of goods that each problem refers to.

1. 1 Boy Scout's coat \$3.25, 1 Scout's cap \$1.15, 1 pr. trousers \$1.75, 1 belt \$.60, 1 pr. leggings \$1.00, and 1 pr. shoes \$2.50.

2. 1 catcher's mit \$4.50, 1 catcher's mask \$3.25, 1 catcher's pad \$3.00, 3 baseman's mits at \$2.25 apiece, 5 bats at \$.45 apiece, and $\frac{1}{2}$ doz. baseballs at \$9.50 per dozen.

3. 3 sewing caps at \$.75 each, 4 aprons at \$.80 each, 5 scarfs at \$1.25 each, 8 caps at \$1.10 each, 1 doz. doilies \$2.25, $\frac{1}{2}$ doz. napkins at \$4.50 per dozen, and 1 sweater \$4.00.

4. 1 set of Dickens \$15.50, 1 set of Scott \$21.00, 1 set of Parkman \$27.00, 1 set of Thackeray \$12.50, 1 set of Hawthorne \$19.00, and 1 set of Lowell \$21.25.

5. 5 hunting coats at \$4.25 apiece, 5 fishing rods at \$2.40 apiece, 5 caps at \$1.25 apiece, 5 shotguns at \$11.25 apiece, 5 belts at \$2.50 apiece, 10 fish lines at \$.35 apiece, 6 silk fish lines at \$.85 apiece, and 4 gun cases at \$5.50 apiece.

6. 4 gal. molasses at \$.48 per gallon, 4 lb. coffee at \$.29 per pound, 8 boxes crackers at \$.09 per box, 12 bars of soap at \$.06 per bar, 25 lb. sugar at \$.08 $\frac{1}{2}$ per pound, 1 sack flour at \$3.85, 3 lb. cocoa at 18¢ per pound, 5 cans soup at 12¢ per can, 10 lb. powdered sugar at \$.12 per pound.

DIVISION OF DECIMALS

1. DIVIDING A DECIMAL, OR A MIXED DECIMAL, BY A WHOLE NUMBER

If there is only one figure in the divisor,

Divide as in short division.

Write the point in the quotient directly above the point in the dividend.

$$\begin{array}{r} .2735 \\ 3 \overline{)8.205} \end{array}$$

$$\begin{array}{r} 2.94 \\ 6 \overline{)17.64} \end{array}$$

$$\begin{array}{r} 4.009 \\ 8 \overline{)32.072} \end{array}$$

$$\begin{array}{r} 14.2 \\ 9 \overline{)127.8} \end{array}$$

WRITTEN PRACTICE

Divide :

1. 12.25 by 5.

6. 2.796 by 6.

11. .0154 by 7.

2. 25.74 by 6.

7. 3.875 by 5.

12. .00712 by 8.

3. 38.78 by 7.

8. .4564 by 7.

13. .00168 by 7.

4. 51.44 by 8.

9. 456.4 by 7.

14. .0018 by 9.

5. 514.4 by 8.

10. 112.08 by 8.

15. .9045 by 9.

If there are two or more figures in the divisor :

Before beginning to divide, write a decimal point for the quotient directly above the decimal point in the given dividend.

Then divide as if the numbers were whole numbers.

Give aloud the steps taken in each of the illustrations :

$$\begin{array}{r} 1.52 \\ 32 \overline{)48.64} \\ \underline{32} \\ 166 \\ \underline{160} \\ 64 \\ \underline{64} \end{array}$$

$$\begin{array}{r} 7.37 \\ 35 \overline{)257.95} \\ \underline{245} \\ 129 \\ \underline{105} \\ 245 \\ \underline{245} \end{array}$$

$$\begin{array}{r} .0051 \text{ Quotient.} \\ 24 \overline{).1231} \\ \underline{120} \\ 31 \\ \underline{24} \\ 7 \text{ Remainder.} \end{array}$$

Result, $.0051\frac{7}{24}$.

WRITTEN PRACTICE

Divide :

- | | | |
|-----------------|------------------|-------------------|
| 1. 14.64 by 12. | 6. 59.29 by 49. | 11. 192.24 by 54. |
| 2. 22.69 by 39. | 7. 4.091 by 69. | 12. 22.311 by 67. |
| 3. 23.52 by 42. | 8. .7344 by 72. | 13. 6.3204 by 69. |
| 4. 530.4 by 52. | 9. 697.7 by 88. | 14. 749.07 by 87. |
| 5. 3.968 by 62. | 10. 8.636 by 98. | 15. 89.001 by 99. |

Find, to three decimal places, the following quotients :

- | | | | |
|--------------------------|----------------------|-------------------------|-------------------------|
| 16. $3\overline{)5.000}$ | 19. $3\overline{)8}$ | 22. $5\overline{).67}$ | 25. $12\overline{)7}$ |
| 17. $4\overline{)7.000}$ | 20. $7\overline{)6}$ | 23. $8\overline{).03}$ | 26. $110\overline{)17}$ |
| 18. $6\overline{)5.000}$ | 21. $8\overline{)9}$ | 24. $7\overline{).005}$ | 27. $125\overline{)6}$ |

II. DIVIDING A WHOLE NUMBER BY A DECIMAL, OR BY A MIXED DECIMAL

You have learned that both the dividend and the divisor may be multiplied by the same number without affecting the quotient. Thus :

$$\frac{12}{3} = \frac{12 \times 5}{3 \times 5} = ? \qquad \frac{15}{.03} = \frac{15 \times 100}{.03 \times 100} = \frac{1500}{3} = ?$$

In the division of 15 by .03 we made the divisor a whole number.

Give aloud the steps taken in each of the following :

	640	Quotient.		39000	Quotient.
$.4\overline{)256}$	$4.\overline{)2560}$		$.005\overline{)195}$	$5.\overline{)195000}$	

Therefore, to divide a whole number by a decimal, or by a mixed decimal :

Make the divisor a whole number by moving the decimal point a sufficient number of places to the right.

Move the decimal point in the dividend the same number of places to the right.

Then divide as if the numbers were whole numbers.

WRITTEN PRACTICE

Divide:

- | | | |
|----------------|-------------------|-------------------|
| 1. 1728 by .3. | 6. 4296 by .06. | 11. 17280 by .03. |
| 2. 1504 by .4. | 7. 6016 by .004. | 12. 1728 by .003. |
| 3. 1816 by .5. | 8. 7285 by .05. | 13. 9016 by .07. |
| 4. 2106 by .3. | 9. 8035 by .005. | 14. 9000 by .05. |
| 5. 2466 by .6. | 10. 18160 by .05. | 15. 14480 by .04. |

If the divisor has two or more figures, the division is made by the same process.

Illustrations:

Divide 3255 by 2.5

$$\begin{array}{r}
 1302. \quad \text{Result.} \\
 25 \overline{)32550.} \\
 \underline{25} \\
 75 \\
 \underline{75} \\
 50 \\
 \underline{50} \\
 0
 \end{array}$$

Divide 78 by 3.25

$$\begin{array}{r}
 24. \quad \text{Result.} \\
 325 \overline{)7800.} \\
 \underline{650} \\
 1300 \\
 \underline{1300} \\
 0
 \end{array}$$

WRITTEN PRACTICE

Divide:

- | | | |
|-----------------|------------------|-------------------|
| 1. 2280 by 1.9. | 6. 6006 by 7.7. | 11. 22646 by 6.7. |
| 2. 2415 by .35. | 7. 5076 by .54. | 12. 32100 by .75. |
| 3. 3225 by 4.3. | 8. 8514 by .66. | 13. 63411 by 6.9. |
| 4. 4378 by 4.6. | 9. 5360 by 5.9. | 14. 82650 by .95. |
| 5. 4453 by .73. | 10. 9880 by .76. | 15. 74820 by .87. |

III. DIVIDING A DECIMAL BY A DECIMAL

The methods that we have used are readily applied to this case.

$2.5 \overline{)12.25}$ may be changed to the form $25 \overline{)122.5}$.
 $.0035 \overline{)175}$ may be changed to the form $35 \overline{)1750}$. etc.

To divide a decimal by a decimal :

Make the divisor a whole number by moving the decimal point a sufficient number of places to the right. Move the decimal point in the dividend the same number of places to the right, and then divide as if both numbers were whole numbers.

WRITTEN PRACTICE

Divide :

- | | | |
|-------------------|---------------------|----------------------|
| 1. 24.15 by .35. | 6. 24.192 by 3.2. | 11. 106.950 by 7.5. |
| 2. 50.76 by 5.4. | 7. 320.04 by 4.2. | 12. 1084.05 by .85. |
| 3. 5.360 by 5.9. | 8. 3200.4 by .42. | 13. 26.2580 by 7.6. |
| 4. 85.14 by .066. | 9. 462.16 by .53. | 14. 5075.57 by .87. |
| 5. 988 by .0076. | 10. 54.25 by .062. | 15. 72.7846 by .098. |
| 16. 34.26 by 7.3. | 20. 625.6 by .099. | |
| 17. 6.324 by .37. | 21. 5.8194 by .397. | |
| 18. 3.985 by 7.1. | 22. .28637 by 34.1. | |
| 19. .7126 by .83. | 23. .05493 by .027. | |

EVERYDAY USES FOR DIVISION OF DECIMALS

UNITED STATES MONEY

I. To Find the Cost of an Article when the Cost of a Number of Articles is Known

If the 15 volumes in a set of books cost \$37.50, how much does one volume cost?

2.50	
15 $\overline{) \$37.50}$	The dividend is a number of dollars.
30	The division finds one of the 15 equal parts of that number.
75	Hence, the quotient is a number of dollars.
<u>75</u>	Therefore, 1 book costs \$2.50. <i>Result.</i>

WRITTEN PRACTICE

Find the cost of :

1. 1 book when 25 books cost \$125.
2. 1 rug when 15 rugs cost \$225.
3. 1 map when 24 maps cost \$192.
4. 1 plow when 30 plows cost \$450.
5. 1 doz. knives when 18 doz. cost \$450.
6. 1 ton of coal when 65 tons cost \$357.50.
7. If 10 books cost \$35, find the cost of 3 books.
8. If 24 sweaters cost \$99.60, find the cost of 5 sweaters.
9. If 75 rugs cost \$1117.50, find the cost of 4 rugs.
10. If 48 cameras cost \$201.60, find the cost of 25 cameras.

II. To Find the Number of Articles that can be Bought for a Given Sum when the Cost of a Single Article is Known

The same principle may be applied in finding the time in which a certain sum can be paid by paying a small sum at regular intervals. This is called paying on the "installment plan."

WRITTEN PRACTICE

1. How many yards of carpet can be bought for \$27.50, if one yard cost \$2.50?
2. How many hats can be bought for \$175, if one hat costs \$3.50?
3. How many books are there in a set costing \$17.50, if one book in the set costs \$3.50?

4. The year's coal supply for a family cost \$121.50, and the price paid per ton was \$6.75. How many tons of coal were bought?

5. A dealer paid \$450 for a lot of rugs. If he paid \$22.50 apiece for them, how many rugs did he buy?

6. In how many weeks can a boy save enough to buy a Liberty Bond costing \$100, if he saves \$2.50 each week?

7. How many weeks should it take you to pay for a sewing machine costing \$45, if you pay \$1.50 each week until it is paid for?

8. In how many weeks should you pay for a phonograph costing \$90, if you pay \$1.50 each week until it is paid for?

9. A man bought a mowing machine, paying \$10 down and \$5 each week until it was paid for. In how many weeks did he complete the payments if the machine cost \$65?

10. A family purchased a player-piano costing \$450, and agreed to pay \$7.50 each week until it should be paid for. How many weeks should they take to pay for it?

11. A young woman wishes to save \$75 for Christmas spending money. Find, to the nearest cent, the sum that she must save each week for forty-eight weeks, to save the desired amount.

12. A farmer received \$351 for 156 barrels of apples. How much did he have left after paying 40 cents for each barrel used and 30 cents per barrel for packing and shipping?

13. A woman found that her player-piano, together with 80 rolls of music, had cost her \$626. If the average price of the rolls was \$1.50, and if her payments on the piano had been \$2.75 each week, how many weeks did it take her to pay for the piano?

MEASURES

I. LENGTH

Each member of the class should be supplied with a ruler one foot long; two or three yardsticks at the blackboard will be very helpful.

Name lengths that are measured by inches. By feet. By rods. By miles.

Table of Length Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

Equivalent Measures: 1 mile = 320 rods = 1760 yards = 5280 feet

The sign ' is used for "ft." and the sign " is used for "in."

Changing Given Units to Lower Units

Change 5 yd. 2 ft. 3 in. to inches.

5 yd. = 5×3 ft. = 15 ft.

15 ft. + 2 ft. = 17 ft.

17 ft. = 17×12 in. = 204 in.

204 in. + 3 in. = 207 in. Result.

To change yards to feet, multiply by 3.

To change feet to inches, multiply by 12.

As each product is obtained add to it the given units of the same kind before performing the next multiplication.

To change miles to rods, multiply the number of miles by 320.

To change rods to yards, multiply the number of rods by $5\frac{1}{2}$.

To change yards to feet, multiply the number of yards by 3.

To change feet to inches, multiply the number of feet by 12.

WRITTEN PRACTICE

Change :

- | | |
|----------------------------------|-------------------------|
| 1. 5 ft. 3 in. to inches. | 3. 4 yd. 2 ft. to feet. |
| 2. 6 ft. 5 in. to inches. | 4. 5 yd. 1 ft. to feet. |
| 5. 5 yd. 2 ft. 5 in. to inches. | |
| 6. 3 yd. 1 ft. 10 in. to inches. | |
| 7. 5 rd. 1 yd. to yards. | 9. 2 miles to rods. |
| 8. 6 rd. 2 yd. to yards. | 10. 5 miles to yards. |

Changing Given Units to Higher Units

Change 27 inches to feet and inches.

27 inches $\div$ 12 inches = 2, with a remainder of 3 inches.

Since 12 inches = 1 foot, 2 is the number of feet.

Therefore, 27 inches = 2 feet 3 inches. *Result.**To change inches to feet, divide the number of inches by 12.**To change feet to yards, divide the number of feet by 3.**To change yards to rods, divide the number of yards by $5\frac{1}{2}$.**To change rods to miles, divide the number of rods by 320.*

WRITTEN PRACTICE

Change :

- 39 inches to feet and inches.
- 77 inches to feet and inches.
- 19 feet to yards and feet.
- 56 feet to yards and feet.
- 105 yards to rods and yards.
- 245 yards to rods and yards.
- 640 rods to miles.
- 800 rods to miles and rods.
- 10,560 feet to miles.
- 20,000 feet to miles and feet.

II. LIQUID MEASURE

Name articles that are measured by the pint. By the quart. By the gallon.

Table of Liquid Measure

2 pints (pt.)	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel
2 barrels	= 1 hogshead

Equivalent Measures: 1 gallon = 4 quarts = 8 pints.

Changing Given Units to Lower Units

To change gallons to quarts, multiply the number of gallons by 4.

To change quarts to pints, multiply the number of quarts by 2.

WRITTEN PRACTICE

Change:

1. 25 qt. to pints.
2. 37 qt. to pints.
3. 25 gal. to quarts.
4. 37 gal. to quarts.
5. 15 gal. to pints.
6. 24 gal. to pints.
7. 30 gal. to pints.
8. 45 gal. to pints.
9. 3 gallons, 3 quarts to quarts.
10. 5 gallons, 1 quart to quarts.
11. 11 quarts, 1 pint to pints.
12. 15 quarts, 1 pint to pints.
13. 2 gallons, 1 quart, 1 pint to pints.
14. 4 gallons, 2 quarts, 1 pint to pints.
15. 5 gallons, 3 quarts, 1 pint to pints.
16. 8 gallons, 1 quart, 1 pint to pints.

Changing Given Units to Higher Units

To change pints to quarts, divide the number of pints by 2.

To change quarts to gallons, divide the number of quarts by 4.

WRITTEN PRACTICE

Change :

- | | |
|---------------------------|-------------------------------|
| 1. 34 pints to quarts. | 6. 175 quarts to gallons. |
| 2. 45 pints to quarts. | 7. 126 gallons to barrels. |
| 3. 60 pints to quarts. | 8. 189 gallons to barrels. |
| 4. 100 quarts to gallons. | 9. 50 barrels to hogsheads. |
| 5. 120 quarts to gallons. | 10. 252 gallons to hogsheads. |

III. WEIGHT MEASURE

Name articles that are measured by the ounce. By the pound. By the ton.

Table of Weight Measure

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 100-weight (cwt.)
2000 pounds	= 1 ton (T.)

Equivalent Measures: 1 ton = 20 hundredweight = 2000 pounds.

Changing Given Units to Lower Units

To change pounds to ounces, multiply the number of pounds by 16.

To change tons to pounds, multiply the number of tons by 2000.

WRITTEN PRACTICE

Change :

1. 15 pounds to ounces.
2. 5 pounds, 8 ounces to ounces.
3. 5 tons to pounds.
4. 3 tons, 500 pounds to pounds.
5. 35 pounds, 15 ounces to ounces.
6. 12 tons, 135 pounds to pounds.

Changing Given Units to Higher Units

To change ounces to pounds, divide the number of ounces by 16.

To change pounds to tons, divide the number of pounds by 2000.

WRITTEN PRACTICE

Change :

- | | |
|--------------------------|---------------------------|
| 1. 48 ounces to pounds. | 5. 4000 pound to tons. |
| 2. 64 ounces to pounds. | 6. 6000 pound to tons. |
| 3. 80 ounces to pounds. | 7. 10,000 pounds to tons. |
| 4. 112 ounces to pounds. | 8. 14,000 pounds to tons. |
9. 54 ounces to pounds and ounces.
 10. 65 ounces to pounds and ounces.
 11. 100 ounces to pounds and ounces.
 12. 250 ounces to pounds and ounces.
 13. 2500 pounds to tons and pounds.
 14. 6100 pounds to tons and pounds.
 15. 3750 pounds to tons, hundredweights, and pounds.
 16. 5875 pounds to tons, hundredweights, and pounds.

IV. TIME MEASURE

What measures of time are used in telling time by the clock? In speaking of one's age? In timing the fast runs on the playground, like the 100-yard dash and the 220-yard hurdles?

Table of Time Measure

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)

We also use :

52 weeks, 1 day = 1 year.

365 days = 1 year.

366 days = 1 Leap Year.

As a rule, a leap year comes once in four years.

An old rhyme gives :

Thirty days hath September,

April, June, and November ;

February has twenty-eight alone,

All the rest have thirty-one,

Excepting leap year, that's the time

When February's days are twenty-nine.

Changing Given Units to Lower Units

To change minutes to seconds, multiply the number of minutes by 60.

To change hours to minutes, multiply the number of hours by 60.

To change days to hours, multiply the number of days by 24.

WRITTEN PRACTICE

Change :

- | | |
|-------------------------------|------------------------------|
| 1. 10 minutes to seconds. | 5. 7 hr. 24 min. to minutes. |
| 2. 7 min. 25 sec. to seconds. | 6. 5 hr. 11 min. to minutes. |
| 3. 12 min. 5 sec. to seconds. | 7. 2 days to hours. |
| 4. 15 hours to minutes. | 8. 3 da. 11 hr. to hours. |

Changing Given Units to Higher Units

To change seconds to minutes, divide the number of seconds by 60.

To change minutes to hours, divide the number of minutes by 60.

To change hours to days, divide the number of hours by 24.

WRITTEN PRACTICE

Change :

1. 135 seconds to minutes.
2. 200 seconds to minutes.
3. 145 minutes to hours.
4. 250 minutes to hours.
5. 40 hours to days.
6. 65 hours to days.
7. 100 hours to days.
8. 40 hours to days, and to minutes.

V. DRY MEASURE

Name articles that are measured by the bushel. By the peck. By the dry quart.

Table of Dry Measure

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

Equivalent Measures : 1 bu. = 4 pk. = 32 qt. = 64 pt.

Changing Given Units to Lower Units

To change quarts to pints, multiply the number of quarts by 2.

To change pecks to quarts, multiply the number of pecks by 8.

To change bushels to pecks, multiply the number of bushels by 4.

WRITTEN PRACTICE

Change :

- | | |
|------------------------|--------------------------|
| 1. 15 quarts to pints. | 5. 20 bushels to pecks. |
| 2. 75 quarts to pints. | 6. 35 bushels to pecks. |
| 3. 15 pecks to quarts. | 7. 25 bushels to quarts. |
| 4. 45 pecks to quarts. | 8. 16 bushels to pints. |

Changing Given Units to Higher Units

To change pints to quarts, divide the number of pints by 2.

To change quarts to pecks, divide the number of quarts by 8.

To change pecks to bushels, divide the number of pecks by 4.

WRITTEN PRACTICE

Change :

- | | |
|------------------------------------|-----------------------------------|
| 1. 18 pints to quarts. | 4. 25 pints to quarts and pints. |
| 2. 40 quarts to pecks. | 5. 50 quarts to pecks and quarts. |
| 3. 24 pecks to bushels. | 6. 30 pecks to bushels and pecks. |
| 7. 64 quarts to bushels. | |
| 8. 240 pints to bushels and pecks. | |

VI. MISCELLANEOUS MEASURES

(a) Counting Measures :

12 units = 1 dozen.

12 dozen = 1 gross. (1 gross = 144 units.)

The dozen is constantly used, but the gross is used but little in modern business.

(b) Paper Measures :

24 sheets = 1 quire.

500 sheets = 1 ream.

The quire is used only in counting folded note-paper.

The ream is in general use, but a large amount of paper is sold by the pound.

EVERYDAY USES FOR MEASURES

I. Using the Inch, Foot, Yard, Rod, and Mile

1. If the length of your step is 27 inches, how many inches do you walk in taking 30 steps? How many feet do you walk in taking 30 such steps?

2. Estimate the length of your schoolroom in feet. Then measure it, and compare your estimate with the actual length.

3. A garden is 35 feet long and 18 feet wide. If there is a fence around it, what is the length of the fence?

4. A boy must go over the running track on a playground eight times in order to go 1 mile. How many feet in the distance around this track?

5. On a railroad track the ties are so placed that the distance from center to center is 2 feet. Counting the end ties, how many ties are there in this track if it is four miles long?

6. A boy finds that the distance around the wheel on his father's automobile is just ten feet. How many times does the wheel turn around in going over a distance of one mile?

7. An ocean liner steams continuously from 8.30 in the morning to 5.30 in the afternoon, and the speed averages 20 "knots," or nautical miles, per hour during that time. How many nautical miles did the ship steam in that time?

8. On a certain day a manufacturer of the cloth used in making window-shades produced 54,000 yards of it. If a window-shade is 7 ft. 6 in. long, how many shades could be made from this quantity?



II. Using the Pint, the Quart, and the Gallon

1. How many times will the contents of the quart measure fill the pint measure? How many times will the contents of the gallon measure fill the pint measure?

2. If Tom measures 45 quarts of sirup with a gallon measure, how many gallons should there be? How many quarts will there be left over?

3. Mary measured some milk with her pint measure. If she filled the measure exactly 16 times, how many quarts of milk did she measure?

4. Jack found that a quantity of sirup filled his quart measure 12 times, and that enough was left over to fill it just half full. How many pints of sirup did he measure?

5. If Mary fills the gallon measure by dipping water with the pint measure, how many times must she pour a pint of water into the gallon measure to fill it?

6. If Jack uses his quart measure in filling a 3-gallon can with oil, how many quarts of oil must he pour into it? How many pints of oil will fill the same can?

7. A grocer wishes to bottle 15 gallons of olive oil, using bottles that hold $\frac{1}{2}$ pint each. How many bottles must he use?

8. The gasoline tank in an automobile holds 14 gallons, and the owner has a storage tank that holds 360 gallons. If the storage tank is full, how many times can he fill the small tank from it?

9. A dealer wishes to put 250 gallons of maple sirup into cans and have one half of the sirup in pint cans and the other half of it in quart cans. How many cans of each size must he use?

10. A housekeeper finds that she uses an average of $2\frac{1}{2}$ quarts of milk and $\frac{1}{2}$ pint of cream daily in a year. If the milk costs 10 cents a quart, and the cream costs 12 cents a half pint, find the total cost of both in one year.

11. A chauffeur finds that his car requires a gallon of gasoline for each 12 miles it runs, and that a quart of oil is required for each 16 gallons of gasoline used. If gasoline costs him 26 cents a gallon, and oil 20 cents a quart, how much do both gasoline and oil cost for a trip of 480 miles?

III. Using the Ounce, the Pound, and the Ton

1. A grocer sells 250 pounds of sugar in packages of 5 pounds each. How much does he receive for this sugar at 45 cents a package?

2. Find the combined weight of a truck and its load, if the truck weighs 1750 pounds, and the load consists of 15 cases weighing 120 pounds each.

3. A load of coal weighing 2240 pounds is delivered in baskets that hold 40 pounds each. How many such baskets of coal in this load of coal?

4. A grocer bought 6 cases of spices, averaging 10 pounds to the case, and he wishes to put this whole quantity of spice into bags that hold 4 ounces each. How many bags must he provide?

5. A retail coal dealer bought 2 cars of coal, each car holding 125,000 pounds. In moving this coal how many trips must be made by a truck that takes 2 tons to each load?

6. A truck contains 24 bags of grain, and the average weight of these bags of grain is 125 pounds. The truck weighs 3350 pounds. Find the total weight of both the truck and the load. (This is called Gross Weight.)

7. A dealer puts 150 pounds of rice into bags that hold $2\frac{1}{2}$ pounds each. If these bags of rice sell for 10 cents each, find the total amount the dealer received for the 150 pounds of rice. If this rice costs the dealer \$3.00, how much is his profit on it?

IV. Using the Second, Minute, Hour, Day, and Year

1. If Donald earned \$.50 on each week day in March, in a year in which that month had five Sundays, how much did he earn in the month?

2. A young man paid \$5.25 a week for his board and room. At that rate how much should he pay during the three months of April, May, and June?

3. If a motorman is paid \$.42 an hour, and works continuously from 8 in the morning till 5 in the afternoon, how much should he be paid?

4. The hours in a shop are from 8 A.M. to 5 P.M., with an hour out for lunch each day. In this shop how many working hours are there in six working days?

5. How many minutes will a clock lose in the month of March, if it loses 2 minutes each day?

6. If your watch should gain 3 seconds a day, how many minutes would it gain in the month of April?

7. Tom's mother roasted a turkey for 2 hours and 40 minutes. If she took it from the oven at 2 P. M., at what time did she put it into the oven?

8. Ben's father agreed to pay him 15 cents an hour for weeding the garden. How much should Ben receive for working from 1 P. M. to 5.30 P. M.?

9. A carpenter works from 8 A. M. to 5 P. M., with an hour out for dinner. If he is paid \$.55 an hour, how much does he receive for a day's work?

10. Jack rode in an automobile from 8 A. M. to 2.30 P. M., and in that time he travelled 117 miles. What was the average rate that he travelled per hour?

11. Suppose you should waste 20 minutes each day for a year. How many hours would this waste amount to? How many days of wasted time would it amount to?

12. How many seconds are there in 1 hour? How many minutes are there in 1 day? How many hours are there in 1 week? How many hours are there in 1 year?

13. If a train is running 60 miles an hour we say it is "going a mile a minute." Can you explain the expression? How many feet does a train go in 1 second, if it is going 1 mile in 1 minute?

14. In cases where there are only a few wires, the telegraph poles are set 160 feet apart. Can you show that a train that passes 33 of these poles each minute is "going a mile a minute"?



V. Using the Pint, Quart, Peck, and Bushel

1. How many times will the peck measure fill the pint measure? How many times will the bushel measure fill the pint measure?

2. How many times must Marion fill the peck measure if she uses it to measure $12\frac{1}{2}$ bushels of potatoes?

3. If Marion measures 50 quarts of peas with a peck measure, how many pecks will there be, and how many quarts of peas will be left over?

4. If Esther uses her quart measure to measure 2 bushels of oats, how many times will she fill her quart measure?

5. If Tom and Marion and Esther should each fill their measures with shelled corn, how many quarts of corn would the three measures contain?

6. If Marion and Esther each fill their measures three times and empty them into Tom's basket, will they fill his basket?

7. Tom filled his measure from a bin of oats; Marion then filled her measure from the same bin; and Esther found enough left to fill her measure exactly five times. How many quarts of oats were there in the bin?

8. The 500 trees in an orchard yield an average of $6\frac{1}{2}$ bushels of apples per tree. If these apples are sold by the peck for an average price of \$.80 per peck, how much is received for them?

9. A grocer purchased navy beans at \$3.50 a bushel, and sold them at 15 cents a quart. How much did he gain if he bought and sold a total of 7 bushels and 16 quarts?

10. A dealer bought 500 bushels of potatoes for \$2 per bushel, and sold them in baskets holding $\frac{5}{8}$ bushel at \$1.60 a basket. If the baskets cost 5 cents apiece, find the total profit on the sale.

11. A dishonest merchant used a so-called bushel basket that lacked two quarts of holding a bushel. If he sold 120 bushels of potatoes and used this basket to measure them before being discovered, and received \$2.00 a bushel for them, how much did his customers lose by his dishonesty?

VI. Miscellaneous Uses for Measures

1. The sides of a room are each 18 feet 6 inches long, and the ends are each 12 feet long. What is the total distance around the room?

2. A farmer packed 500 bushels of peaches in baskets that held $\frac{5}{8}$ of a bushel apiece. How many baskets did he use?

3. 150 gallons of maple sirup are to be sold in cans holding 1 pint each. How many such cans will be needed?

4. Every time that the wheels of an automobile turn around, the car moves ahead 9 feet, 6 inches. How many feet has the car moved when the wheels have turned 250 times?

5. The gas tank in an automobile holds 12 gallons, and in going 1800 miles this tank was filled 15 times. How many miles did the car average to 1 gallon of gasoline?

6. A truck, with its load of 24 bags of grain weighing 200 pounds apiece, weighs 4 tons. What is the weight of the truck?

7. The actual number of weeks in the school year of a certain school was 36, and there were 5 days in each of these school weeks. If the school was in session from 9 A.M. to 1 P.M. each day, how many hours were there in the school year?

8. A ream of paper is cut, folded and sold in quires. If the ream cost \$1.80, and the cost of cutting it is \$.10, find the cost of a single quire of the paper. Disregard the small portion of the ream that is less than 1 quire.

9. A gross of pencils that cost a dealer \$4.00 was sold at 5 cents apiece. What profit did this dealer make on the whole gross of pencils? If this dealer sells an average of 30 gross of pencils each year at that rate of profit, how much profit does he make in the year?

10. A man was offered a gross of tablets for \$11.25, but refused to buy so many at one time. During the year his office used a whole gross of them, but paid 10 cents apiece for them. How much could he have saved by buying the whole gross of tablets when they were offered to him?

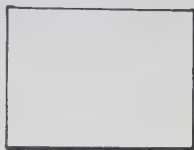
11. A dealer in canned goods buys his stock in cases of one dozen cans each. He sells the cans at an average price of 12 cents apiece, and sells 100 cases of them in 1 year. If these cans cost him \$.90 a dozen, and if the freight charge is 24 cents a case, how much is his profit on the 100 cases?

DRAWING TO SCALE

The oblong represents the floor of a room 20 feet long and 15 feet wide.

1 inch in the drawing represents 20 feet in the length of the floor, hence, the drawing is 1 inch long.

Also, if 20 feet are represented by 1 inch, 15 feet are represented by $\frac{15}{20}$ or $\frac{3}{4}$ of 1 inch.



Hence, the drawing on a scale of 1 inch to 20 feet is 1 inch long and $\frac{3}{4}$ of one inch wide.

The oblong is said to be *Drawn on a Scale of 20 Feet to 1 Inch*, and each inch of length in the drawing stands for a length of twenty feet in the object.

You can find innumerable instances of the uses for drawing to scale. The maps in your geography, the pocket maps of states and cities which are found at all stationery stores, and all of the drawings and blueprints used in machine shops, are drawn to scale.

ORAL PRACTICE

Using the Foot-Rule

1. If the lines, *a*, *b*, and *c*, are drawn on a scale of 50 feet to 1 inch:

a _____

b _____

c _____

What distance is represented by the line *a*?

What distance is represented by the line *b*?

What distance is represented by the line *c*?

2. If the lines m , n , and p are drawn on a scale of 100 feet to 1 inch:

m _____
 n _____
 p _____

What distance is represented by the line m ?

What distance is represented by the line n ?

What distance is represented by the line p ?

3. If the lines x , y , and z are drawn on a scale of 100 feet to $\frac{1}{2}$ inch:

x _____
 y _____
 z _____

What distance is represented by the line x ?

What distance is represented by the line y ?

What distance is represented by the line z ?

4. If you make a drawing on a scale of 10 feet to 1 inch, how long will you draw a line to represent:

- | | | |
|--------------|--------------|---------------|
| (a) 30 feet? | (d) 45 feet? | (g) 120 feet? |
| (b) 50 feet? | (e) 75 feet? | (h) 150 feet? |
| (c) 70 feet? | (f) 95 feet? | (i) 275 feet? |

5. If you make a drawing on a scale of 40 feet to 1 inch, what is the length represented on your drawing by a line

- | | | |
|-----------------|------------------------------|-----------------------|
| (a) 4 in. long? | (d) $4\frac{1}{2}$ in. long? | (g) 1 ft. 3 in. long? |
| (b) 5 in. long? | (e) $7\frac{1}{4}$ in. long? | (h) 2 ft. 6 in. long? |
| (c) 7 in. long? | (f) $8\frac{3}{4}$ in. long? | (i) 3 ft. 9 in. long? |

DRAWING PRACTICE

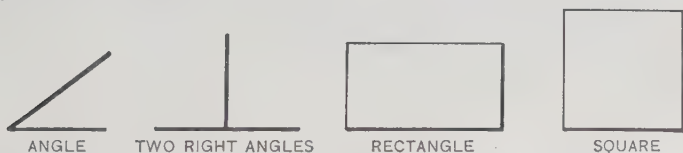
Draw oblongs on some convenient scale to represent the following dimensions :

- | | |
|------------------------|--------------------------|
| 1. 8 feet by 10 feet. | 5. 15 yards by 25 yards. |
| 2. 12 feet by 10 feet. | 6. 18 yards by 32 yards. |
| 3. 25 feet by 20 feet. | 7. 16 rods by 35 rods. |
| 4. 30 feet by 45 feet. | 8. 24 rods by 45 rods. |

PRACTICAL MEASUREMENTS

USING THE MEASURES OF LENGTH

When two straight lines meet as in the figure at the left, they make an **Angle**.



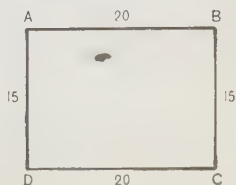
When two straight lines meet so that two angles are formed, and these two angles are equal, each of them is called a **Right Angle**.

A figure that has four sides and four right angles is called a **Rectangle**. Sometimes such a figure is said to be an **Oblong**.

If a figure has four equal sides and four right angles, it is called a **Square**.

The total length of all the lines that bound a figure, or the distance around it, is called the **Perimeter** of the figure. In the figure, $ABCD$, if $AB=20$, $BC=15$, $CD=20$, and $DA=15$, the perimeter equals

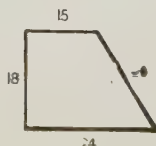
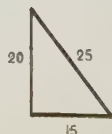
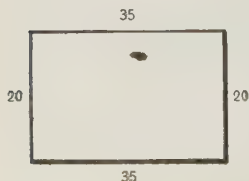
$$20+15+20+15=70.$$



WRITTEN PRACTICE

Find the perimeter of

1. A rectangle 15 in. long, and 10 in. wide.
2. A rectangle 24 ft. long, and 15 ft. wide.
3. A rectangle 15 ft. 6 in. long, and 10 ft. 8 in. wide.
4. A square, each of whose sides is 15 inches long.
5. A square, 90 feet on each side. (A regulation baseball diamond.)
6. Find the number of yards in the perimeter of a field 100 feet long and 50 feet wide.
7. Find the number of rods in the perimeter of a square field whose sides are each 150 feet.
8. How much picture molding will it take to go around a room 30 feet long and 20 feet wide, and how much will it cost at 15¢ a foot?
9. A field is 140 rd. long and 110 rd. wide. It is inclosed by a barbed wire fence of five strands of wire, the wire costing $4\frac{1}{2}$ ¢ a rod for each strand. What was the total cost of the wire for this fence?
10. A picture 4 feet long and 2 feet wide is framed with a molding that is 3 inches wide. For the corners the maker allows four times the width of the molding and adds that amount to the length needed. Find the total cost of the molding used to frame this picture at 20 cents a foot.
11. Find the perimeter of each of the following figures :



USING THE MEASURES OF SURFACE

You have learned the common measures of surface.

$$144 \text{ sq. in.} = 1 \text{ sq. ft.} \quad 9 \text{ sq. ft.} = 1 \text{ sq. yd.}$$

If a rectangle is 4 inches long and 3 inches wide, and if we use the square inch to measure its surface, we have four rows of squares, with three squares in each row. That is:



(4 times 3) square inches of surface.

Or, the unit of measure, 1 square inch, is used 4×3 times.

We say, therefore: The Area = (4×3) sq. in. = 12 sq. in.

Table of Surface Measures

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

WRITTEN PRACTICE

How many square inches are there in a rectangle:

1. 5 in. long and 3 in. wide?
2. 8 in. long and 4 in. wide?
3. 6 in. long and 5 in. wide?
4. 10 in. long and 4 in. wide?
5. 13 in. long and 8 in. wide?
6. 18 in. long and 12 in. wide?

How many square feet are there in a rectangle :

7. 5 ft. long and 3 ft. wide?
8. 10 ft. long and 7 ft. wide?
9. 15 ft. long and 12 ft. wide?
10. 15 yd. long and 9 yd. wide?
11. 35 yd. long and 10 yd. wide?
12. 50 yd. long and 42 yd. wide?

13. The floor of a room is 32 ft. long and 15 ft. wide. At 5¢ a sq. ft., how much will it cost to scrape this floor?

14. If a gallon of floor varnish, which costs \$3.25, covers 180 square feet, what will it cost to varnish the floor of the room in example 13?

15. How many square rods are there in the area of a field 150 rods long and 80 rods wide? How many acres are there in this area?

16. The sidewalks on a street 450 feet long are 6 feet wide. What was the cost of the sidewalks on both sides of the street at \$2.10 a square yard?

17. A farmer has a potato field 40 rods long and 20 rods wide. If he plants 12 bushels of potatoes to the acre, how many bushels will he need?

18. The paving in a city street cost \$2.40 a square yard. The street is 1 mile long and 60 feet wide. What was the cost of paving the street?

19. A room is 28 feet long, 15 feet wide, and 9 feet high. A painter paints the four walls and the ceiling for \$.20 a square yard. Find the total amount that is paid him for painting this room, no allowance being made for openings.

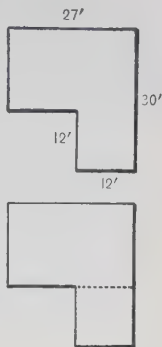
20. A room is 27 feet long, 18 feet wide, and 10 feet high. The area of the doors and windows is 135 square feet. Find the cost of painting the walls and ceiling at \$.25 a sq. yd.

FINDING THE AREA OF IRREGULAR-SHAPED SURFACES

When a given surface has right angles but is not a rectangle or a square, it is usually possible to consider the surface as made up of smaller surfaces that have familiar shapes.

The upper figure at the right is neither a rectangle nor a square, but if a line is drawn across it as indicated by the dots, we have the lower figure and in it a square and a rectangle.

From the known lengths and widths we may now calculate the entire area.



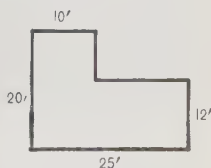
The area of the rectangle (18×27) sq. ft. = 486 sq. ft.

The area of the square (12×12) sq. ft. = 144 sq. ft.

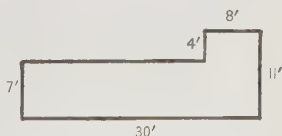
Adding these two areas, the area of the given figure = 630 sq. ft.

WRITTEN PRACTICE

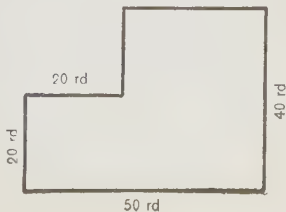
1. The figure at the right represents the shape of a floor. Find the number of square feet in its area.



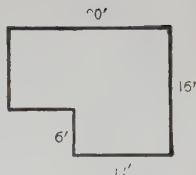
2. The floor of a hallway is shaped like the figure at the right. Find the number of square feet in its area.



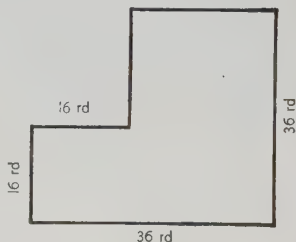
3. A field on a farm is shaped like the figure at the right. There are 160 sq. rd. in an acre. Find the number of acres in this field.



4. At \$1.50 a square yard, how much will the linoleum cost to cover a kitchen floor shaped like the figure at the right?



5. A field of potatoes is shaped like the figure at the right. If the yield from this field was 150 bushels to the acre, how many bushels of potatoes did the whole field yield?



USING MEASURES OF VOLUME

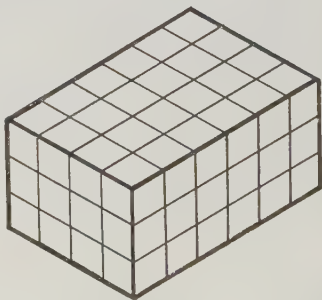
You have learned that a solid contains as many cubic units of volume as the product of its three dimensions, each expressed in the same unit of measure.

In the figure :

The length = 6 inches,

The width = 4 inches,

The height = 3 inches.



The Volume = $(6 \times 4 \times 3)$ cubic inches = 72 cubic inches.

You have learned also, that

A cube 12 inches long, 12 inches wide, and 12 inches high, has a volume $(12 \times 12 \times 12)$ cubic inches = 1728 cubic inches.
Or,

$$1728 \text{ cubic inches (cu. in.)} = 1 \text{ cubic foot (cu. ft.)}$$

Also, since 3 feet = 1 yard,

A cubic yard contains as many cubic feet as $(3 \times 3 \times 3)$ cubic feet. Or,

$$27 \text{ cubic feet} = 1 \text{ cubic yard (cu. yd.)}$$

Illustrations :

1. A contractor dug a cellar 40 ft. long, 30 ft. wide, and 9 ft. deep. How many cubic yards of dirt did he remove?

The volume of the dirt removed $(40 \times 30 \times 9)$ cu. ft. = 10,800 cu. ft.

Since 1 cu. yd. = 27 cu. ft., $10,800 \div 27 = 400$, the number of cubic yards.

2. A farmer has a grain bin 12 feet long, 5 feet wide, and 4 feet deep. If a bushel of grain equals approximately $1\frac{1}{4}$ cubic feet, how many bushels of grain are there in the bin when it is level full?

The volume of the bin = $(12 \times 5 \times 4)$ cu. ft. = 240 cu. ft.

Since 1 bushel = $1\frac{1}{4}$ cu. ft., or 1.25 cu. ft., $240 \div 1.25 = 192$, the number of bushels of grain in the bin when it is level full.

WRITTEN PRACTICE

1. A cellar is 35 feet long, 18 feet wide, and 6 feet deep. How many cubic yards of dirt were removed in digging it?

2. A grain bin is 12 feet long, 6 feet wide, and 3 feet deep. How many cubic feet of grain will the bin hold when it is level full?

3. A ditch is 1800 feet long, 2 feet wide, and 6 feet deep. How many cubic yards of dirt were removed in digging it? If the cost of removing this dirt averaged \$.60 a cubic yard, what did it cost to dig the ditch?

4. When wheat was selling at \$2.25 a bushel, a farmer sold the contents of a bin that was 12 feet long, 6 feet wide, and 5 feet deep, and which was level full of wheat. How much did he receive for this wheat? (A cubic foot = $\frac{1}{4}$ bushel.)

5. Some authorities estimate that a ton of loose hay in the mow occupies 400 cubic feet. If that estimate is used in calculating the amount of hay in a mow 30 feet long, 12 feet wide, and 20 feet deep, how many tons of hay are there in the mow?

6. A farmer has a water storage tank in the attic of his house, and this tank is a cubical iron box whose inside dimensions are each 5 feet. If a cubic foot of water weighs $62\frac{1}{2}$ pounds, what is the weight of the water in this tank when it is full?

GENERAL REVIEW

ORAL

1. Give all of the prime numbers from 1 to 75.
2. Give all of the exact one figure divisors of 5280.
3. Give the prime factors of 20, 30, 45, 60, 75, 90, 100.
4. Which is the greater value: $\frac{3}{4}$ of $\frac{2}{3}$ of \$24, or $\frac{2}{3}$ of $\frac{3}{4}$ of \$24?
5. Which would you prefer to own: a $\frac{3}{8}$ share of a motor-boat, or a $\frac{2}{5}$ share of the same boat?
6. What fractional part of a square yard is the area of a rug which is 3 feet long and 2 feet wide?
7. If 2 acres of land yield 300 bushels of potatoes, how many bushels of potatoes should 3 such acres yield?
8. If you know the area of a field and also its length, how can you find the width of the field?
9. If you know the volume of a box, and the length and the width of it, how can you find the depth of it?

10. If $\frac{1}{4}$ of a pound of tea costs 15 cents, how much should be paid for 1 pound of it? How much for 3 pounds?

11. For drill in quick oral work try the following. Take the number 50; divide it by 10, multiply the result by 3, add 3, divide by 6, multiply by 20, divide by 15, subtract 2, multiply by 10, and subtract 20. What is your final result?

12. Add 3 to both the numerator and the denominator of the fraction $\frac{2}{7}$. What is your result? Reduce the result to lowest terms. Compare it with the given fraction. Does adding the same number to both the numerator and the denominator of a fraction change the value of that fraction?

13. Subtract 1 from both the numerator and the denominator of the fraction $\frac{2}{7}$. What is your result? From this result and from that which you obtained in exercise 12 tell what you have learned about the effect of adding a number to, or subtracting a number from, both terms of a fraction.

14. Take the number 27, and perform orally and in order each of the following operations as each new result is obtained. Divide by 3, add 6, multiply by 2, divide by 5, subtract 4, multiply by 12, divide by 8, add 7. What is your result? (Make an example of this kind, and ask some one to solve it.)

15. Take the result obtained in exercise 14, that is, 10, and perform in reverse order the steps asked for in that exercise; but subtract where you formerly added, add where you formerly subtracted, multiply where you formerly divided, and divide where you formerly multiplied. Thus: From 10 subtract 7, then multiply by 8, divide by 12, etc. What is your result?

GENERAL REVIEW

EVERYDAY USES FOR ELEMENTARY ARITHMETIC

1. How much more than $\frac{3}{8}$ of 72 is $\frac{5}{8}$ of 60?
2. Find the dividend if the given divisor is 25 and the given quotient is 16.
3. Find the minuend when the given subtrahend is 214 and the given difference is 329.
4. What fractional part of a bushel is 3 pk. 1 qt.? What fractional part of a mile is 2 rods?
5. If the sum of two numbers is 3407, and one of them is 1588, what is the other number?
6. Find the dividend when the given divisor is 24, the quotient 19, and the remainder 13.
7. How many hair ribbons, each $1\frac{1}{8}$ yd. long, can be cut from a piece of ribbon $4\frac{1}{2}$ yd. long?
8. A city block in a certain locality is 528 feet long. What fraction of a mile is the length of this block?
9. The area of the floor of a room is 192 square feet, and the room is 16 feet long. How wide is the room?
10. If one of three numbers is 160, the two others are equal, and the sum of the three is 384, find the other two numbers.
11. When a train is running at the rate of 60 miles an hour, how many feet does it run in one second?
12. A rural mail carrier went over his route 307 times in a year. If his route is $27\frac{1}{2}$ miles long, how far did he travel?
13. At $37\frac{1}{2}$ cents a pound how much must a druggist pay for 35 boxes of candy, if each box holds 5 pounds?
14. Add $3\frac{7}{100}$ and $4\frac{3}{1000}$; then write them in decimal form and add again. Which process is the shorter one?

15. Draw a plan of the floor of a schoolroom which is 48 feet long and 32 feet wide, using a scale of 8 feet to 1 inch.

16. How many hours does Barbara study in the 27 evenings of a school month, if her average time to an evening is 2 hr. 40 min.?

17. Emeline wanted a hat that was marked \$3, but she waited until it was marked " $\frac{1}{4}$ off." How much did she save by waiting?

18. The area of the bottom of a pasteboard box is 24 square inches. If the box will hold just 72 cubic inches of sugar, how many inches deep is it?

19. A motor truck is built to carry $2\frac{1}{2}$ tons. If a barrel of flour contains 196 pounds of flour, and the barrel weighs 15 pounds, how many barrels of flour will the truck carry safely?

20. The sum of three numbers is 128. One of the numbers is twice as large as the other, and the third number is 68. Find each of the two numbers whose relation is given.

21. If you are on a train which runs continuously at the rate of 40 miles an hour, how many miles should you travel between 3 P.M. and 5.15 P.M.? Between 6.05 P.M. and 7.15 P.M.?

22. A farmer feeds his cows so that they get 3 pounds of corn to each 7 pounds of hay. If each cow gets 40 pounds of feed daily, how many pounds of corn and how many pounds of hay does she get?

23. Multiply 27 by 35. Then write the numbers in Roman notation and try to multiply them. Why is it that you find the task so difficult? Is the addition of 30 and 57 a simpler problem? Try to add 39 and 41 expressed in Roman numerals.

24. Pearl earned \$4.50 for the Y. M. C. A. War Fund, $\frac{2}{3}$ as much for the K. of C. Fund, and $\frac{5}{8}$ as much for the Red Cross. How much did she earn for each of the last two funds, and how much did she earn in all?

25. The length of a common brick is 8 inches, the width is 4 inches, and the thickness is 2 inches. Find the total area of the surface of such a brick in square inches. What fractional part of a square foot is the total area of a brick?

26. Divide 3724 by 25 and test your work. How would you change the dividend so that there will be no remainder? Can you suggest two possible ways to do this? How do you test your work when there is no remainder?

27. In planting an acre of beans a farmer used $\frac{1}{2}$ bushel of seed, and the yield from the acre was 10 bushels. What fractional part of the crop was the amount of seed he used? What decimal part of the crop was the seed used?

28. Two trains start from points 720 miles apart and go towards each other. If they start at the same time, and if each train averages 45 miles an hour, in how many hours will they meet? How far will each train run?

29. In making an apron Helen uses $2\frac{1}{2}$ yd. of gingham costing 24 cents a yard, 3 yd. of braid costing 8 cents a yard, and a spool of thread costing 6 cents. If she sells this apron for \$1.25, what profit does she make?

30. In 1917 the national wealth of the United States was estimated at \$187,739,000,000. If the population of the United States at that time was 100,000,000, what was the average wealth per person?

31. Jean made a doll's blanket that was 30 inches long and 20 inches wide. How much did the ribbon for binding the edge of the blanket cost, if the price per yard was 24¢?

32. How many houses, each 30 feet wide, can be built on one side of a street that is 720 feet long? How many houses 22.5 feet wide can be built on the same ground?

33. The average yield of wheat in the United States from 1906 to 1916 was 13.2 bushels per acre. At this rate of production, how many bushels of wheat may be expected from 19,500,000 acres?

34. How many small boxes, each 4 in. long, 3 in. wide, and 2 in. deep, can be placed in a packing-box whose dimensions are 24 in. by 9 in. by 6 in.? (Use cancellation.)

35. Elmer spent $\frac{2}{3}$ of $\frac{5}{8}$ of his earnings on clothing, and $\frac{5}{6}$ of $\frac{1}{2}$ of them on his vacation. If he earned \$48, what sum did he pay for his clothing? What sum did he spend on his vacation? If he saved the remainder, how much did he save?

36. A valuable cow gave 8560 pounds of milk in a year, and it was found that each pound of milk gave, on an average, .042 pound of butter fat. How many pounds of butter fat did she produce in a year?

37. Find the area of the four walls of your schoolroom. Then find the combined area of the doors and the windows in it. What fractional part of the whole area of the walls is the total area of the doors and windows in the room?

38. Harry finds that he walks just $\frac{5}{8}$ of a mile in going from his home to his school. If he attends school 165 days in a year, how far does he walk in going to and returning from school?

39. The average consumption of fat per person per year in the United States is estimated at 3.42 lb. If the population is 100,000,000, how many tons of fat are consumed per year in the United States?

40. Sixteen Camp Fire Girls shared equally the cost of the lunch they took on a trip to the woods. If the sandwiches cost \$1.25, the fruit 60¢, the olives 40¢, the cheese 35¢, and the lemonade 60¢, how much was each girl's share of the cost?

41. Tom added some numbers and made a mistake of 1 in the hundreds' column, and Gertrude added the same numbers and made a mistake of 1 in the tens' column. Tom's mistake was how many times as great as Gertrude's mistake?

42. Jack saved 50 cents each week from January 1 to June 30, and then earned \$28 a month through July, August, and September. If he saved .75 of his earnings from July 1 to September 30, and placed the sum in the bank with the earlier savings, how much did he have in the bank?

43. A school district paid \$45 a month for carrying 21 school children to school and home again on each of the 23 school days in that month. Find to the nearest tenth of a cent the cost of carrying one child one way. How does the cost compare with an ordinary trolley fare of 5 cents for the same distance?

44. In one week Jack received marks as follows :

	MON.	TUE.	WED.	THU.	FRI.
Reading	90	85	94	91	88
Spelling	96	100	90	100	100
Arithmetic	95	96	91	99	87
History	90	80	90	75	60
Geography	88	84	81	87	88

Find his average mark on each of the five days, and his average mark for the week.

SIXTH YEAR

REVIEW

REVIEW OF ADDITION

ORAL PRACTICE

Any of the following groups may be used for speed drills.

- | | | |
|---------------------|-----------------------|-----------------|
| 1. $2+3+5+4$. | 6. $4+7+6+9$. | 11. $9+3+8+5$. |
| 2. $3+4+6+5$. | 7. $5+8+3+9$. | 12. $6+7+9+5$. |
| 3. $3+5+7+6$. | 8. $8+6+9+4$. | 13. $6+8+7+3$. |
| 4. $4+6+5+8$. | 9. $9+1+3+6$. | 14. $9+6+4+8$. |
| 5. $5+7+9+7$. | 10. $7+5+9+8$. | 15. $9+7+8+9$. |
| 16. $3+2+4+7+5+3$. | 22. $8+4+3+5+6+9+2$. | |
| 17. $4+6+5+8+7+6$. | 23. $7+8+6+5+9+8+4$. | |
| 18. $5+3+4+6+5+7$. | 24. $9+3+6+7+5+8+5$. | |
| 19. $6+5+9+8+7+6$. | 25. $7+4+7+6+5+8+3$. | |
| 20. $5+8+7+9+4+5$. | 26. $6+8+9+5+4+7+8$. | |
| 21. $8+5+9+3+9+1$. | 27. $9+7+3+9+6+5+7$. | |

Add:

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
28.	13	14	15	16	13	11	12	17
	<u>5</u>	<u>6</u>	<u>8</u>	<u>7</u>	<u>3</u>	<u>8</u>	<u>5</u>	<u>8</u>
29.	14	23	24	22	21	25	18	23
	<u>7</u>	<u>8</u>	<u>5</u>	<u>3</u>	<u>6</u>	<u>5</u>	<u>8</u>	<u>6</u>
30.	21	23	35	44	62	51	32	28
	<u>12</u>	<u>17</u>	<u>6</u>	<u>8</u>	<u>9</u>	<u>14</u>	<u>12</u>	<u>19</u>
31.	36	33	44	47	52	43	59	51
	<u>16</u>	<u>24</u>	<u>19</u>	<u>27</u>	<u>14</u>	<u>39</u>	<u>29</u>	<u>35</u>

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
32.	54	85	65	56	52	61	72	77
	<u>34</u>	<u>37</u>	<u>49</u>	<u>49</u>	<u>46</u>	<u>53</u>	<u>58</u>	<u>59</u>
33.	66	62	87	72	91	83	79	77
	<u>48</u>	<u>57</u>	<u>67</u>	<u>64</u>	<u>79</u>	<u>75</u>	<u>63</u>	<u>26</u>
34.	58	45	38	58	67	54	92	85
	<u>21</u>	<u>22</u>	<u>27</u>	<u>33</u>	<u>64</u>	<u>53</u>	<u>74</u>	<u>47</u>
35.	92	71	95	66	64	79	82	89
	<u>49</u>	<u>34</u>	<u>25</u>	<u>43</u>	<u>52</u>	<u>73</u>	<u>78</u>	<u>67</u>
36.	35	38	64	41	73	55	74	56
	<u>27</u>	<u>32</u>	<u>26</u>	<u>19</u>	<u>57</u>	<u>45</u>	<u>34</u>	<u>26</u>
37.	98	86	55	35	82	93	80	68
	<u>17</u>	<u>59</u>	<u>37</u>	<u>24</u>	<u>39</u>	<u>30</u>	<u>44</u>	<u>58</u>
38.	43	69	84	97	80	79	77	78
	<u>32</u>	<u>45</u>	<u>68</u>	<u>50</u>	<u>38</u>	<u>49</u>	<u>69</u>	<u>76</u>
39.	97	69	97	56	88	98	99	97
	<u>92</u>	<u>65</u>	<u>47</u>	<u>55</u>	<u>79</u>	<u>77</u>	<u>84</u>	<u>69</u>

Add, observing combinations that make ten :

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
40.	5	8	7	9	5	4	8	3	4
	4	2	7	1	3	4	7	5	8
	6	5	8	8	2	6	9	7	3
	<u>7</u>	<u>6</u>	<u>2</u>	<u>2</u>	<u>9</u>	<u>8</u>	<u>6</u>	<u>9</u>	<u>5</u>
41.	6	8	6	7	5	8	9	6	8
	5	6	2	8	7	5	6	8	9
	7	4	8	4	7	7	8	9	7
	<u>4</u>	<u>6</u>	<u>4</u>	<u>6</u>	<u>8</u>	<u>9</u>	<u>5</u>	<u>4</u>	<u>7</u>

BLACKBOARD PRACTICE

Add:

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
1.	34	31	44	85	25	75	47
	76	24	65	55	84	37	54
	52	68	51	38	76	88	36
	<u>38</u>	<u>27</u>	<u>49</u>	<u>22</u>	<u>37</u>	<u>82</u>	<u>75</u>
2.	27	65	87	47	56	76	83
	81	54	32	68	75	38	34
	72	31	71	52	84	51	78
	<u>35</u>	<u>78</u>	<u>64</u>	<u>35</u>	<u>85</u>	<u>97</u>	<u>65</u>
3.	23	37	57	62	58	41	78
	42	42	32	33	67	79	87
	18	51	43	25	77	85	46
	<u>37</u>	<u>19</u>	<u>50</u>	<u>75</u>	<u>83</u>	<u>96</u>	<u>94</u>

Add, and test by adding in the opposite direction:

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
4.	34	36	53	87	72	50	88
	62	45	46	26	58	89	40
	35	24	67	71	47	77	78
	78	57	71	58	84	41	97
	<u>47</u>	<u>75</u>	<u>38</u>	<u>66</u>	<u>39</u>	<u>69</u>	<u>49</u>
5.	71	65	16	46	64	54	48
	24	54	55	98	89	87	76
	56	84	48	69	95	98	89
	67	73	57	74	47	79	95
	53	69	96	77	76	85	58
	<u>45</u>	<u>42</u>	<u>24</u>	<u>45</u>	<u>53</u>	<u>47</u>	<u>77</u>

	(a)	(b)	(c)	(d)	(e)	(f)	
6.	74	57	94	56	43	86	
	65	85	73	45	87	58	
	37	86	65	78	67	48	
	18	86	52	49	84	85	
	28	48	95	58	63	96	
	47	52	85	96	76	68	
	<u>74</u>	<u>36</u>	<u>37</u>	<u>57</u>	<u>68</u>	<u>87</u>	
7.	143	467	354	543	584	757	
	818	143	247	515	764	895	
	423	325	865	418	543	615	
	315	265	654	865	787	768	
	<u>212</u>	<u>576</u>	<u>346</u>	<u>745</u>	<u>654</u>	<u>674</u>	
8.	129	676	476	677	897	654	
	231	565	141	876	578	786	
	313	447	518	987	464	895	
	445	563	765	758	978	985	
	<u>776</u>	<u>654</u>	<u>576</u>	<u>834</u>	<u>876</u>	<u>998</u>	
9.	4567	10.	5161	11.	8574	12.	6476
	4756		2536		4670		8762
	5364		2005		8907		9087
	7586		7685		4611		1987
	<u>1986</u>		<u>6756</u>		<u>5667</u>		<u>7181</u>
13.	46576	14.	64775	15.	75648	16.	57764
	75648		83388		41176		90018
	45116		58870		50019		58435
	65784		72435		75364		11980
	87756		69283		58700		50000
	21314		51708		31411		87999

SUBTRACTION

157

17.	3685576	18.	68757	19.	4855000	20.	65
	756487		9986114		98000		10075
	5776		87		100		78654
	118000		7798		1900		6015
	768		116759		65000		1000000
	7588		8		17500		345000
	87000		67850		6400		171
	675		77		400		19905
	1125364		7543		1980000		1150000
	678767		8777		8000		500
	888		564000		700		5
	765		655		11150000		25450000
	7765		10000		37000		1750000
	<u>15468110</u>		<u>108</u>		<u>240000</u>		<u>185180</u>

REVIEW OF SUBTRACTION

ORAL PRACTICE

This exercise includes all of the subtraction combinations.

Subtract :

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
1.	22	35	46	57	36	57	48
	<u>12</u>	<u>13</u>	<u>24</u>	<u>12</u>	<u>31</u>	<u>25</u>	<u>13</u>
2.	36	68	69	48	59	79	59
	<u>26</u>	<u>54</u>	<u>32</u>	<u>35</u>	<u>51</u>	<u>48</u>	<u>46</u>
3.	47	69	78	97	89	88	97
	<u>41</u>	<u>24</u>	<u>31</u>	<u>76</u>	<u>23</u>	<u>67</u>	<u>57</u>
4.	32	44	55	46	44	53	36
	<u>19</u>	<u>18</u>	<u>27</u>	<u>28</u>	<u>35</u>	<u>37</u>	<u>19</u>

	(a)	(b)	(c)	(d)	(e)	(f)
5.	453° <u>286</u>	672 <u>394</u>	481 <u>387</u>	565 <u>378</u>	874 <u>596</u>	313 <u>204</u>
6.	852 <u>543</u>	981 <u>798</u>	982 <u>499</u>	908 <u>499</u>	361 <u>284</u>	472 <u>316</u>
7.	841 <u>586</u>	801 <u>643</u>	931 <u>772</u>	600 <u>485</u>	501 <u>219</u>	876 <u>590</u>
8.	839 <u>475</u>	905 <u>698</u>	870 <u>709</u>	560 <u>190</u>	989 <u>798</u>	976 <u>889</u>

BLACKBOARD PRACTICE

Subtract and test:

1.	64 <u>35</u>	57 <u>38</u>	36 <u>17</u>	42 <u>28</u>	50 <u>32</u>	82 <u>59</u>	73 <u>47</u>	56 <u>28</u>	78 <u>59</u>
2.	86 <u>54°</u>	57 <u>39</u>	97 <u>79</u>	86 <u>58</u>	49 <u>39</u>	87 <u>59</u>	90 <u>61</u>	77 <u>59</u>	91 <u>73</u>
3.	364 <u>324</u>	576 <u>245</u>	465 <u>267</u>	477 <u>409</u>	577 <u>376</u>	364 <u>299</u>	765 <u>569</u>	768 <u>709</u>	657 <u>589</u>
4.	4327 <u>2579</u>	5364 <u>4388</u>	5009 <u>3119</u>	6570 <u>5467</u>	8091 <u>7188</u>	5700 <u>4469</u>	6571 <u>4188</u>		
5.	16457 <u>12756</u>	24539 <u>19667</u>	46679 <u>36645</u>	60008 <u>38997</u>	86755 <u>76809</u>	99811 <u>76099</u>			
6.	85769 <u>54677</u>	85769 <u>71098</u>	75640 <u>57770</u>	65000 <u>39999</u>	87902 <u>63987</u>	90000 <u>41345</u>			
7.	100675 <u>67548</u>	186754 <u>86758</u>	265776 <u>199887</u>	346678 <u>277788</u>	788801 <u>699099</u>				

Subtraction of United States Money

BLACKBOARD PRACTICE

Subtract and test :

1. $\begin{array}{r} \$45.75 \\ 19.25 \\ \hline \end{array}$	2. $\begin{array}{r} \$50.80 \\ 32.54 \\ \hline \end{array}$	3. $\begin{array}{r} \$62.50 \\ 45.65 \\ \hline \end{array}$	4. $\begin{array}{r} \$75.00 \\ 67.97 \\ \hline \end{array}$
--	--	--	--

5. $\begin{array}{r} \$87.75 \\ 33.76 \\ \hline \end{array}$	6. $\begin{array}{r} \$92.07 \\ 60.08 \\ \hline \end{array}$	7. $\begin{array}{r} \$110.87 \\ 82.19 \\ \hline \end{array}$	8. $\begin{array}{r} \$135.90 \\ 89.75 \\ \hline \end{array}$
--	--	---	---

9. $\begin{array}{r} \$180.09 \\ 97.66 \\ \hline \end{array}$	10. $\begin{array}{r} \$256.08 \\ 154.18 \\ \hline \end{array}$	11. $\begin{array}{r} \$375.10 \\ 287.86 \\ \hline \end{array}$	12. $\begin{array}{r} \$567.91 \\ 430.08 \\ \hline \end{array}$
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13. $\begin{array}{r} \$675.40 \\ 191.91 \\ \hline \end{array}$	14. $\begin{array}{r} \$875.07 \\ 350.56 \\ \hline \end{array}$	15. $\begin{array}{r} \$896.11 \\ 579.08 \\ \hline \end{array}$	16. $\begin{array}{r} \$925.00 \\ 770.19 \\ \hline \end{array}$
---	---	---	---

17. $\begin{array}{r} \$1250.00 \\ 964.75 \\ \hline \end{array}$	18. $\begin{array}{r} \$1967.42 \\ 1534.78 \\ \hline \end{array}$	19. $\begin{array}{r} \$2463.80 \\ 1363.81 \\ \hline \end{array}$
--	---	---

- | | |
|------------------------------|----------------------------------|
| 20. $\$3546.88 - \$2536.80.$ | 27. $\$334575.97 - \$364.08.$ |
| 21. $\$3500.00 - \$2109.11.$ | 28. $\$118900.00 - \$1189.00.$ |
| 22. $\$4650.95 - \$3988.15.$ | 29. $\$254000.50 - \$35467.18.$ |
| 23. $\$7800.00 - \$5673.16.$ | 30. $\$450000.75 - \$39111.17.$ |
| 24. $\$8100.00 - \$7750.76.$ | 31. $\$1345756.00 - \54109.11 |
| 25. $\$9012.34 - \$8676.08.$ | 32. $\$3546800.00 - \$8765.47.$ |
| 26. $\$9000.00 - \$3456.78.$ | 33. $\$836457.87 - \$675811.00.$ |

REVIEW OF MULTIPLICATION

For purposes of review the Multiplication tables are given in a condensed form. The tables should be memorized.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144
13	26	39	52	65	78	91	104	117	130	143	156
14	28	42	56	70	84	98	112	126	140	154	168
15	30	45	60	75	90	105	120	135	150	165	180

Using the Table. The numbers at the extreme left may be regarded as multipliers, and the top horizontal line as multiplicands. Each horizontal line, therefore, is a table for the number beginning that line. Thus:

To find the product of 8×9 .

8 (from the left column) times 9 (from the top line) gives 72 (on a line with 8, and under 9).

ORAL PRACTICE

Give orally the following products:

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. 4×14 . | 4. 7×19 . | 7. 5×20 . | 10. 8×16 . |
| 2. 3×17 . | 5. 4×43 . | 8. 3×32 . | 11. 9×15 . |
| 3. 8×21 . | 6. 6×17 . | 9. 7×41 . | 12. 4×42 . |

BLACKBOARD PRACTICE

Copy and find the products of the following :

1. $\begin{array}{r} 154 \\ \underline{3} \end{array}$	$\begin{array}{r} 175 \\ \underline{4} \end{array}$	$\begin{array}{r} 154 \\ \underline{7} \end{array}$	$\begin{array}{r} 162 \\ \underline{8} \end{array}$	$\begin{array}{r} 153 \\ \underline{6} \end{array}$	$\begin{array}{r} 109 \\ \underline{7} \end{array}$	$\begin{array}{r} 187 \\ \underline{5} \end{array}$	$\begin{array}{r} 192 \\ \underline{7} \end{array}$
2. $\begin{array}{r} 234 \\ \underline{4} \end{array}$	$\begin{array}{r} 209 \\ \underline{5} \end{array}$	$\begin{array}{r} 218 \\ \underline{6} \end{array}$	$\begin{array}{r} 265 \\ \underline{4} \end{array}$	$\begin{array}{r} 269 \\ \underline{7} \end{array}$	$\begin{array}{r} 278 \\ \underline{6} \end{array}$	$\begin{array}{r} 280 \\ \underline{5} \end{array}$	$\begin{array}{r} 285 \\ \underline{8} \end{array}$
3. $\begin{array}{r} 329 \\ \underline{5} \end{array}$	$\begin{array}{r} 334 \\ \underline{6} \end{array}$	$\begin{array}{r} 357 \\ \underline{3} \end{array}$	$\begin{array}{r} 309 \\ \underline{7} \end{array}$	$\begin{array}{r} 364 \\ \underline{6} \end{array}$	$\begin{array}{r} 375 \\ \underline{8} \end{array}$	$\begin{array}{r} 387 \\ \underline{6} \end{array}$	$\begin{array}{r} 380 \\ \underline{8} \end{array}$
4. $\begin{array}{r} 458 \\ \underline{4} \end{array}$	$\begin{array}{r} 473 \\ \underline{5} \end{array}$	$\begin{array}{r} 476 \\ \underline{6} \end{array}$	$\begin{array}{r} 481 \\ \underline{7} \end{array}$	$\begin{array}{r} 485 \\ \underline{8} \end{array}$	$\begin{array}{r} 491 \\ \underline{8} \end{array}$	$\begin{array}{r} 492 \\ \underline{7} \end{array}$	$\begin{array}{r} 499 \\ \underline{9} \end{array}$
5. $\begin{array}{r} 508 \\ \underline{3} \end{array}$	$\begin{array}{r} 519 \\ \underline{4} \end{array}$	$\begin{array}{r} 523 \\ \underline{5} \end{array}$	$\begin{array}{r} 531 \\ \underline{5} \end{array}$	$\begin{array}{r} 542 \\ \underline{6} \end{array}$	$\begin{array}{r} 557 \\ \underline{6} \end{array}$	$\begin{array}{r} 564 \\ \underline{7} \end{array}$	$\begin{array}{r} 571 \\ \underline{7} \end{array}$
6. $\begin{array}{r} 623 \\ \underline{3} \end{array}$	$\begin{array}{r} 630 \\ \underline{5} \end{array}$	$\begin{array}{r} 647 \\ \underline{7} \end{array}$	$\begin{array}{r} 655 \\ \underline{6} \end{array}$	$\begin{array}{r} 667 \\ \underline{8} \end{array}$	$\begin{array}{r} 673 \\ \underline{7} \end{array}$	$\begin{array}{r} 687 \\ \underline{9} \end{array}$	$\begin{array}{r} 688 \\ \underline{7} \end{array}$
7. $\begin{array}{r} 734 \\ \underline{5} \end{array}$	$\begin{array}{r} 743 \\ \underline{6} \end{array}$	$\begin{array}{r} 758 \\ \underline{7} \end{array}$	$\begin{array}{r} 769 \\ \underline{6} \end{array}$	$\begin{array}{r} 777 \\ \underline{7} \end{array}$	$\begin{array}{r} 783 \\ \underline{8} \end{array}$	$\begin{array}{r} 788 \\ \underline{7} \end{array}$	$\begin{array}{r} 790 \\ \underline{8} \end{array}$

BLACKBOARD PRACTICE

Find the product of :

- | | | |
|----------------------|----------------------------|--------------------------|
| 1. $30 \times 273.$ | 5. $100 \times 327.$ | 9. $1000 \times 342.$ |
| 2. $40 \times 368.$ | 6. $200 \times 450.$ | 10. $50 \times \$12.45.$ |
| 3. $60 \times 394.$ | 7. $500 \times 798.$ | 11. $60 \times \$48.05.$ |
| 4. $70 \times 409.$ | 8. $700 \times 904.$ | 12. $90 \times \$57.80.$ |
| 13. $38 \times 564.$ | 17. $100 \times \$110.10.$ | |
| 14. $46 \times 738.$ | 18. $200 \times \$1400.$ | |
| 15. $53 \times 927.$ | 19. $300 \times \$5610.$ | |
| 16. $69 \times 869.$ | 20. $2000 \times \$19000.$ | |

21. 1458 <u>34</u>	22. 1627 <u>42</u>	23. 1925 <u>53</u>	24. 3407 <u>66</u>	25. 4005 <u>87</u>
26. 5014 <u>112</u>	27. 7403 <u>245</u>	28. 7043 <u>507</u>	29. 8970 <u>392</u>	30. 6578 <u>829</u>
31. \$2100 <u>32</u>	32. \$4579 <u>65</u>	33. \$8708 <u>87</u>	34. \$4117 <u>398</u>	35. \$5000 <u>904</u>
36. \$43.02 <u>7.00</u>	37. \$36.80 <u>5.00</u>	38. \$6784 <u>707</u>	39. \$78.28 <u>8.14</u>	40. \$52.68 <u>9.40</u>

How to Plan the Solution of a Problem. In the solution of the problems that follow you will find the following steps exceedingly helpful.

1. *Read the problem carefully.*
2. *Decide just what steps you must take to solve the problem.*
3. *Indicate those steps by writing upon the blackboard or upon paper the operations you intend to perform.*

Illustration :

A painter received 45¢ each hour for a day of 8 hours. How much should he receive for 12 days of labor of the same number of hours each, and at the same rate per hour ?

1. For 8 hours of labor at 45¢ each he receives

$$8 \times 45¢ \text{ or } 8 \times \$.45$$

2. For 12 days of the same length, and at the same rate, he will receive

$$12 \times 8 \times \$.45. \text{ (The work you plan to do.)}$$

This practice of attempting to see in advance just what you must do before you actually perform the steps is called “visualizing” a problem.

MULTIPLICATION IN BUSINESS AND IN THE HOME

WRITTEN APPLICATIONS

The Business Man's Problems in Multiplication

1. How many pounds of sugar will a grocer sell in 26 days, if his average daily sales are 65 pounds?
2. A merchant paid his clerk 30 cents an hour for a day of 10 hours. How much did he pay him in 26 such days?
3. A supply house sold 150 adjustable desks to a school board, at \$4.25 each. What was the amount of the sale?
4. A standard case of eggs contains 30 dozen, and a dealer buys 23 cases at an average price of 39¢ a dozen. How much do they cost him?
5. A commission man finds that his earnings average \$65 per week. At this rate how much are his total earnings for 48 weeks?
6. A mine owner finds that his coal costs him \$3.25 per ton at the mine. At this rate what is the cost at the mine of a pile of coal containing 1575 tons?

The Farmer's Problems in Multiplication

7. A fence is built to inclose a field. The distance around the field is 1020 feet. What does the fence cost at \$.42 a foot?
8. A herd of 42 cows yields an average of 13 quarts of milk each for a period of 40 days. What is the total amount of milk produced by the herd in that time?
9. A farmer finds that 5 pounds of corn daily are needed to feed each of his 75 cows. How many pounds of corn will he need to feed all of them for 60 days?
10. A farm laborer is paid \$28 a month from the beginning of April to the end of October. What amount is paid to him in that time?

11. 8 thousand shingles are required for a barn roof, and they are bought at a cost of \$3.25 a thousand. Adding a hauling charge of \$1.25, find the cost of the shingles.

12. A field of 54 acres was planted to potatoes with 11 bushels of seed to the acre, and the yield was 130 bushels to the acre. How many bushels of seed were planted? How many bushels of potatoes were produced?

The Housekeeper's Problems in Multiplication

13. What is the cost of 15 tons of coal at \$6.75 per ton?

14. A maid received \$6.50 per week. What did she receive in a year, or fifty-two weeks, at that rate?

15. A woman bought 54 quarts of strawberries, for which she paid 18¢ a quart. How much did she pay for them?

16. A family used 2 quarts of milk daily for sixty days. At 8¢ a quart, how much did this milk cost?

17. A family uses 12 pounds of meat each week. At an average price of \$.22 a pound how much will their meat cost them in a year?

18. 24 yards of carpet at \$1.85 a yard were used for a room. How much did this carpet cost?

19. The fruit for 100 jars of preserves cost a housekeeper 9¢ per jar, and the jars cost 4¢ each. What was the total cost of the fruit and the jars?

20. Painted linoleum for a kitchen costs 50¢ a square yard. If the floor of this kitchen requires 32 yards, how much will it cost to cover it with this linoleum?

21. A family moving into a new house must buy 28 electric light bulbs at 30¢ each, 4 at 40¢ each, and 3 at 55¢ each. How much will all of these bulbs cost?

REVIEW OF DIVISION

BLACKBOARD PRACTICE

Find the quotients of the following divisions :

- | | | | |
|-----------------------|------------------------|-------------------------|----------------------|
| 1. $248 \div 11$. | 5. $816 \div 51$. | 9. $4858 \div 14$. | 13. $8694 \div 21$. |
| 2. $861 \div 21$. | 6. $854 \div 61$. | 10. $3495 \div 15$. | 14. $5725 \div 25$. |
| 3. $961 \div 31$. | 7. $923 \div 71$. | 11. $5780 \div 20$. | 15. $8903 \div 29$. |
| 4. $697 \div 41$. | 8. $972 \div 81$. | 12. $5706 \div 18$. | 16. $9078 \div 34$. |
| 17. $9718 \div 43$. | 24. $29889 \div 41$. | 31. $174522 \div 59$. | |
| 18. $9888 \div 52$. | 25. $81190 \div 46$. | 32. $179279 \div 61$. | |
| 19. $9891 \div 63$. | 26. $61054 \div 49$. | 33. $1569165 \div 65$. | |
| 20. $11178 \div 27$. | 27. $82728 \div 54$. | 34. $1380988 \div 62$. | |
| 21. $23264 \div 32$. | 28. $86681 \div 61$. | 35. $2333331 \div 77$. | |
| 22. $25650 \div 38$. | 29. $94770 \div 78$. | 36. $2753542 \div 94$. | |
| 23. $32136 \div 39$. | 30. $105112 \div 56$. | 37. $242591 \div 97$. | |

BLACKBOARD PRACTICE

Find the quotients, and the remainders, if any, of :

- | | |
|--------------------------|---------------------------|
| 1. $236625 \div 375$. | 14. $1126248 \div 562$. |
| 2. $243412 \div 604$. | 15. $1259268 \div 202$. |
| 3. $403452 \div 136$. | 16. $1966707 \div 621$. |
| 4. $458763 \div 111$. | 17. $2560272 \div 212$. |
| 5. $464370 \div 345$. | 18. $3114180 \div 365$. |
| 6. $504944 \div 209$. | 19. $4122841 \div 321$. |
| 7. $530660 \div 628$. | 20. $4729104 \div 216$. |
| 8. $620630 \div 265$. | 21. $5997654 \div 342$. |
| 9. $648186 \div 506$. | 22. $6646416 \div 453$. |
| 10. $757442 \div 413$. | 23. $7564326 \div 462$. |
| 11. $866970 \div 195$. | 24. $8701044 \div 348$. |
| 12. $982581 \div 321$. | 25. $976517 \div 241$. |
| 13. $1056240 \div 326$. | 26. $11216736 \div 232$. |

- | | |
|---------------------------|----------------------------|
| 27. $11234275 \div 325$. | 32. $17412096 \div 416$. |
| 28. $13977718 \div 206$. | 33. $190222244 \div 406$. |
| 29. $14434434 \div 248$. | 34. $19422156 \div 314$. |
| 30. $14896404 \div 387$. | 35. $22693488 \div 372$. |
| 31. $16571295 \div 507$. | 36. $23103018 \div 397$. |

BLACKBOARD PRACTICE

Find the quotient, and the remainder, if any, of :

- | | |
|------------------------|--------------------------|
| 1. $23219 \div 60$. | 9. $470000 \div 451$. |
| 2. $53708 \div 69$. | 10. $769877 \div 638$. |
| 3. $68008 \div 63$. | 11. $1056577 \div 326$. |
| 4. $200204 \div 81$. | 12. $1469238 \div 509$. |
| 5. $319409 \div 104$. | 13. $2576923 \div 611$. |
| 6. $345688 \div 60$. | 14. $4900627 \div 801$. |
| 7. $413345 \div 131$. | 15. $5000000 \div 931$. |
| 8. $697896 \div 164$. | 16. $6050000 \div 909$. |

WRITTEN APPLICATIONS

The Business Man's Problems in Division

1. A grocer pays an annual rental of \$540. How much does he pay per month?
2. If he pays \$5512 each year for clerk hire, how much does he pay for clerk hire each month?
3. A consignment of flour consisting of 225 barrels cost \$1080. How much did each barrel cost?
4. A grocer put 1920 pounds of flour in sacks holding 24 pounds each. How many sacks did he need?
5. A wholesale dealer buys 24,000 cakes of soap in boxes holding 150 cakes each. How many boxes of soap does he buy?

6. A dealer in clothing pays \$1800 for 144 suits. How much does he pay for each suit?

7. A book publisher received \$744 for forty-eight sets of Dickens's Works. How much did he receive for each set?

8. A jeweler paid \$2150 for 576 watches. What was the average cost of these watches?

9. The total amount paid for lighting a store for one year was \$212.56. What was the average amount paid per month?

10. A merchant finds that his total cash sales for a period of 28 business days were \$4846.80. What were his average daily sales during that time?

11. A local newspaper charges a merchant \$1170 for an advertising space in 26 issues. What was the amount paid for each issue?

12. A merchant buys a cash register for \$225, and agrees to pay for it in 18 equal monthly payments. How much is each of his monthly payments?

13. A piano dealer sells a piano for \$420, and is to receive payment in monthly installments of \$12 each. In how many months should the amount be paid?

14. A manufacturer employs 208 men, and his payroll for 6 working days is \$3744. What is the average amount paid to each man for a day's work?

15. A steel coal car can carry 100,000 pounds of coal. How many tons of coal can be carried by a train of 35 such cars?

16. The freight bill for a shipment of 85 tons of coal was \$130.90. What was the freight charge per ton?

17. The total profits of a merchant for twelve years amounted to \$39,720. What was his average annual profit during that time?

18. A merchant paid \$720 rent, \$5600 for clerk hire, and \$180 for heat and light in one year. What was his average expense per week for these four items?

The Farmer's Problems in Division

19. A field of 38 acres produced 8904 bushels of potatoes. What was the average yield in bushels per acre?

20. During the month of July, a dairy made 1160 pounds of butter. How much was the average daily production of butter from this dairy?

21. If the dairy in the foregoing problem had 15 cows, what was the average daily yield of butter from each cow?

22. A farm of 78 acres was sold for five thousand eight hundred fifty dollars. What was the average price per acre?

23. A dairyman feeds his herd 560 pounds of hay each day, and he has 28 tons of hay. How many days will this amount of hay feed his herd?

24. A farmer lays a pipe from a spring to one of his barns, the distance being 1856 feet. How many lengths of pipe, each 12 feet long, does he use?

25. A bundle of shingles will cover 250 square feet of surface. How many bundles will be required to cover a roof whose area is 7200 square feet?

26. The feed bills for a dairy farm amounted to \$1267.20 in one year. What was the average cost for feed per month?

27. If 25 cows were fed on the farm referred to in problem 26, what was the average cost for each cow per month?

28. A farmer received \$1935 for hay, and he used the money to purchase young cattle at \$45 each. How many head did he buy?

29. A farmer bought 215 acres of land for \$13,437.50, and paid one half of the purchase price in cash. What amount did he pay in cash per acre?

30. What is the average yield per acre when a field of 42 acres produces 103,320 pounds of timothy? (Give the yield in tons per acre.)

31. A concrete foundation for a barn cost \$469.80. If the foundation contained 3132 cubic feet, what was the cost per cubic foot?

32. An orchard produced 3600 sound apples, and these were packed in boxes holding 48 apples each. How many boxes were packed?

33. A poultryman received \$1134 from the sale of 1575 broilers. What was the average price received for them?

34. The 64 trees in an orchard produced an average of 3 bushels of apples, and \$220.80 was received for them. What was the average price per bushel?

35. A dairyman feeds an average of 16 pounds of silage daily to each of his cows for 90 days. If in that time he feeds 30,240 pounds, how many cows does he feed?

The Housekeeper's Problems in Division

36. 9 tons of coal cost a housekeeper \$45. What was the cost of the coal per ton?

37. A housewife bought a 25-pound sack of sugar for \$2.00. What was the price of the sugar per pound?

38. For the 32 window shades in a house the owner paid \$25.60. What was the average cost of these shades?

39. A grocer offered a housekeeper 150 cakes of soap for \$5.25. How much was this per cake?

40. A housekeeper bought thirty-five yards of carpet for \$122.50. How much did she pay per yard?

41. A woman paid \$468 to her servants in 52 weeks. How much were the average weekly payments?

42. The milk used by a family for one year cost \$58.24. What was the average cost per week?

43. A supply of wood which lasted two years cost a housekeeper \$145.60. What was the average weekly cost for wood during that time?

44. The meat bill for a family in the month of January was \$27.90. What was the average daily expense for meat?

45. The total living expenses for a family for one year amounted to \$803.00. What was the average daily living expense of this family during that year?

46. A coal bill for 90 days was \$40.50. What was the average daily coal bill during this time?

47. A housewife paid a grocer in one year \$318.24. What was the average amount paid per week?

48. A woman employed a seamstress and paid her \$21.60 for working 9 hours per day for 8 days. How much did the seamstress receive per hour?

49. What is the average monthly rent, if the total amount paid in a year is \$480?

50. The rent paid by a housekeeper for a year amounted to \$520. Since there are 365 days in a year, what was the rent per day?

REVIEW OF FRACTIONS

ORAL PRACTICE

Add at sight :

1. $\frac{1}{2} + \frac{1}{3}$; $\frac{1}{3} + \frac{1}{4}$; $\frac{1}{4} + \frac{1}{5}$; $\frac{1}{5} + \frac{1}{6}$; $\frac{1}{6} + \frac{1}{7}$.
2. $\frac{1}{2} + \frac{1}{5}$; $\frac{1}{3} + \frac{1}{7}$; $\frac{1}{5} + \frac{1}{9}$; $\frac{1}{7} + \frac{1}{10}$; $\frac{1}{9} + \frac{1}{10}$.
3. $2 + \frac{2}{3}$; $3 + \frac{4}{5}$; $4 + \frac{5}{6}$; $5 + \frac{7}{8}$; $6 + \frac{8}{9}$.
4. $\frac{2}{3} + 5$; $\frac{3}{5} + 6$; $\frac{4}{5} + 7$; $\frac{7}{8} + 9$; $\frac{8}{9} + 10$.
5. $\frac{3}{5} + 6$; $8 + \frac{5}{9}$; $9 + \frac{9}{10}$; $\frac{7}{10} + 10$; $\frac{5}{11} + 12$.

Subtract at sight :

6. $\frac{1}{2} - \frac{1}{3}$; $\frac{1}{2} - \frac{1}{5}$; $\frac{1}{3} - \frac{1}{4}$; $\frac{1}{5} - \frac{1}{7}$; $\frac{1}{4} - \frac{1}{9}$.
7. $\frac{3}{5} - \frac{1}{2}$; $\frac{3}{4} - \frac{1}{3}$; $\frac{3}{4} - \frac{1}{2}$; $\frac{5}{6} - \frac{1}{2}$; $\frac{3}{4} - \frac{2}{3}$.
8. $\frac{5}{6} - \frac{3}{4}$; $\frac{4}{5} - \frac{2}{3}$; $\frac{5}{7} - \frac{3}{5}$; $\frac{10}{11} - \frac{1}{3}$; $\frac{12}{13} - \frac{3}{4}$.
9. $3 - \frac{2}{3}$; $4 - \frac{3}{4}$; $5 - \frac{4}{5}$; $6 - \frac{3}{4}$; $7 - \frac{3}{10}$.
10. $2\frac{1}{2} - \frac{3}{5}$; $2\frac{1}{2} - \frac{3}{4}$; $3\frac{1}{2} - \frac{1}{5}$; $4\frac{1}{2} - \frac{1}{3}$; $5\frac{1}{3} - \frac{5}{6}$.

Multiply at sight :

11. $\frac{1}{2} \times \frac{1}{3}$; $\frac{1}{3} \times \frac{1}{4}$; $\frac{1}{4} \times \frac{1}{5}$; $\frac{1}{5} \times \frac{1}{6}$; $\frac{1}{6} \times \frac{1}{7}$.
12. $\frac{1}{3} \times \frac{2}{5}$; $\frac{1}{4} \times \frac{3}{7}$; $\frac{1}{5} \times \frac{7}{8}$; $\frac{2}{5} \times \frac{4}{7}$; $\frac{2}{7} \times \frac{5}{3}$.
13. $\frac{5}{6} \times \frac{6}{7}$; $\frac{6}{7} \times \frac{7}{8}$; $\frac{7}{8} \times \frac{8}{9}$; $\frac{8}{9} \times \frac{9}{10}$; $\frac{9}{10} \times \frac{10}{11}$.
14. $2\frac{1}{2} \times 4$; $3\frac{1}{2} \times 5$; $4\frac{1}{3} \times 6$; $5\frac{1}{2} \times 8$; $6\frac{1}{4} \times 10$.
15. $3\frac{1}{2} \times \frac{3}{7}$; $4\frac{1}{3} \times \frac{3}{5}$; $3\frac{1}{5} \times \frac{5}{7}$; $4\frac{1}{5} \times \frac{5}{7}$; $5\frac{1}{5} \times \frac{10}{13}$.

Divide at sight :

16. $\frac{4}{5} \div 2$; $\frac{8}{9} \div 4$; $\frac{9}{10} \div 3$; $\frac{8}{13} \div 4$; $\frac{16}{19} \div 8$.
17. $\frac{4}{5} \div 3$; $\frac{8}{9} \div 3$; $\frac{9}{10} \div 7$; $\frac{8}{13} \div 5$; $\frac{16}{19} \div 3$.
18. $10 \div \frac{2}{3}$; $10 \div \frac{5}{6}$; $14 \div \frac{7}{9}$; $15 \div \frac{5}{7}$; $18 \div \frac{3}{10}$.
19. $10 \div \frac{3}{2}$; $10 \div \frac{6}{5}$; $14 \div \frac{9}{7}$; $15 \div \frac{7}{5}$; $18 \div \frac{10}{13}$.
20. $3\frac{1}{2} \div 7$; $5\frac{1}{3} \div 8$; $6\frac{1}{4} \div 5$; $7\frac{1}{3} \div 11$; $10\frac{1}{2} \div 7$.
21. $10\frac{2}{5} \div 13$; $12\frac{5}{6} \div 11$; $13\frac{1}{4} \div 11$; $16\frac{1}{4} \div 13$.
22. $2\frac{1}{2} \div \frac{3}{5}$; $3\frac{1}{2} \div \frac{7}{5}$; $4\frac{1}{5} \div \frac{7}{2}$; $4\frac{2}{3} \div 1\frac{1}{6}$; $6\frac{1}{2} \div 3\frac{1}{4}$.

BLACKBOARD PRACTICE

Find the sum of:

- | | | |
|-----------------------------------|--------------------------------------|--|
| 1. $\frac{1}{2} + \frac{1}{3}$. | 16. $\frac{1}{4} + \frac{1}{5}$. | 31. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$. |
| 2. $\frac{2}{3} + \frac{1}{5}$. | 17. $\frac{1}{7} + \frac{1}{9}$. | 32. $\frac{1}{4} + \frac{1}{2} + \frac{1}{5}$. |
| 3. $\frac{1}{4} + \frac{1}{5}$. | 18. $\frac{1}{9} + \frac{1}{10}$. | 33. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$. |
| 4. $\frac{1}{5} + \frac{1}{6}$. | 19. $\frac{1}{10} + \frac{1}{12}$. | 34. $\frac{1}{3} + \frac{1}{5} + \frac{1}{6}$. |
| 5. $\frac{1}{6} + \frac{1}{8}$. | 20. $\frac{2}{9} + \frac{3}{8}$. | 35. $\frac{2}{3} + \frac{3}{5} + \frac{5}{6}$. |
| 6. $\frac{1}{8} + \frac{1}{9}$. | 21. $\frac{3}{10} + \frac{7}{12}$. | 36. $\frac{3}{10} + \frac{5}{12} + \frac{3}{4}$. |
| 7. $\frac{2}{3} + \frac{1}{4}$. | 22. $\frac{4}{11} + \frac{5}{6}$. | 37. $\frac{4}{15} + \frac{1}{10} + \frac{5}{6}$. |
| 8. $\frac{2}{3} + \frac{3}{4}$. | 23. $\frac{5}{11} + \frac{3}{10}$. | 38. $\frac{11}{10} + \frac{3}{8} + \frac{4}{5}$. |
| 9. $\frac{3}{4} + \frac{2}{5}$. | 24. $\frac{5}{12} + \frac{1}{15}$. | 39. $\frac{5}{12} + \frac{5}{6} + \frac{5}{8}$. |
| 10. $\frac{3}{5} + \frac{5}{6}$. | 25. $\frac{3}{14} + \frac{5}{21}$. | 40. $\frac{7}{15} + \frac{3}{10} + \frac{11}{30}$. |
| 11. $\frac{1}{6} + \frac{4}{5}$. | 26. $\frac{4}{15} + \frac{3}{20}$. | 41. $\frac{9}{16} + \frac{3}{32} + \frac{9}{4}$. |
| 12. $\frac{2}{5} + \frac{3}{8}$. | 27. $\frac{5}{18} + \frac{4}{9}$. | 42. $\frac{7}{18} + \frac{5}{27} + \frac{7}{9}$. |
| 13. $\frac{4}{7} + \frac{5}{8}$. | 28. $\frac{11}{18} + \frac{5}{6}$. | 43. $\frac{6}{11} + \frac{3}{22} + \frac{7}{44}$. |
| 14. $\frac{6}{7} + \frac{5}{6}$. | 29. $\frac{13}{20} + \frac{7}{30}$. | 44. $\frac{19}{13} + \frac{4}{39} + \frac{11}{78}$. |
| 15. $\frac{5}{3} + \frac{3}{5}$. | 30. $\frac{5}{21} + \frac{9}{35}$. | 45. $\frac{6}{11} + \frac{5}{22} + \frac{17}{44}$. |

Find the results in the following subtractions:

- | | | |
|-----------------------------------|-------------------------------------|---------------------------------------|
| 46. $\frac{2}{3} - \frac{1}{4}$. | 56. $\frac{8}{9} - \frac{3}{4}$. | 66. $\frac{11}{12} - \frac{5}{8}$. |
| 47. $\frac{2}{3} - \frac{1}{2}$. | 57. $\frac{9}{10} - \frac{4}{5}$. | 67. $\frac{15}{16} - \frac{3}{8}$. |
| 48. $\frac{2}{3} - \frac{1}{5}$. | 58. $\frac{11}{12} - \frac{5}{9}$. | 68. $\frac{4}{5} - \frac{5}{12}$. |
| 49. $\frac{3}{5} - \frac{1}{2}$. | 59. $\frac{3}{4} - \frac{2}{3}$. | 69. $\frac{7}{8} - \frac{7}{10}$. |
| 50. $\frac{3}{5} - \frac{1}{4}$. | 60. $\frac{3}{4} - \frac{3}{5}$. | 70. $\frac{7}{10} - \frac{5}{12}$. |
| 51. $\frac{4}{5} - \frac{1}{3}$. | 61. $\frac{4}{5} - \frac{5}{7}$. | 71. $\frac{8}{15} - \frac{3}{10}$. |
| 52. $\frac{5}{6} - \frac{3}{4}$. | 62. $\frac{4}{5} - \frac{5}{8}$. | 72. $\frac{15}{16} - \frac{11}{12}$. |
| 53. $\frac{5}{6} - \frac{1}{2}$. | 63. $\frac{5}{6} - \frac{4}{5}$. | 73. $\frac{19}{24} - \frac{5}{36}$. |
| 54. $\frac{6}{7} - \frac{1}{2}$. | 64. $\frac{7}{8} - \frac{3}{7}$. | 74. $\frac{9}{10} - \frac{7}{8}$. |
| 55. $\frac{7}{8} - \frac{2}{3}$. | 65. $\frac{8}{9} - \frac{7}{8}$. | 75. $\frac{19}{36} - \frac{11}{30}$. |

Find the results in the following subtractions :

76. $4 - \frac{2}{3}$.

81. $4\frac{3}{4} - 1\frac{1}{2}$.

86. $7\frac{4}{5} - 3\frac{1}{2}$.

77. $5 - \frac{3}{5}$.

82. $4\frac{3}{5} - 1\frac{1}{3}$.

87. $4\frac{4}{5} - 2\frac{1}{3}$.

78. $6 - \frac{5}{6}$.

83. $5\frac{3}{4} - 1\frac{2}{5}$.

88. $5\frac{5}{6} - 2\frac{3}{5}$.

79. $9 - \frac{5}{7}$.

84. $5\frac{7}{8} - 2\frac{3}{4}$.

89. $5\frac{6}{7} - 3\frac{2}{3}$.

80. $10 - \frac{7}{8}$.

85. $6\frac{7}{9} - 2\frac{1}{3}$.

90. $6\frac{5}{8} - 3\frac{3}{7}$.

Multiply :

91. $2 \times \frac{3}{8}$.

96. $3 \times \frac{7}{12}$.

101. $7 \times \frac{13}{28}$.

92. $2 \times \frac{5}{6}$.

97. $4 \times \frac{5}{18}$.

102. $8 \times \frac{13}{32}$.

93. $3 \times \frac{7}{9}$.

98. $5 \times \frac{13}{20}$.

103. $8 \times \frac{5}{12}$.

94. $3 \times \frac{7}{15}$.

99. $5 \times \frac{14}{15}$.

104. $12 \times \frac{13}{28}$.

95. $4 \times \frac{9}{16}$.

100. $6 \times \frac{13}{18}$.

105. $15 \times \frac{17}{25}$.

Find :

106. $\frac{2}{3}$ of 12.

111. $\frac{9}{10}$ of 72.

116. $\frac{7}{10}$ of 52.

107. $\frac{3}{4}$ of 16.

112. $\frac{11}{12}$ of 80.

117. $\frac{9}{21}$ of 56.

108. $\frac{5}{6}$ of 34.

113. $\frac{13}{16}$ of 84.

118. $\frac{10}{17}$ of 68.

109. $\frac{5}{7}$ of 56.

114. $\frac{11}{15}$ of 66.

119. $\frac{13}{20}$ of 75.

110. $\frac{7}{8}$ of 60.

115. $\frac{17}{24}$ of 90.

120. $\frac{16}{35}$ of 84.

Multiply :

121. $\frac{2}{3} \times \frac{3}{4}$.

126. $\frac{5}{7} \times \frac{14}{15}$.

131. $\frac{11}{13} \times \frac{26}{33}$.

122. $\frac{2}{5} \times \frac{5}{6}$.

127. $\frac{6}{7} \times \frac{49}{54}$.

132. $\frac{15}{17} \times \frac{34}{35}$.

123. $\frac{3}{4} \times \frac{8}{15}$.

128. $\frac{7}{8} \times \frac{64}{63}$.

133. $\frac{16}{21} \times \frac{35}{48}$.

124. $\frac{3}{5} \times \frac{7}{12}$.

129. $\frac{9}{10} \times \frac{28}{33}$.

134. $\frac{20}{27} \times \frac{36}{55}$.

125. $\frac{4}{5} \times \frac{17}{16}$.

130. $\frac{14}{15} \times \frac{33}{36}$.

135. $\frac{21}{35} \times \frac{45}{84}$.

Multiply :

136. $4 \times 8\frac{1}{2}$.

140. $12 \times 15\frac{2}{5}$.

144. $15 \times 10\frac{7}{15}$.

137. $6 \times 9\frac{5}{6}$.

141. $24 \times 21\frac{5}{8}$.

145. $21 \times 13\frac{5}{14}$.

138. $7 \times 10\frac{2}{3}$.

142. $12 \times 18\frac{11}{12}$.

146. $24 \times 21\frac{11}{16}$.

139. $8 \times 14\frac{2}{3}$.

143. $16 \times 20\frac{13}{16}$.

147. $32 \times 25\frac{7}{20}$.

Multiply :

- | | | |
|----------------------------------|---------------------------------|----------------------------------|
| 148. $2\frac{1}{3} \times 15$. | 153. $4\frac{3}{8} \times 16$. | 158. $6\frac{1}{2} \times 30$. |
| 149. $3\frac{1}{2} \times 22$. | 154. $5\frac{4}{7} \times 21$. | 159. $7\frac{1}{5} \times 42$. |
| 150. $4\frac{3}{8} \times 40$. | 155. $4\frac{5}{6} \times 21$. | 160. $8\frac{1}{6} \times 72$. |
| 151. $6\frac{8}{9} \times 63$. | 156. $5\frac{3}{5} \times 15$. | 161. $4\frac{1}{10} \times 20$. |
| 152. $5\frac{9}{10} \times 22$. | 157. $6\frac{2}{9} \times 15$. | 162. $10\frac{5}{4} \times 60$. |

Multiply :

- | | | |
|---|---|--|
| 163. $2\frac{1}{4} \times 3\frac{1}{3}$. | 168. $4\frac{8}{5} \times 7\frac{1}{2}$. | 173. $5\frac{2}{3} \times 4\frac{1}{5}$. |
| 164. $3\frac{1}{3} \times 4\frac{5}{5}$. | 169. $5\frac{5}{8} \times 11\frac{1}{9}$. | 174. $6\frac{1}{6} \times 2\frac{1}{7}$. |
| 165. $3\frac{1}{5} \times 4\frac{1}{4}$. | 170. $6\frac{7}{8} \times 12\frac{4}{11}$. | 175. $7\frac{3}{5} \times 2\frac{2}{19}$. |
| 166. $3\frac{1}{6} \times 5\frac{3}{5}$. | 171. $7\frac{3}{7} \times 24\frac{1}{2}$. | 176. $9\frac{2}{7} \times 4\frac{7}{13}$. |
| 167. $3\frac{4}{7} \times 6\frac{1}{5}$. | 172. $10\frac{2}{9} \times 48\frac{1}{4}$. | 177. $10\frac{5}{8} \times 10\frac{2}{17}$. |

Find the quotient of :

- | | | |
|--------------------------------|--------------------------------|--------------------------------|
| 178. $\frac{10}{11} \div 5$. | 183. $\frac{18}{25} \div 8$. | 188. $\frac{16}{21} \div 12$. |
| 179. $\frac{15}{16} \div 5$. | 184. $\frac{24}{25} \div 10$. | 189. $\frac{22}{27} \div 18$. |
| 180. $\frac{18}{25} \div 6$. | 185. $\frac{32}{45} \div 24$. | 190. $\frac{22}{27} \div 11$. |
| 181. $\frac{20}{27} \div 12$. | 186. $\frac{28}{42} \div 84$. | 191. $\frac{35}{42} \div 14$. |
| 182. $\frac{16}{15} \div 8$. | 187. $\frac{42}{49} \div 56$. | 192. $\frac{21}{50} \div 35$. |

Divide :

- | | | |
|-------------------------------|--|---|
| 193. $6\frac{9}{10}$ by 3. | 203. 5 by $\frac{3}{4}$. | 213. $5\frac{3}{4}$ by $5\frac{1}{8}$. |
| 194. $8\frac{5}{8}$ by 4. | 204. 7 by $\frac{4}{5}$. | 214. $6\frac{1}{4}$ by $1\frac{7}{8}$. |
| 195. $9\frac{6}{7}$ by 6. | 205. 10 by $\frac{6}{9}$. | 215. $7\frac{1}{3}$ by $2\frac{3}{4}$. |
| 196. $10\frac{3}{5}$ by 7. | 206. 12 by $\frac{8}{9}$. | 216. $3\frac{4}{5}$ by $2\frac{8}{15}$. |
| 197. $12\frac{2}{3}$ by 6. | 207. 64 by $\frac{7}{8}$. | 217. $8\frac{2}{5}$ by $1\frac{1}{3}$. |
| 198. $36\frac{5}{8}$ by 7. | 208. $\frac{15}{16}$ by $\frac{5}{12}$. | 218. $7\frac{7}{10}$ by $1\frac{1}{2}$. |
| 199. $60\frac{6}{7}$ by 9. | 209. $2\frac{3}{5}$ by $\frac{3}{4}$. | 219. $5\frac{7}{10}$ by $1\frac{4}{15}$. |
| 200. $64\frac{9}{10}$ by 10. | 210. $3\frac{4}{7}$ by $\frac{4}{5}$. | 220. $6\frac{1}{11}$ by $1\frac{3}{35}$. |
| 201. $90\frac{9}{13}$ by 12. | 211. $5\frac{3}{5}$ by $\frac{3}{5}$. | 221. $7\frac{1}{12}$ by $1\frac{5}{8}$. |
| 202. $120\frac{8}{17}$ by 14. | 212. $8\frac{5}{8}$ by $\frac{7}{8}$. | 222. $8\frac{7}{11}$ by $2\frac{1}{2}$. |

Estimating Results. In many cases you will find it helpful to estimate the answer to a problem before you begin the actual work of finding the correct answer.

Illustration :

How much must I pay for $19\frac{1}{2}$ quarts at 15¢ a quart?

To multiply $19\frac{1}{2}$ by 15¢ *mentally* is difficult.

But $19\frac{1}{2}$ is *nearly 20*;

Hence, the answer is nearly 20×15 ¢, or *nearly \$3.00*.

Having made this estimate you begin the actual work with the knowledge that your result must be a little less than \$3.00. You are not merely *guessing the answer*. You are making a close estimate of it. You know when you begin the solution that the result will not be far from \$3.00.

In the following exercises practice estimating your answers.

WRITTEN APPLICATIONS

THE USE OF FRACTIONS IN EVERYDAY LIFE

The Business Man's Problems in Fractions

1. Three cans of oil contain $3\frac{5}{8}$ gallons, $4\frac{3}{4}$ gallons, and $11\frac{7}{8}$ gallons respectively. How many gallons of oil do these three cans contain?

2. Find the cost of seventeen and five eighths yards of muslin at twenty-eight and one half cents per yard.

3. $2\frac{1}{2}$ pounds of rice at 10 cents a pound, $12\frac{1}{2}$ pounds of sugar at $8\frac{1}{2}$ cents a pound, and 16 pounds of ham at $36\frac{1}{2}$ cents a pound are paid for with a ten-dollar bill. How much change should the customer receive?

4. A barrel of flour weighs 196 pounds and costs a grocer \$12.50. If he puts the flour into 14 sacks of equal weight, and sells each sack for 9 cents per pound, how much profit does he make on the barrel of flour?

5. Four pieces of cloth measuring $4\frac{1}{4}$ yards, $6\frac{5}{8}$ yards, $2\frac{1}{3}$ yards, and $5\frac{1}{2}$ yards respectively were sold at \$.50 per yard. How much was received for the cloth?

6. How much change should be received from a ten-dollar bill given in payment for $7\frac{1}{2}$ yards of muslin at 60 cents per yard, and $10\frac{5}{8}$ yards of denim at 25 cents per yard, and 7 spools of thread at 5 cents each?

7. A carpenter is engaged at the rate of \$3.50 for eight hours' work. In ten days he works the following number of hours: 8, $7\frac{3}{4}$, $8\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{3}{4}$, $8\frac{1}{2}$, 7, 5, and $6\frac{3}{4}$. How much should he receive?

8. A dealer bought 7580 bushels of wheat at \$1.87 per bushel, and sold it at a cash profit of \$341.10. How much did he receive per bushel? How much did he receive for all of the wheat?

9. Mr. Brown owned $\frac{5}{7}$ of a business, and sold $\frac{3}{4}$ of his share to Mr. Johnson. Mr. Johnson sold $\frac{1}{3}$ of his share to a friend for \$5000. At this rate what was the value of each man's share?

10. A music dealer bought 150 phonographs at \$63 each. He sold seven fifteenths of the lot at \$90 each, two fifths of them at \$75 each, but on the remainder he lost \$10 each. How much did he gain on the whole transaction?

The Farmer's Problems in Fractions

11. If one ton of hay is worth \$12 $\frac{1}{2}$, how much are 5 $\frac{1}{2}$ tons of hay worth?

12. A pasture lot at \$95 per acre cost a farmer \$684. How many acres did he buy?

13. On a field of $4\frac{7}{8}$ acres, $84\frac{1}{2}$ bushels of wheat were raised. What was the average yield per acre?

14. Four loads of hay weigh, including the wagon, $1\frac{7}{8}$ tons, $1\frac{5}{8}$ tons, $2\frac{1}{8}$ tons, and $1\frac{3}{4}$ tons respectively. If the weight of the wagon is $\frac{3}{4}$ of a ton, what is the total weight of the hay in the four loads?

15. A farmer finds that he can plow $4\frac{2}{3}$ furrows across a field in one hour, and that he must plow 454 furrows to plow the whole field. How many hours' work will this plowing require?

16. A dairy averages a production of 7200 pounds of milk for each of the three summer months, and is credited at the creamery with an average of 296 pounds of butter fat for the same three months. What fraction of the milk production is the production of butter fat?

17. A farmer brought to a general store $34\frac{1}{2}$ dozen eggs and $18\frac{1}{2}$ pounds of butter. For the eggs he received 48 cents per dozen and for the butter 42 cents per pound. What amount should be paid him for both?

18. A farmer is offered \$12.50 per hundred pounds live weight for a hog weighing 260 pounds, or $14\frac{1}{2}$ cents per pound dressed weight. The dressed weight of the hog proves to be 208 pounds. Which is the better offer for the farmer, and how much better is it?

19. On a dairy farm two men are employed at $\$2\frac{1}{4}$ per day; two other men are employed at \$28 per month with board that costs the farmer $\frac{2}{3}$ of a dollar a day for each man; and a boy is employed at \$10 per month including board costing at the rate estimated for the man. What is the total cost for the labor of all five for six months, beginning April 1st?

20. A farmer sows a field of twenty-five acres to oats. He sows one half of the field broadcast and the other half with a drill. The first half produced $48\frac{1}{2}$ bushels to the acre, and the second half $54\frac{3}{4}$ bushels to the acre. How much is the gain per acre by using the drill? What fraction of the smaller crop is the gain?

The Housekeeper's Problems in Fractions

21. A housekeeper bought 20 quarts of strawberries at 20¢ a quart. How much did she pay for them?

22. A cup holds one fourth of a quart. How many cupfuls of molasses are there in a jug which will hold $9\frac{3}{4}$ gallons, when it is only two thirds full?

23. 3 loaves of bread can be made from 2 quarts of flour, and a quart of flour weighs $1\frac{1}{7}$ pounds. How many loaves can be made from a barrel of flour? (A barrel of flour weighs 196 pounds.)

24. A housewife bought 710 pounds of ice in June and 820 pounds in July. What was the total cost of ice for both months at the rate of $\$ \frac{2}{5}$ per hundred pounds?

25. A woman bought a dress, the price of which had been reduced from \$16 to $\frac{3}{4}$ of that amount, and paid $\frac{1}{10}$ of the purchase price for alteration. What was the total cost of the dress?

26. A housekeeper bought $\frac{1}{8}$ of a bushel of new potatoes for \$.45, and five weeks later bought the same quantity for \$.20. How much had the price per bushel been reduced in that time?

27. A woman pays her cook \$8 per week, her maid \$6 per week, and her expense for food averages \$31.50 per week. What fractional part of the total expenses for one week are paid to the cook? To the maid? For food?

28. A housekeeper finds from her accounts that she spent last year \$1350 for rent, service, light, and food. The rent was \$35 per month, the service \$5 per week, the light \$3 per month, and the balance was spent for food. What fractional part of the total expense was spent for each of the four items?

29. A clerk receives \$22.50 per week, and spends one fifth of his salary for board, one tenth of it for his room, one third of it for clothing, and saves the remainder. How much does he save each week? How much does he save in one year? What fractional part of his earnings for one year does he save?

30. The clerk in the preceding example secures another position at a salary of \$26.00 per week. Because of his increased prosperity he moves to a room that costs him one fifth of his salary per week, and he spends one fourth of his salary for clothing, and saves the remainder. He makes no change in other expenses. Does he save as much as formerly?

31. A man rents a house for \$25 per month for one year. His friend buys a similar house for \$3200, and his expenses for the same year are: interest on money invested, $\frac{1}{20}$ of purchase price; taxes, $\frac{1}{160}$ of purchase price; repairs, $\frac{1}{100}$ of purchase price. How much more does it cost the man for rent than it costs his friend for these expenses?

32. A lady desires to carpet a living room, and to cover the floor of a bathroom. She can buy 40 yards of Wilton carpet at $\$2\frac{1}{4}$ per yard for the living room, and 8 yards of printed linoleum at $\$2\frac{3}{4}$ per yard for the bathroom; or she can buy a cheaper grade of carpet for the room at $\$1\frac{3}{4}$ per yard and the best inlaid linoleum for the bathroom at $\$1\frac{1}{2}$ per yard. How much will each plan cost?

Miscellaneous Applications of Fractions

33. How much should you pay for 4 days' board if the rate per day is $\$1\frac{1}{4}$?

34. How much should you pay for 4 days' board if the rate per week is $\$8\frac{3}{4}$?

35. How much should you pay for 5 days' board if the rate per week is $\$5\frac{1}{2}$?

36. How much should you pay for 3 weeks and 4 days if the price of board is $\$8\frac{1}{2}$ per week?

37. A gas company charges \$1.10 per thousand cubic feet of gas consumed. How much should be paid at this rate for $\frac{3}{4}$ thousand cubic feet of gas?

38. A farmer sold $\frac{3}{8}$ of his land and had 165 acres left. How many acres did he originally have? What fraction of the original farm did he have left?

39. How many bushels of onions at $\$1\frac{1}{4}$ per bushel must a farmer give a grocer in exchange for a barrel of sugar weighing 224 pounds at $6\frac{1}{4}$ cents per pound?

40. Two and three quarters yards of carpet cost \$5.50. At the same rate, how much will $16\frac{1}{2}$ yards cost?

41. How many brass rods, each $\frac{7}{8}$ of a foot in length, can be cut from 36 brass rods each 14 feet in length?

42. A quart of milk weighs 2.15 pounds. How many quarts of milk are there in three cans of milk weighing $97\frac{1}{2}$ pounds, $88\frac{3}{4}$ pounds, and 91 pounds respectively; the total weight of the three empty cans being 54 pounds?

43. How many tons of coal worth \$7.50 can be bought with $32\frac{1}{2}$ cords of wood worth \$9 per cord? At the same rate, what part of a ton of coal can be bought with $\frac{2}{3}$ of a cord of wood?

44. An automobile is running at the rate of $31\frac{1}{4}$ miles per hour, and its engine averages 15 miles to each gallon of gasoline used. In how many hours will the car run 120 miles at this rate, and how much gasoline will it use?

45. A building lot was sold for \$60,000. One fourth of it was sold to two men, one of whom paid three times as much as the other. How much did each pay for his share?

46. A farmer owns four sevenths of the stock in a creamery, his brother owns one fifth of it, and a friend owns the remainder of it and values his stock at \$800. At the same rate what is the value of all of the stock?

47. Two automobiles met after one of them had run for $3\frac{3}{4}$ hours at a rate of $18\frac{1}{2}$ miles per hour; and the other for $2\frac{4}{5}$ hours at $24\frac{1}{2}$ miles per hour. How many miles apart were they when they started?

48. A room is $15\frac{3}{4}$ feet long and $11\frac{5}{8}$ feet wide. How much will it cost to put a picture molding around the room if the molding costs $9\frac{3}{4}$ cents per foot, and 3 hours' labor for putting it up is charged at \$.45 an hour?

49. A boy's expenses at a boarding school amount to \$500 in one year. $\frac{1}{4}$ of the amount is paid for tuition, $\frac{2}{5}$ of it for board, $\frac{1}{8}$ of it for clothing, and the rest for incidentals. Find the sum spent for each item.

50. A owned $\frac{2}{5}$ of an office building, and sold $\frac{1}{3}$ of his interest to B. C bought $\frac{1}{4}$ of B's interest for \$30,000. At the rate C paid, find the value of the building, and the value of the shares A and B each own.

SAVING AND INVESTING MONEY

The Value of Saving. Almost any elderly person can tell you of many instances in which men have become wealthy through the habit of saving. Thrift and saving on the part of the founders have been the means by which many of our largest enterprises have made their beginning. Biographies of successful men are filled with interesting stories which point to *thrift* as the foundation of their successes.

Savings in the United States. Until recently the Savings Banks in this country offered the principal opportunity for saving, and in 1918 a total of \$5,471,579,948 was on deposit with these banks to the credit of 11,379,553 persons. But this large number does not represent the total of persons who save, for there are many safe opportunities for investment outside of savings banks.

In 1911 the United States Government offered a new opportunity for saving by establishing the Postal Savings Plan. By this plan deposits of one dollar are accumulated through the saving of dimes, and the government guarantees the payment of the deposits. In six years this plan accumulated a total of \$131,954,696, representing a saving of more than one billion, three hundred million dimes.

In 1918 the War-Savings Plan was offered by the Government.

The War-Savings Plan

This plan was designed by the United States Government to encourage men, women, and children to form habits of saving, and also to provide money needed for increased government expenses. Under this plan you lend your small savings to your government by the purchase of Thrift Stamps and War-Savings Stamps.

Thrift Stamps

Thrift Stamps cost twenty-five cents each, and may be bought at any post office, bank, or other authorized agency. For the convenience of the investor a "Thrift Card" is furnished free to each investor as often as his purchases fill the sixteen spaces which each card provides. When all of the spaces are filled the Thrift Card may be exchanged for a *War-Savings Stamp*.

Thrift Stamps bear no interest. War-Savings Stamps bear interest. The sooner you fill up and exchange your Thrift Stamps for War-Savings Stamps, the more interest you will receive.

1 Affix the first 25-cent THRIFT STAMP here.	5 The first principle of money- making is money saving.	9 Many a mickle makes a muckle.	13 Save and have..
2 Your second stamp here.	6 Don't put off till to-morrow.	10 Saving creates independence.	14 Great oaks from little acorns grow.
3 If you want to succeed, save.	7 A penny saved is a penny gained.	11 Thrift begins with little savings.	15 Waste not; want not.
4 Thrift is the power to save.	8 All fortunes have their foundations laid in thrift.	12 Money placed at interest works day and night— in wet and dry weather.	16 Learn economy and you start on the road to success.

The payment or exchange for War-Savings Stamps must, until further public notice, be made during the year 1919.

Thrift Stamps help you to save small sums until you have saved enough to buy a War-Savings Stamp. Thrift Stamps may be lost, hence they should be attached to a Thrift Card which bears your name and address. Like postage stamps, they may be used by anyone who finds them.

War-Savings Stamps

A War-Savings Stamp is a government security which promises the investor the return of his money, together with interest from the day he purchases it to the day it is paid. On January 1, 1924, a War-Savings Stamp will be worth \$5.00.



The prices of War-Savings Stamps vary from month to month, as indicated in the following table :

January	\$4.12	May	\$4.16	September	\$4.20
February	4.13	June	4.17	October	4.21
March	4.14	July	4.18	November	4.22
April	4.15	August	4.19	December	4.23

For convenience, and to guard against loss, the government provides a **War-Savings Certificate** which is furnished free when you purchase your first stamp. Other stamps purchased from time to time are attached to this card, which holds twenty stamps. A complete War-Savings Certificate represents an investment of \$80, plus the cost of the Stamps in the months in which your investments were made.

Information as to the value of your investment at any time before the Certificate is due is printed upon the Certificate. In this way investors who may be compelled to ask for the return of their investment before January 1, 1924, are informed of its value in any particular month. *War-Savings Stamps* may be registered, as detailed in the Certificate, thus protecting the investor against loss of any kind.

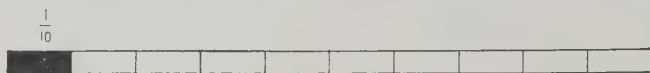
WRITTEN APPLICATIONS

Find the cost of :

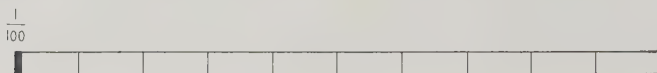
1. 20 War-Savings Stamps in June.
2. 25 War-Savings Stamps in July.
3. 50 War-Savings Stamps in August.
4. 60 War-Savings Stamps in November.
5. 10 War-Savings Stamps in April and 30 in September.
6. 20 War-Savings Stamps in May, 30 in June, and 50 in July.
7. How much will 100 War-Savings Stamps cost, if 26 are bought in October, 42 in November, and 32 in December?
8. If Franklin's father gives him as a birthday present 100 War-Savings Stamps in November, how much will they be worth in January, 1924?

DECIMALS

If we divide any given length into ten equal parts, each part is "one tenth of the ten," "one tenth" of the whole length.



If we divide this one tenth into ten equal parts, each part is one tenth of one of the ten, or "one tenth of one tenth," or "one hundredth" of the whole length.



In the same manner we divide "one hundredth" into ten equal parts, each of which is "one tenth of one hundredth," or "one thousandth" of the unit or the thing divided.

Decimal Fractions are expressions for common fractions whose denominators are 10, 100, 1000, or any other and higher power of 10.

The Decimal Point (.) is written before the tenths' place in all decimals.

Thus, in decimal form :

$\frac{1}{10}$ is written .1 and is read "one tenth."

$\frac{1}{100}$ is written .01 and is read "one hundredth."

$\frac{1}{1000}$ is written .001 and is read "one thousandth."

$\frac{1}{10000}$ is written .0001 and is read "one ten-thousandth."

Decimals of more than four places are not of frequent occurrence.

Common fractions may have any number for a denominator, while decimal fractions, or "decimals," as they are called, have only 10, 100, 1000, etc., as denominators, and *these denominators in decimal forms are never written.*

Numbers that represent whole units are called **Whole Numbers, Integral Numbers, or Integers.**

Place Value in Decimals

In whole numbers we know that

1 hundred = 10 tens,

1 ten = 10 units.

That is, the value of any place as we read towards the right is $\frac{1}{10}$ of the place immediately preceding it. The same relation holds in decimals, and we can readily see that

1 unit = 10 tenths,

1 tenth = 10 hundredths,

1 hundredth = 10 thousandths.

Hence, the place value relation of one tenth continues through whole numbers, and also through decimals.

Integers occupy places at the left of the decimal point.

Decimals occupy places at the right of the decimal point.

TABLE

6	8	2	7	1	.	3	4	2	5
Ten Thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths

The number in the table is read :

"Sixty-eight thousand two hundred seventy-one and three thousand four hundred twenty-five ten-thousandths."

A mixed decimal like 16.25 means "16 units *and* 25 hundredths of 1 unit." We read it "Sixteen and twenty-five hundredths," the *and* meaning *plus* just as in $2\frac{3}{4}$ we read "2 and $\frac{3}{4}$ " meaning 2 units plus $\frac{3}{4}$ of 1 unit.

I. Reading Decimals

Decimals are read exactly as we read integers, and the name of the lowest decimal place is the name of the decimal.

Thus: .2 is read "two tenths."

.24 is read "twenty-four hundredths."

.245 is read "two hundred forty-five thousandths."

ORAL PRACTICE

Read :

1.	.3	.5	.9	.12	.27	.56	.87	.98	.112
2.	.7	.9	.15	.30	.54	.71	.92	.106	.154
3.	.8	.14	.29	.09	.86	.91	.057	.092	.0012
4.	.9	.19	.36	.72	.08	.104	.008	.017	.0107
5.	.13	.29	.52	.87	.09	.005	.019	.035	.0305
6.	.21	.42	.79	.112	.114	.006	.018	.042	.0042

II. Writing Decimals

Decimals are written as if they were whole numbers, with the decimal point immediately before the tenths' place.

Thus: Four tenths is written .4

Forty-five hundredths is written .45

Sometimes the required denomination necessitates the use of one or more zeros.

Thus: Forty-five thousandths is written .045

Twenty-four ten-thousandths is written .0024

In writing a decimal the last figure of the number given must stand in the place that names the kind of decimal.

BLACKBOARD PRACTICE

Write in figures :

1. Fourteen hundredths; twenty-nine hundredths; forty-six hundredths; sixty-nine hundredths; one hundred twenty-five thousandths.

2. Twenty-seven hundredths; twenty-seven thousandths; two hundred seven thousandths; one hundred nine thousandths; two hundred one thousandths.

3. Sixty-one hundredths; seventy-nine thousandths; eleven ten-thousandths; two hundred seventy-six ten-thousandths; two hundred one ten-thousandths.

4. Nineteen thousandths; one hundred nine thousandths; one hundred nine ten-thousandths; eleven ten-thousandths; seventy-seven ten-thousandths.

5. Two hundred eleven thousandths; two hundred eleven ten-thousandths; two thousand eleven ten-thousandths; four thousand four ten-thousandths.

6. Eleven hundred thirty-seven ten-thousandths; three thousand seven hundred eleven ten-thousandths; two thousand five hundred nine hundred-thousandths.

7. 359 thousandths.

8. 108 ten thousandths.

9. 9 hundred thousandths.

10. 12 and 12 hundredths.

11. 351 and 17 hundredths.

12. 19 and 19 ten thousandths.

13. 243 and 19 thousandths.

14. 7008 and 11 hundred thousandths.

15. 7 thousand and 7 thousandths.

16. 35 hundred and 35 hundredths.

17. 10 thousand and 10 thousandths.

Fractions whose denominators are multiples of 10 may readily be changed to decimals. Thus:

$$\frac{7}{10} = .7, \frac{17}{100} = .017, \frac{213}{1000} = .0213, \text{ etc.}$$

If the given fraction is an improper fraction the resulting decimal is a mixed decimal, that is, partly integral and partly decimal. Thus:

$$\frac{17}{10} = 1.7, \frac{1769}{1000} = 1.769, \frac{23451}{100} = 234.51, \text{ etc.}$$

BLACKBOARD PRACTICE

Change to decimals:

1. $\frac{3}{10}$; $\frac{5}{10}$; $\frac{6}{10}$; $\frac{7}{10}$; $\frac{8}{10}$; $\frac{9}{10}$.
2. $\frac{3}{100}$; $\frac{5}{100}$; $\frac{6}{100}$; $\frac{7}{100}$; $\frac{8}{100}$; $\frac{9}{100}$.
3. $\frac{19}{100}$; $\frac{23}{100}$; $\frac{27}{100}$; $\frac{37}{100}$; $\frac{51}{100}$; $\frac{63}{100}$; $\frac{77}{100}$.
4. $\frac{127}{1000}$; $\frac{359}{1000}$; $\frac{127}{10000}$; $\frac{579}{10000}$; $\frac{1551}{10000}$; $\frac{31177}{100000}$.

Change to mixed decimals:

5. $\frac{13}{10}$; $\frac{19}{10}$; $\frac{23}{10}$; $\frac{31}{10}$; $\frac{47}{10}$; $\frac{53}{10}$.
6. $\frac{121}{100}$; $\frac{135}{100}$; $\frac{231}{100}$; $\frac{351}{100}$; $\frac{479}{100}$; $\frac{625}{100}$.
7. $\frac{1225}{1000}$; $\frac{2557}{1000}$; $\frac{3775}{1000}$; $\frac{5700}{1000}$; $\frac{7590}{1000}$.
8. $\frac{12525}{10000}$; $\frac{45580}{10000}$; $\frac{125000}{10000}$; $\frac{375400}{100000}$.

III. Changing Decimal Forms

Similar Decimals are decimals that have the same number of decimal places. Thus:

.25 and .54 are similar decimals.

.145 and .032 and .007 are similar decimals.

It follows, therefore, that

.35 and .145 are dissimilar decimals.

.12, .017, .5, and .0214 are dissimilar decimals.

We often make decimals similar by annexing zeros.

Thus: .15 and .142 are by definition dissimilar.

But .150 and .142 are by definition similar.

The change in the form of the first decimal does not change its value, for

$$.15 = \frac{15}{100} = \frac{15 \times 10}{100 \times 10} = \frac{150}{1000} = .150. \quad (\text{See page 47.})$$

That is, annexing zeros at the right of a decimal changes the name *but not the value* of the decimal:

BLACKBOARD PRACTICE

Change to similar decimals :

- | | | |
|-------------|----------------|------------------------|
| 1. .3, .19 | 6. .7, .097 | 11. .12, .002, .0014 |
| 2. .5, .46 | 7. .15, .271 | 12. .35, .071, .0104 |
| 3. .7, .08 | 8. .34, .207 | 13. .45, .1007, .05411 |
| 4. .8, .018 | 9. .62, .408 | 14. .004, .11, .00007 |
| 5. .1, .003 | 10. .75, .0011 | 15. .8, .008, .00008 |

To Change a Decimal to a Common Fraction

Illustrations :

$$.35 = \frac{35}{100} = \frac{7}{20}. \quad (\text{Reduced to lowest terms.})$$

$$.37\frac{1}{2} = 37\frac{1}{2} \div 100 = \frac{75}{2} \div 100 = \frac{75}{2} \times \frac{1}{100} = \frac{75}{200} = \frac{3}{8}.$$

Write the figures of the decimal for the numerator.

Write the denomination of the decimal for the denominator.

Reduce the resulting fraction to its lowest terms.

BLACKBOARD PRACTICE

Change to fractions, each in its lowest terms :

- .3, .4, .5, .6, .9, .12, .16, .22, .36
- .7, .8, .2, .16, .24, .32, .42, .45, .56, .68
- .15, .19, .25, .65, .87, .112, .135, .225, .455

Change to mixed numbers :

- 1.25, 1.35, 1.45, 1.60, 1.72, 1.85, 1.98, 2.25, 2.30
- 1.125, 1.175, 1.225, 1.625, 2.225, 2.250, 5.125, 7.625
- 25.05, 30.12, 36.25, 45.12, 55.55, 75.125, 80.375, 90.625

UNITED STATES MONEY

United States Money is founded on the decimal plan, the principal unit being the Dollar. Then,

Since 100 cents = 1 dollar, 1 cent = .01 dollar;
and since 10 dimes = 1 dollar, 1 dime = .1 dollar.

Any number of dollars and cents may be written in dollars and decimal parts of the dollar.

$$\$2.57 = 2.57 \text{ dollars; etc.}$$

$$\$15.12\frac{1}{2} = 15.125 \text{ dollars.}$$

Any number of cents may be written in dollars and decimals of the dollar.

$$687 \text{ cents} = \$6.87; \text{ etc.}$$

$$745.5 \text{ cents} = \$7.45\frac{1}{2}.$$

ADDITION OF DECIMALS

For convenience in adding, the numbers are written so that units of the same order and decimals of the same kind stand in vertical columns.

Illustrations:

10.15	5.019	.0019	.37
4.72	0.42	.72	1.414
1.08	0.012	1.5	12.1127
<u>15.95</u> Sum.	<u>5.451</u> Sum.	<u>2.2219</u> Sum.	<u>13.8967</u> Sum.

BLACKBOARD PRACTICE

Copy and add:

1. 3.12	2. 7.1	3. 9.05	4. .5	5. 10.4
4.14	4.13	12.001	4.81	.002
5.18	.09	7.42	11.004	40.0143
<u>6.27</u>	<u>8.04</u>	<u>.008</u>	<u>.0089</u>	<u>49.5837</u>

6. 45.12	7. 8.001	8. .9
61.07	16.4	41.4
12.53	32.71	5.16
.946	41.02	.00001
9.46	.007	.1776
<u>19.005</u>	<u>7.002</u>	<u>16.26239</u>

9. 3.00042	10. 9.401	11. 2.14
.19	.004	3.151
76.4	106.8	8.02
3.009	2409.785	6.413
.77	474.01	51.7
<u>.09991</u>	<u>2000.</u>	<u>18.09</u>

Write in vertical columns and in the proper order and add :

12. $31.12 + 4.113 + .416 + .041 + .004 + .00002$.
13. $27.3 + 61.14 + .004 + 51.142 + 31.09 + 4.0004$.
14. $.009 + .0001 + 54 + .186 + 114.77 + .00021$.
15. $71.19 + .4116 + 100.1 + 2174.01 + 800.4002 + 1331.0033$.
16. $300.41 + 542.004 + 6211.301 + 4.0009 + 12.416 + .00015$.

17.	18.	19.	20.	21.
\$115.12	\$217.19	\$ 69.19	\$187.12	\$ 64.10
86.74	304.00	110.00	.10	35.42
119.00	8.90	14.50	11.90	67.44
89.05	11.79	7.99	.87	98.81
181.09	600.88	19.00	776.00	54.92
55.08	78.00	11.88	34.00	167.81
234.19	500.00	250.00	1.10	73.00
65.99	.97	44.90	45.67	.80
70.00	7.16	665.10	110.00	56.00
<u>560.05</u>	<u>10.00</u>	<u>34.00</u>	<u>465.05</u>	<u>141.74</u>

WRITTEN APPLICATIONS

1. On the Lackawanna Railroad, the distance from New York City to the Delaware Water Gap is 88.7 miles; from the Water Gap to Scranton, 56.1 miles; and from Scranton to Buffalo, 265.6 miles. How many miles in the distance from New York to Buffalo?

2. A surveyor measures the sides of a field, and finds that the first is 2134.05 ft.; the second, 1187.9 ft.; the third, 1008.75 ft.; and the fourth, 2087.16 ft. What is the total distance in feet around the field?

3. A dairyman owns five farms, the first containing 110.56 acres; the second, 98.54 acres; the third, 145.125 acres; the fourth, 120.251 acres; and the fifth, 114.6 acres. How many acres of land are there in the five farms?

4. A farm cost \$4750; the purchaser bought twenty head of stock for \$1700, modern machinery for \$892.15, a pair of draft horses for \$625, and built a new barn for \$2170.50. What is the total of these five amounts?

5. The four sides of a field measure respectively 27.15 rods, 31.1 rods, 54.25 rods, and 38.45 rods. How many rods of wire will be needed to inclose the field with a five-wire fence?

6. During a period of five consecutive years a cow's yield of butter fat was 312.4, 290.5, 307.6, 341.8, and 312.7 pounds respectively. What was her total yield of butter fat for this entire time?

7. The grocery bills of a certain family for the first six months of a year were \$24.76, \$27.55, \$29.54, \$28.07, \$26.17, and \$30.75, respectively. What was the total amount of these grocery bills?

8. The meat bills of the same family during the same six months amounted to \$11.17, \$12.80, \$11.66, \$12.14, \$13.05, and \$10.87 respectively. What was the total amount of these meat bills?

9. The family mentioned in examples 7 and 8 paid for milk during the six months a total of \$18.35, for ice \$9.10, for eggs \$17.10, and for rent \$30 per month. What was the total amount paid by this family for groceries, meat, milk, ice, eggs, and rent for the six months?

SUBTRACTION OF DECIMALS

Write the numbers so that units of the same order and decimals of the same kind stand in vertical columns.

Illustrations:

1. $12.7 - 5.3 = ?$ 2. $14.161 - 3.12 = ?$ 3. $9.41 - 3.0126 = ?$

12.7

5.3

7.4 *Result.*

14.161

3.120

11.041 *Result.*

9.4100

3.0126

6.3974 *Result.*

After a little practice we should be able to subtract without writing zeros in the minuend. Thus:

4. 9.41

3.0126

6.3974 *Result.*

5. $50.$

4.573

45.427 *Result.*

6. $275.$

1.006

273.994 *Result.*

BLACKBOARD PRACTICE

Subtract:

1. 15.51 2. 18.54 3. 35.172 4. 51.019 5. 116.432

6.32

9.67

19.024

27.112

97.119

6. 67.15 7. 83.07 8. 116.9 9. 71.009 10. 85.010

12.015

41.155

7.115

9.3

1.97681

Find the remainder in each of the following :

11. $15 - 8.12$; $42 - .0716$; $42 - .00716$.
12. $54.1 - 0.019$; $101.171 - 0.0041$; $167 - 0.0032$.
13. $79.01 - 16.014$; $217.1 - 71.04$; $240.4 - 119.01019$.
14. $93.51 - 32.009$; $32.041 - 0.0089$; $311.117 - 11.118$.
15. $112.011 - 17.11$; $51.03 - 19.0299$; $619.07 - 618.0699$.
16. $157.05 - 8.002115$; $711.0098 - 710.0099$; $821.03 - 245.9909$.

WRITTEN APPLICATIONS

1. If I pay \$14.19 for merchandise, how much change should I receive from a twenty-dollar bill?

2. From a farm containing 242.25 acres, 96.5 acres were sold. How many acres are left of the original farm?

3. 365.25 acres are sold from an island containing 512.15 acres of land. How many acres of the island were not sold?

4. A man bought a suit of clothes for \$20.00, a hat for \$2.75, a pair of shoes for \$3.45, a box of collars for \$1.50, and a necktie for \$0.75. If he gave three ten-dollar bills in payment, how much change should he receive?

5. A man's total income for one year amounted to \$3175.50. His living expenses were \$1475.60, his payments for insurance \$318.90, and payment for interest on loans \$179.62. How much of his income was left after paying these three items?

6. A man purchased a house and paid for it in four payments. The first payment was \$525.00 ; the second, \$674.18 ; the third, \$680.95 ; and the fourth, \$725.10. In addition to the purchase price, he spent \$417.15 on repairs and improvements, and then sold it for \$3750.00. How much profit did he make?

7. A truck gardener rents a plot of ground near a city for \$50, and during the season pays out as follows: seed, \$5.48; fertilizer, \$18.75; cartage, \$4.75; labor, \$17.50; tools, \$3.80, and spraying material, \$5.45. If he receives for his produce a total of \$412.19, how much is left for his labor after paying these expenses?

MULTIPLICATION OF DECIMALS

When we find the products of fractions whose denominators are 10, 100, 1000, etc. the denominator of the result is governed by the number of tens multiplied together.

Thus: $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$; $\frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$; $\frac{1}{100} \times \frac{1}{1000} = \frac{1}{100000}$, etc.

In the same manner the product of decimals, $.1 \times .1$, $.01 \times .01$, $.001 \times .001$, etc., is governed by the number of places in the decimals multiplied.

Thus:

$$.1 \times .1 = .01; .1 \times .01 = .001; .01 \times .001 = .00001; \text{etc.}$$

To multiply a decimal by a decimal:

Multiply as in whole numbers.

Point off as many places in the product as there are decimal places in both the multiplier and multiplicand.

Prefix zeros if necessary to complete the required number of decimal places in the product.

Illustrations:

125	125	125	1.25	.125
<u>1.5</u>	<u>.015</u>	<u>.15</u>	<u>.15</u>	<u>.0015</u>
625	625	625	625	625
<u>125</u>	<u>125</u>	<u>125</u>	<u>125</u>	<u>125</u>
187.5	1.875	18.75	.1875	.0001875

When the multiplier is a multiple of 10 it is possible to shorten the process by multiplying as follows :

Illustrations :

$$\begin{array}{r} 2.57 \\ \underline{2} \\ 5.14 \end{array}$$

$$\begin{array}{r} 2.57 \\ \underline{20} \\ 51.4 \end{array}$$

$$\begin{array}{r} 2.57 \\ \underline{200} \\ 514. \end{array}$$

$$\begin{array}{r} 2.57 \\ \underline{2000} \\ 5140. \end{array}$$

Since each decimal place is one tenth of the preceding place, the removal of the decimal point

One place to the right multiplies by 10;

Two places to the right multiplies by 100;

Three places to the right multiplies by 1000; etc.

Hence, in the above illustrations we multiply in each case by 2, and move the decimal point as many places to the right as there are zeros in the multiplier.

BLACKBOARD PRACTICE

Copy and multiply :

$$\begin{array}{r} 1. \ 125 \\ \underline{2.5} \end{array}$$

$$\begin{array}{r} 2. \ 125 \\ \underline{.25} \end{array}$$

$$\begin{array}{r} 3. \ 125 \\ \underline{.025} \end{array}$$

$$\begin{array}{r} 4. \ 12.5 \\ \underline{.25} \end{array}$$

$$\begin{array}{r} 5. \ 1.25 \\ \underline{.025} \end{array}$$

$$\begin{array}{r} 6. \ .0125 \\ \underline{.025} \end{array}$$

$$\begin{array}{r} 7. \ 27.15 \\ \underline{.212} \end{array}$$

$$\begin{array}{r} 8. \ 31.45 \\ \underline{.014} \end{array}$$

$$\begin{array}{r} 9. \ 10.07 \\ \underline{.0019} \end{array}$$

$$\begin{array}{r} 10. \ 100.7 \\ \underline{.0015} \end{array}$$

$$\begin{array}{r} 11. \ 21.001 \\ \underline{4.009} \end{array}$$

$$\begin{array}{r} 12. \ 3.5001 \\ \underline{.02041} \end{array}$$

$$13. \ 12.7 \times .15.$$

$$14. \ 13.6 \times .24.$$

$$15. \ 15.9 \times .31.$$

$$16. \ 27.5 \times .57.$$

$$17. \ 34.1 \times .65.$$

$$18. \ 45.8 \times .81.$$

$$19. \ 24.8 \times 32.1.$$

$$20. \ 31.6 \times 3.21.$$

$$21. \ 40.7 \times 4.82.$$

$$22. \ 58.1 \times .603.$$

$$23. \ 67.9 \times 7.11.$$

$$24. \ 92.3 \times .117.$$

- | | |
|----------------------------|-------------------------------|
| 25. 27.6×1.2 . | 38. $80.06 \times .0104$. |
| 26. 34.7×1.02 . | 39. $85.18 \times .0051$. |
| 27. $57.01 \times .19$. | 40. 90.014×20.04 . |
| 28. $48.01 \times .012$. | 41. 96.004×42.01 . |
| 29. $64.04 \times .019$. | 42. 115.24×1.121 . |
| 30. $65.12 \times .024$. | 43. 127.011×2.104 . |
| 31. $75.02 \times .322$. | 44. 137.102×37.15 . |
| 32. $79.13 \times .018$. | 45. 145.904×12.12 . |
| 33. 84.07×4.12 . | 46. 146.192×3.211 . |
| 34. 92.12×14.2 . | 47. $164.24 \times .01141$. |
| 35. 92.17×1.04 . | 48. 178.134×121.42 . |
| 36. $84.09 \times .008$. | 49. $137.001 \times .00015$. |
| 37. 105.22×11.7 . | 50. 2411.45×1.0025 . |

WRITTEN APPLICATIONS

1. What is the cost of 3.875 yards of silk at \$3.75 per yard?
2. What is the cost of 254.5 bushels of wheat at $\$1.82\frac{1}{2}$ per bushel?
3. What is the cost of 251.45 acres of land at \$37.50 per acre?
4. A field containing 74.25 acres gives an average yield of 2.75 tons of hay to the acre. What is the total yield of hay?
5. A merchant holds a customer's note for \$1250, but is able to collect only 75 cents on the dollar when it is paid. What amount is collected?
6. A building lot extends 104.35 feet north from a corner on a business street, and is sold for \$112.50 per front foot. How much does the lot sell for?

7. A farmer raises a crop of corn that averages 41.75 bushels to the acre. How much does he receive for the corn at an average price of \$1.125 per bushel?

8. The circumference of a circle is 3.14159 times its diameter. What is the length of the circumference of an automobile wheel whose diameter is 34 inches?

9. The area of a rectangle equals the number of square units in its surface. If a rectangular field is 35.75 yards long and 12.35 yards wide, how many square yards are there in its area?

10. A farmer sold 150 bushels of wheat at \$1.875 per bushel, and 245 bushels of potatoes at \$1.625 per bushel. What amount did he receive from the sale of both?

11. A farm containing 127.5 acres is sold in two parts. One part containing 87.25 acres is sold at \$75 per acre, and the other part is sold at \$100 an acre. What is the total amount received for the farm?

12. An estate amounting to \$12,000 is to be divided among four children. The eldest is to receive .35 of it, the next younger .2 of it, and the remaining portion is to be divided equally between the other two children. How much does each child receive?

13. A man makes a statement of his annual expenses, and he finds that he spends a total of \$4250 in conducting his business; .15 of this expense is for rent, .12 for traveling expenses, .18 is for advertising and the remainder is spent for salaries. How much is spent for each of these four items?

14. 35,000 pounds of potatoes were shipped from a point in Maine to Boston; the freight rate was \$1.11 per hundred pounds, the heating charge \$.04 per hundred pounds, and the insurance on the shipment \$3.75. What was the total cost of shipping the potatoes to Boston?

DIVISION OF DECIMALS

The Relations of Dividend, Divisor, and Quotient

I. If either the dividend or the divisor is multiplied, or divided, by the same number, and the other remains unchanged, the quotient will be changed.

Illustrations :

1. $12 \div 6 = 2$. But if 12 is multiplied by 2, $24 \div 6 = 4$.

That is, multiplying only the dividend multiplies the quotient.

2. $12 \div 6 = 2$. But if 12 is divided by 2, $6 \div 6 = 1$.

That is, dividing only the dividend divides the quotient.

3. $12 \div 6 = 2$. But if 6 is multiplied by 2, $12 \div 12 = 1$.

That is, multiplying only the divisor divides the quotient.

4. $12 \div 6 = 2$. But if 6 is divided by 2, $12 \div 3 = 4$.

That is, dividing only the divisor multiplies the quotient.

II. If both the dividend and the divisor are multiplied, or divided, by the same number, the quotient will not be changed.

Illustrations :

5. $12 \div 6 = 2$.

But if 12 and 6 are both multiplied by 2, $24 \div 12 = 2$.

That is, multiplying both the dividend and the divisor by the same number does not change the quotient.

6. $12 \div 6 = 2$.

But if 12 and 6 are both divided by 2, $6 \div 3 = 2$.

That is, dividing both the dividend and the divisor by the same number does not change the quotient.

Application of these principles in Division of Decimals simplifies the work and the placing of the Decimal Point.

Three Methods for Division of Decimals

One good method for dividing with decimals should be thoroughly mastered. Three practical methods are given as follows.

First Method

Multiply both dividend and divisor by 10, or by the multiple of 10 that will make the dividend a whole number.

1. Divide 12.95 by 3.5.

Multiplying both dividend and divisor by 10, the example becomes $129.5 \div 35$.

$$\begin{array}{r} 3.7 \text{ Result.} \\ 35 \overline{)129.5} \\ \underline{105} \\ 245 \\ \underline{245} \end{array}$$

The division is carried out exactly as in whole numbers.

The decimal point in the quotient is placed exactly above the decimal point in the dividend.

2. Divide .05025 by 10.05.

Multiplying both dividend and divisor by 100, we have

$$\begin{array}{r} .005 \text{ Result.} \\ 1005 \overline{)5.025} \\ \underline{5025} \end{array}$$

3. Divide .01296 by .036.

Multiplying both dividend and divisor by 1000, we have

$$\begin{array}{r} .36 \text{ Result.} \\ 36 \overline{)12.96} \\ \underline{108} \\ 216 \\ \underline{216} \end{array}$$

Second Method

By means of a caret written in the dividend, mark off from the right of the decimal point as many places as there are decimal places in the divisor.

Divide as in integers, and place the decimal point in the quotient immediately upon using all of the figures of the dividend at the left of the caret.

1. Divide 3.136 by .56.

$$\begin{array}{r}
 5.6 \text{ Result.} \\
 .56 \overline{)3.13\underset{\wedge}{6}} \\
 \underline{2\ 80} \\
 336 \\
 \underline{336}
 \end{array}$$

2. Divide 46.5 by .1875.

$$\begin{array}{r}
 248. \text{ Result.} \\
 .1875 \overline{)46.5000\underset{\wedge}{}} \\
 \underline{37\ 50} \\
 9000 \\
 \underline{7500} \\
 15000 \\
 \underline{15000}
 \end{array}$$

Third Method

Divide as in whole numbers, and point off as many places in the quotient as the number of decimal places in the dividend exceeds the number of decimal places in the divisor.

1. Divide 37.525 by 2.5.

$ \begin{array}{r} 1501 \\ 2.5 \overline{)37.52\bar{5}} \\ \underline{25} \\ 125 \\ \underline{125} \\ 25 \\ \underline{25} \end{array} $	<i>With whole numbers</i> the quotient would result as given in the process at the left. Number of decimal places in the dividend..... 3 Number of decimal places in the divisor..... <u>1</u> Difference in the numbers..... 2 Point off <i>two places</i> in the quotient obtained. Hence, the required quotient is 15.01 <i>Result.</i>
---	--

2. Divide .8192 by 320.

$ \begin{array}{r} 256 \\ 320 \overline{).819\bar{2}} \\ \underline{640} \\ 1792 \\ \underline{1600} \\ 1920 \\ \underline{1920} \end{array} $	<i>With whole numbers</i> the quotient would result as given in the process at the left. Number of decimal places in the dividend..... 5 Number of decimal places in the divisor..... <u>0</u> Difference in the numbers..... 5 Point off <i>five places</i> in the quotient obtained. Hence, the required quotient is .00256 <i>Result.</i>
--	--

If the number of decimal places in the divisor exceeds the number of decimal places in the dividend, annex as many zeros to the quotient as the number of decimal places in the dividend exceeds the number of decimal places in the divisor.

3. Divide 1296 by .027.

48	<i>With whole numbers</i> the quotient would result as given	
.027 $\overline{)1296}$	in the process at the left.	
108	Number of decimal places in the divisor.....	3
216	Number of decimal places in the dividend.....	0
216	Difference in the numbers.....	3

Annex three zeros to the quotient obtained.

Hence, the required quotient is 48000. *Result.*

Inexact Division. Division in decimals is not always exact, and in such cases we continue the division until the number of places asked for has been obtained in the quotient, annexing the sign + to show that the work is incomplete. Thus, $.052 \div 1.5$ to four decimal places gives .0346, with a remainder $\frac{2}{3}$. The $\frac{2}{3}$ not being a complete division, we write the answer .0346+.

BLACKBOARD PRACTICE

Find the quotient of:

- | | | |
|------------------------------|-------------------------------|----------------------------|
| 1. $2.56 \div .08$. | 8. $14.52 \div 1.2$. | 15. $42.25 \div .025$. |
| 2. $4.64 \div .08$. | 9. $14.52 \div .12$. | 16. $4.225 \div .0025$. |
| 3. $6.75 \div 2.5$. | 10. $1.452 \div .00012$. | 17. $104225 \div .00025$. |
| 4. $8.47 \div .07$. | 11. $26.39 \div 1.3$. | 18. $86.015 \div .05$. |
| 5. $84.7 \div .07$. | 12. $2.639 \div .13$. | 19. $86.015 \div 500$. |
| 6. $847 \div .007$. | 13. $.02639 \div 13$. | 20. $1240.93 \div 3100$. |
| 7. $.847 \div .007$. | 14. $.02639 \div .013$. | 21. $1.24093 \div .0031$. |
| 22. $.003105 \div 15$. | 29. $4.7952 \div .00648$. | |
| 23. $23.75 \div .0095$. | 30. $151.008 \div .07952$. | |
| 24. $.008575 \div .0049$. | 31. $321.711 \div .000015$. | |
| 25. $.01058 \div .00046$. | 32. $.203412 \div .000201$. | |
| 26. $.0053282 \div .0777$. | 33. $300.303 \div .001875$. | |
| 27. $17.78028 \div .14574$. | 34. $.000123 \div .01875$. | |
| 28. $.0025 \div .0000125$. | 35. $.00003133 \div 3.0125$. | |

Find, to three decimal places, the following quotients:

- | | | |
|----------------------|-------------------------|--------------------------|
| 36. $15 \div 17$. | 41. $124 \div .13$. | 46. $1500 \div 11.3$. |
| 37. $15 \div .017$. | 42. $.0145 \div .07$. | 47. $1875 \div .27$. |
| 38. $19 \div 4.2$. | 43. $54.5 \div .0011$. | 48. $202.5 \div 165.1$. |
| 39. $57 \div 12.5$. | 44. $200 \div .0175$. | 49. $.02025 \div 1651$. |
| 40. $77 \div .25$. | 45. $240.7 \div 11.5$. | 50. $2450 \div 11.775$. |

Find the quotient of:

- | | |
|----------------------------|---------------------------------|
| 51. $\$450 \div 15$. | 59. $\$1250 \div \4.125 . |
| 52. $\$3250 \div 125$. | 60. $\$244.85 \div \4.15 . |
| 53. $\$4500 \div 360$. | 61. $\$175.25 \div \25.00 . |
| 54. $\$15,625 \div 62.5$. | 62. $\$175.25 \div \0.25 . |
| 55. $\$450.90 \div 270$. | 63. $\$175.25 \div \0.025 . |
| 56. $\$450 \div \0.75 . | 64. $\$175.25 \div \2.50 . |
| 57. $\$500 \div \1.25 . | 65. $\$1752.50 \div \1250 . |
| 58. $\$625 \div \0.25 . | 66. $\$2750.50 \div \110.02 . |

WRITTEN APPLICATIONS

1. The wheels on an automobile are 8.8 feet in circumference. How many turns does each wheel make in going one mile?

2. The product of two numbers is 30.04812, and one of the numbers is 0.437. What is the other number?

3. A painter receives \$3.75 for a day's work. At this rate how many days must he work to receive \$101.75?

4. A farmer receives \$16.25 per ton for his hay, and sells his crop for \$877.50. How many tons of hay does he sell?

5. The English pound is equivalent to \$4.8665 in United States money. What is the value in English pounds of \$100 in United States money?

6. A railroad track rises 2.25 feet in each 100 feet for a distance of 2 miles. What is the total rise in the track in that distance?

7. A passenger train travels a distance of 1 mile in 42.5 seconds. Find, to the nearest hundredth, the number of feet traveled per second at that rate.

8. How many cords of wood at \$3.25 per cord must be given in payment for 45 tons of coal at \$6.25 per ton, and 25 tons at \$7.50 per ton?

9. A grocer paid \$169.47 for 27 boxes of soap and 19 cases of starch. The soap cost him \$4.25 per box. Find the price of the starch per case.

10. In 6.25 hours an engine pumped 465.5 barrels of water into a tank. Since a barrel contains 31.5 gallons, how many gallons were pumped per hour?

11. In 1915 the railroad fare between Philadelphia and Chicago was \$18.00, but in 1918 the fare over the same route was \$34.13. How much more did it cost to make the round trip between the two cities in 1918 than it did in 1915?

12. The cost of delivering 20 tons of coal by wagon was \$8.80, and in the same time a motor truck delivered 45 tons at a cost of \$10.62. Find the cost per ton of delivering the coal by wagon. Find, also, the cost per ton of delivering it by truck.

13. The freight rate on a shipment of books from Chicago to Jackson, Mississippi, was \$1.57½ per hundredweight. At that rate, and including a war tax of \$.75, what was the total cost of transporting the books between the two cities, if the weight of the shipment was 1535 pounds?

To Change a Fraction to a Decimal

(a) When the Given Fraction can be changed to an Exact Decimal

Illustration :

Change $\frac{5}{8}$ to an equivalent decimal.

A common fraction is an indicated division, hence,

$$\frac{5}{8} = 5 \div 8 = .625. \quad \text{Operation : } 8 \overline{)5.000} \quad \text{Result.}$$

(b) When the Given Fraction cannot be changed to an Exact Decimal

Illustration :

Change $\frac{5}{6}$ to a decimal of three places.

$$\frac{5}{6} = 5 \div 6 = .833\frac{1}{3}. \quad \text{Operation : } 6 \overline{)5.000} \quad \text{Result.}$$

Since a decimal of this kind cannot be divided exactly, it is customary to ask for a certain number of decimal places, and the fractional remainder is affixed to the result.

BLACKBOARD PRACTICE

Change to equivalent decimals :

1. $\frac{3}{5}$; $\frac{4}{5}$; $\frac{3}{6}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{8}{10}$.
2. $\frac{7}{14}$; $\frac{8}{16}$; $\frac{7}{28}$; $\frac{8}{32}$; $\frac{16}{20}$; $\frac{15}{75}$.
3. $\frac{12}{40}$; $\frac{21}{35}$; $\frac{35}{70}$; $\frac{35}{140}$; $\frac{56}{140}$; $\frac{65}{360}$.
4. $\frac{15}{60}$; $\frac{32}{60}$; $\frac{45}{100}$; $\frac{48}{160}$; $\frac{63}{252}$; $\frac{86}{480}$.
5. $\frac{12}{400}$; $\frac{35}{700}$; $\frac{54}{1080}$; $\frac{75}{2250}$; $\frac{90}{2250}$; $\frac{145}{2900}$.

Change to decimals of three places :

6. $\frac{5}{7}$; $\frac{6}{7}$; $\frac{7}{9}$; $\frac{8}{11}$; $\frac{15}{16}$; $\frac{21}{23}$; $\frac{35}{39}$.
7. $\frac{12}{29}$; $\frac{35}{45}$; $\frac{45}{54}$; $\frac{60}{65}$; $\frac{75}{85}$; $\frac{91}{92}$; $\frac{98}{99}$.
8. $\frac{13}{100}$; $\frac{57}{121}$; $\frac{121}{150}$; $\frac{354}{511}$; $\frac{675}{688}$; $\frac{877}{1000}$.

ALIQOT PARTS

An **Aliquot Part** of a given number is a number that will exactly divide the given number.

Thus: 2, 5, 10, 20, 25, and 50 will exactly divide 100.

100 is also exactly divisible by the mixed numbers $6\frac{1}{4}$, $8\frac{1}{3}$, $12\frac{1}{2}$, $16\frac{2}{3}$, and $33\frac{1}{3}$; but as division with these numbers is not a short process, we use their fractional equivalents and multiply. Memorize the following:

$$\begin{array}{lll} 6\frac{1}{4} = 1/16 \text{ of } 100. & 8\frac{1}{3} = 1/12 \text{ of } 100. & 12\frac{1}{2} = 1/8 \text{ of } 100. \\ 16\frac{2}{3} = 1/6 \text{ of } 100. & 33\frac{1}{3} = 1/3 \text{ of } 100. & 37\frac{1}{2} = 3/8 \text{ of } 100. \end{array}$$

Aliquot Parts of \$1.00

In business transactions fractional parts of a dollar frequently occur and a knowledge of these fractional parts and their equivalent decimals is constantly required. You should memorize the following:

TABLE OF EQUIVALENTS	
$1¢ = \frac{1}{100} \text{ of } \1	$25¢ = \frac{1}{4} \text{ of } \1
$2¢ = \frac{1}{50} \text{ of } \1	$33\frac{1}{3}¢ = \frac{1}{3} \text{ of } \1
$4¢ = \frac{1}{25} \text{ of } \1	$37\frac{1}{2}¢ = \frac{3}{8} \text{ of } \1
$5¢ = \frac{1}{20} \text{ of } \1	$50¢ = \frac{1}{2} \text{ of } \1
$6\frac{1}{4}¢ = \frac{1}{16} \text{ of } \1	$62\frac{1}{2}¢ = \frac{5}{8} \text{ of } \1
$8\frac{1}{3}¢ = \frac{1}{12} \text{ of } \1	$66\frac{2}{3}¢ = \frac{2}{3} \text{ of } \1
$10¢ = \frac{1}{10} \text{ of } \1	$75¢ = \frac{3}{4} \text{ of } \1
$12\frac{1}{2}¢ = \frac{1}{8} \text{ of } \1	$80¢ = \frac{4}{5} \text{ of } \1
$16\frac{2}{3}¢ = \frac{1}{6} \text{ of } \1	$83\frac{1}{3}¢ = \frac{5}{6} \text{ of } \1
$20¢ = \frac{1}{5} \text{ of } \1	$87\frac{1}{2}¢ = \frac{7}{8} \text{ of } \1

These fractions are called the **Aliquot Parts** of one dollar.

SHORT METHODS BY MEANS OF ALIQOT PARTS

The advantage of the use of fractions instead of decimals will be seen in the following :

Illustrations :

1. Multiply 2460 by 25

$$25 = \frac{1}{4} \text{ of } 100, \text{ or } \frac{100}{4}.$$

$$2460 \times 25 = \frac{2460 \times 100}{4} = \frac{246000}{4} = 61500. \quad \text{Result.}$$

Operation

$$\begin{array}{r} 61500 \quad \text{Result.} \\ 4 \overline{)246000} \end{array}$$

2. Multiply 3279 by $33\frac{1}{3}$.

$$33\frac{1}{3} = \frac{1}{3} \text{ of } 100, \text{ or } \frac{100}{3}.$$

$$\frac{3279 \times 100}{3} = \frac{327900}{3} = 109300. \quad \text{Result.}$$

Operation :

$$\begin{array}{r} 109300 \quad \text{Result.} \\ 3 \overline{)327900} \end{array}$$

BLACKBOARD PRACTICE

Multiply by the use of fractional equivalents :

1. 2480 by $12\frac{1}{2}$. 7. 10472 by 25. 13. 32000 by $6\frac{1}{4}$.
2. 3440 by $12\frac{1}{2}$. 8. 12800 by $37\frac{1}{2}$. 14. 36732 by $6\frac{1}{4}$.
3. 4236 by $16\frac{2}{3}$. 9. 13542 by $33\frac{1}{3}$. 15. 43980 by $16\frac{2}{3}$.
4. 5256 by 25. 10. 16496 by $62\frac{1}{2}$. 16. 44520 by $83\frac{1}{3}$.
5. 6824 by 25. 11. 18615 by $66\frac{2}{3}$. 17. 45600 by $87\frac{1}{2}$.
6. 7611 by $33\frac{1}{3}$. 12. 21963 by $66\frac{2}{3}$. 18. 48800 by $87\frac{1}{2}$.

ORDER OF OPERATION WHEN ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION ARE COMBINED IN A SINGLE EXPRESSION

1. Addition with Subtraction.

When addition and subtraction occur in succession the work is performed in order, beginning at the left.

Thus: $5+7-3+6=12-3+6=9+6=15$. *Result.*

2. Multiplication or Division with Addition and Subtraction.

When multiplication or division occurs with addition and subtraction the multiplications or divisions must be performed first.

Thus:

$8 \times 3 + 7 \times 2 - 6 \times 3 = 24 + 14 - 18 = 38 - 18 = 20$. *Result.*

Also:

$10 - 8 \div 2 + 15 \div 3 = 10 - 4 + 5 = 6 + 5 = 11$. *Result.*

3. Multiplication and Division in Succession.

When multiplication and division occur in succession the work is performed in order, beginning at the left.

Thus: $7+8 \div 2 \times 3 = 7+4 \times 3 = 7+12 = 19$. *Result.*

The Parenthesis () is often used to indicate that a group of two or more numbers is to be considered as a single number.

BLACKBOARD PRACTICE

Find the result in each of the following:

- | | |
|--|---|
| 1. $12.1+5.4-8.7+.4$. | 5. $2.5 \times 3.0 + 3.5 \times 1.2$. |
| 2. $15.4-3.1+5.6-11.9$. | 6. $5.5 \times 3.5 - 2.4 \times 7.9$. |
| 3. $9.01+3.45-4.1-7.22$. | 7. $5.12 \times 3.7 - 5.15 \div 5.0$. |
| 4. $20.7-11.6-5.7+.09$. | 8. $12.5 \times 15.6 - 72.9 \div .45$. |
| 9. $(2.4 \times 1.5) - (3.5 \div 14) \div (3.6 \times 0.25)$. | |

10. $10.25 - (9.6 \div 1.2) \div (6.26 \div 2.5) - (.025 \div .05).$
11. $4.5 \times 3.2 \div .8 \times 1.5 \div .04 \times 12.5 \div .625 \times .00001.$
12. $(.4 \times .8 \div .016) + (.3 \times .17 \div .06) \div (.1 \times .01 \div .001).$

A valuable exercise in testing addition of decimals is shown at the right. If the work is correct the sum of the results from the vertical additions will equal the sum of those from the horizontal additions.

Illustration:

$$\begin{array}{r}
 12.7 + 13.08 = 25.78 \\
 6.18 + 1.11 = 7.29 \\
 .501 + .75 = 1.251 \\
 15.08 + 14.008 = 29.088 \\
 \hline
 34.461 + 28.948 = 63.409
 \end{array}$$

BLACKBOARD PRACTICE

General Review of Decimals

Add the following, and test as illustrated:

- | | | |
|-----------------|-------------------|------------------|
| 1. $2.3 + 0.45$ | 2. $14.5 + 11.56$ | 3. $12.077 + 87$ |
| $.118 + 12.3$ | $1.08 + 5.8$ | $9.4 + 5.115$ |
| $4.09 + 5.07$ | $.134 + 43.07$ | $3 + 10.33$ |

- | | |
|----------------------------------|-------------------------------------|
| 4. $\$10.50 + \$12.18 + \$14.00$ | 5. $\$101.16 + \$100.16 + \$100.00$ |
| $4.66 + 30.76 + 19.77$ | $45.86 + 149.00 + 78.00$ |
| $45.00 + 32.00 + 27.00$ | $77.00 + 1.98 + 177.00$ |
| $8.98 + 19.87 + 27.99$ | $.95 + 86.00 + 1.40$ |
| $3.75 + 34.00 + 18.20$ | $46.98 + 78.19 + 354.10$ |

- | |
|---|
| 6. $\$ 1.45 + \$10.16 + \$100.65 + \1250.45 |
| $.86 + 1.45 + 45.66 + 456.00$ |
| $2.50 + 23.45 + 110.00 + 356.10$ |
| $.54 + 54.66 + .90$ |
| $10.08 + 66.00 + 15.00$ |
| $1.90 + 155.12$ |
| $3.40 + 113.45$ |
| $54.19 + 45.09$ |
| $20.93 + 10.01 + 50.00$ |

WRITTEN APPLICATIONS

The Use of Decimals in Everyday Life

I. The Business Man's Problems in Decimals

1. A coal dealer bought 45.8 tons of coal at \$5.40 per ton, and 115.7 tons at \$4.85 per ton. What was the total amount paid for both?

2. A grocer bought a quantity of coffee at 21.5¢ a pound, and sold it for \$66.25, making a profit of \$12.50. How many pounds of coffee did he buy?

3. A contractor hired 27 men to work 8 hours per day for 18 days, paying each man \$.25 an hour. If he received \$1125 for their work, what was his profit on their labor?

4. The freight charges on 500 bags of coffee averaging 130 pounds to the bag were 18¢ on each 100 pounds. What was the amount of the freight bill?

5. A contractor bought a quantity of cement, using .15 of it on a reservoir and .25 of it on a retaining wall. He had 57.285 tons left. How many tons did he buy?

6. Two real estate men bought a suburban plot of ground containing 1240 acres. One paid for $.37\frac{1}{2}$ of it, the other paid for the rest of it. How many acres did each buy? How much did each man's share cost at \$180 per acre?

7. A merchant bought 360 pounds of coffee at $26\frac{2}{3}$ cents per pound, and 280 pounds at $22\frac{1}{2}$ cents per pound. He mixed the two lots and sold it all at 32 cents per pound. What was his profit on the transaction?

8. A merchant bought 300 yards of carpet at \$1.10 per yard. The weight of the carpet packed for shipment was 200 pounds, and the freight rate was 70¢ per 100 pounds. What was the cost of the carpet including the freight charge?

9. A merchant offered a clerk \$45 a month for a year, the clerk to pay board at \$3.50 per week. The clerk refused this offer, but accepted another offer of \$35 per month including board. How much did the clerk gain in the year by his choice?

10. A contractor put a new roof on a coal shed. He used 450 pounds of tin at \$16.50 per hundred pounds and figured his profit and labor at \$190.00. The coal dealer supplied the contractor with 62.5 tons of coal at \$5.25 per ton. Which one then owed the other, and how much?

II. The Farmer's Problems in Decimals

11. A farmer offers to exchange hay at \$12.50 a ton for 2000 bushels of corn at $87\frac{1}{2}$ cents per bushel. How many tons of hay must he give in exchange for the corn?

12. The cost of raising potatoes on a farm was found to be \$23.45 per acre. The average yield was 142 bushels per acre and the price received was \$1.55 per bushel. What was the average profit per acre?

13. A cow gave 1440 quarts of milk in 90 days, for which her owner received \$0.06 per quart. Grain for the cow during that time cost $\$0.12\frac{1}{2}$ daily. What was the difference between the amount received for her milk and the cost of her grain?

14. A farmer filled his silo at a cost of \$0.75 per ton, but after purchasing improved machinery he filled it at a cost of \$0.48 per ton. How much does he now save in filling a silo holding 225 tons? How much will his saving amount to over a period of five years, if the average amount of his silage is 220 tons for each of those years?

15. A quart of milk weighs 2.15 pounds. In one year one cow gave 4328 pounds of milk, and another cow, 3412 pounds. The dairyman received $\$0.05\frac{1}{2}$ per quart for the milk, and he paid $\$94.15$ for feed for them. How much profit did he make from these two cows?

16. On a stock farm three men are employed by the year at $\$28$ per month, one man at $\$24$ per month, two men at $\$5.60$ per week, and one at $\$18$ per month. What is the total amount paid to the seven men in a year?

17. A farmer feeds 100 pounds of green fodder daily for each 1000 pounds of weight in his herd. The respective weights of his cows are 1041 lb., 1120 lb., 1060 lb., 940 lb., 890 lb., 960 lb., 1060 lb., and 1010 lb. What amount of green fodder does he feed to this herd?

18. A farmer estimated that he could make $\$450$ profit on a field of potatoes. He paid $\$24$ for seed; $\$32$ for planting; $\$26.50$ for spraying; $\$21$ for cultivating; $\$20$ for digging; and sold the crop of 712 bushels at $\$1.85$ per bushel. Did he make more or less than he estimated, and how much?

19. A farmer builds a new fence 2048 feet long, using posts at $\$0.25$ each, and 4 strands of wire. The posts are set 8 feet apart. One strand of wire weighs 7 pounds per hundred feet, and costs him $\$0.035$ per pound. What is the total cost of wire and posts for this fence?

20. In repairing a barn the owner buys 3400 feet of lumber at $\$28.75$ per thousand; 135 pounds of nails at $\$0.0375$ per pound, and hardware amounting to $\$7.35$. He hires a carpenter for $34\frac{1}{2}$ days at $\$5.75$ per day. What is the total amount he pays for the materials and for the labor?

III. The Housekeeper's Problems in Decimals

21. A woman subscribed for a weekly magazine and a monthly magazine, each at a cost of \$1.50 per year. If during this year she had bought a copy of the former each week at \$.05 a copy, and a copy of the latter each month at \$0.15, how much more would she have paid for both?

22. A family allows \$15.00 per week for the expenses of the kitchen. The grocery bill averages \$.90 daily; the milk bill \$.15 daily, the meat bill \$.40 daily, miscellaneous expenses \$.20 daily, and the cook \$4.00 per week. At these rates does the family keep within the allowance for the year?

23. A housekeeper bought a sewing machine on the installment plan, paying \$1.00 down and \$0.50 per week for a year. She also bought a piano on the same plan, paying \$10.00 down, and \$2.00 per week for $2\frac{1}{2}$ years. If she had paid cash, the sewing machine could have been purchased for \$20 and the piano for \$225. How much could she have saved by paying cash for both?

24. A family consisting of father and mother and three children are keeping house at an average weekly expense for food of \$16.25, and their other fixed expenses are house rent, \$30.00 per month; gas, \$1.85 per month; and coal, 15 tons annually at an average price of \$6.00 per ton. They are planning to leave their house and rent a three-room flat at \$20.00 per month including heat and light. Their board will cost \$5.00 each per week for the father and the mother, and \$3.50 per week for each of the children. How much could they save in one year by making this change?

DENOMINATE NUMBERS

To Measure a Quantity is to find how many times the quantity contains some established unit of measure.

The Unit of Measure is the established unit that we use as a divisor.

Illustrations :

To measure the length of a rope in feet is to find how many times that length contains the unit of measure, 1 foot.

To measure a quantity of grain in pounds is to find how many times that quantity contains the unit of measure, 1 pound.

The different units of measure are established by custom and are chosen for convenience.

For example, it is convenient to measure the length of this page with a small unit of measure, the inch; but the length of the room can be measured more conveniently by using the unit of measure, one foot. Furthermore, neither of these units of measure would be convenient for the measure of a long distance like that from New York to Chicago. For such a distance the convenient unit of measure is the mile.

A Concrete Number that expresses a measure is called a **Denominate Number**.

5 feet, 9 yards, 7 pounds, are denominate numbers, for each expresses a measure.

5 books, 9 men, 7 houses, although concrete numbers, do not express measures, and are not denominate numbers.

A Simple Denominate Number is a denominate number expressing a single measure.

15 feet, 35 minutes, 75 tons, are each simple denominate numbers.

A Compound Denominate Number is a denominate number expressed in two or more measures that are related to each other.

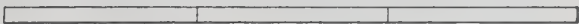
3 feet, 7 inches : 4 yards, 2 feet : 5 hours, 20 minutes — are each compound denominate numbers.

A *denominate number* may be changed from one denomination to another without changing its value.

For example, we may change the measure "1 yard" to the denomination "3 feet," without changing the length.

Furthermore, since 1 foot equals 12 inches, we may again change the measure "1 yard" by writing "36 inches."

1 yard 

3 feet 

36 inches 

It will be observed in the illustration that the length is the same in each case, although we name it differently as we refer to the yard, or to feet, or to inches.

The process of changing a given denominate number from a given denomination to a lower denomination is sometimes called "**Reduction Descending.**"

Illustration :

Change 12 yards, 2 feet, 7 inches to inches.

For this change we use the denominate expressions

1 foot = 12 inches ; 1 yard = 3 feet.

Then to change 12 yards, 2 feet, 7 inches, we work as follows :

Since 1 yard = 3 feet, 12 yards = (12×3) feet = 36 feet.

Adding the given 2 feet :

36 feet + 2 feet = 38 feet.

$$\begin{array}{r}
 12 \text{ yards} \\
 \underline{3} \\
 36 \text{ feet} \\
 \underline{2 \text{ feet}} \\
 38 \text{ feet} \\
 \underline{12} \\
 76 \\
 \underline{38}
 \end{array}$$

Since 1 foot = 12 inches, 38 feet = (38×12) inches = 456 inches.

Adding the given 7 inches :

456 inches + 7 inches = 463 inches.

$$\begin{array}{r}
 456 \text{ inches} \\
 \underline{7 \text{ inches}} \\
 463 \text{ inches}
 \end{array}$$

Therefore : 12 yards, 2 feet, 7 inches = 463 inches. Result.

In practice the work is usually arranged as shown at the right.

From the process we have the following :

To change a denominate number to a lower denomination :

Multiply the number of the highest denomination given by the number of units of the next lower denomination required to make one of the higher.

Add to the product the number of given units of that lower denomination.

With the result proceed in the same manner for the next lower denomination.

Repeat the process with each successive result until the entire given number is changed to the required denomination.

In practical work most reductions of denominate numbers include but two denominations. Because of the ease with which fractions are used, we speak of " $2\frac{3}{4}$ feet" instead of "2 feet 9 inches." Engineers and surveyors use steel measuring tapes marked in feet and tenths of feet, and use the notation "7.5'" instead of "7 feet 6 inches."

MEASURES OF LENGTH

Measures that are used for measuring only length are called **Linear Measures**.

TABLE OF LINEAR MEASURES

The Primary Unit =	1 inch	abbreviated	1 in.
12 inches	= 1 foot	"	1 ft.
3 feet	= 1 yard	"	1 yd.
$5\frac{1}{2}$ yards	= 1 rod	"	1 rd.
also $16\frac{1}{2}$ feet	= 1 rod		
320 rods	= 1 mile	"	1 mi.

It is helpful to remember the equivalents in reverse order.

That is: 1 mile = 320 rods = 1760 yards = 5280 feet.

ORAL PRACTICE

How many inches are there in :	How many feet are there in :	How many yards are there in :
-----------------------------------	---------------------------------	----------------------------------

- | | | |
|------------------|-------------------|-------------------|
| 1. 5 ft.? | 8. 7 yd.? | 15. 12 rd.? |
| 2. 8 ft.? | 9. 9 yd.? | 16. 20 rd.? |
| 3. 10 ft.? | 10. 15 yd.? | 17. 10 rd. 2 yd.? |
| 4. 5 ft. 6 in.? | 11. 6 yd. 2 ft.? | 18. 20 rd. 4 yd.? |
| 5. 6 ft. 9 in.? | 12. 8 yd. 2 ft.? | 19. 30 rd. 5 yd.? |
| 6. 8 ft. 10 in.? | 13. 20 yd. 1 ft.? | 20. 1 mi. 20 rd.? |
| 7. 10 ft. 7 in.? | 14. 6 rd. 6 ft.? | 21. 2 mi. 50 rd.? |

How many rods are there in :

- | | | |
|-------------|--------------------|---------------------|
| 22. 2 mi.? | 25. 2 mi. 50 rd.? | 28. 5 mi. 100 rd.? |
| 23. 5 mi.? | 26. 3 mi. 100 rd.? | 29. 10 mi. 50 rd.? |
| 24. 10 mi.? | 27. 4 mi. 200 rd.? | 30. 10 mi. 300 rd.? |

BLACKBOARD PRACTICE

Change to inches :	Change to feet :	Change to inches :
--------------------	------------------	--------------------

- | | | |
|------------------|------------------|------------------|
| 1. 5 ft. 3 in. | 8. 5 yd. 1 ft. | 15. 2 yd. 2 ft. |
| 2. 7 ft. 4 in. | 9. 7 yd. 2 ft. | 16. 4 yd. 1 ft. |
| 3. 10 ft. 5 in. | 10. 12 yd. 1 ft. | 17. 6 yd. 2 ft. |
| 4. 13 ft. 4 in. | 11. 24 yd. 2 ft. | 18. 10 yd. 2 ft. |
| 5. 24 ft. 1 in. | 12. 10 rd. 3 ft. | 19. 12 yd. 1 ft. |
| 6. 38 ft. 4 in. | 13. 12 rd. 7 ft. | 20. 21 yd. 1 ft. |
| 7. 45 ft. 11 in. | 14. 24 rd. 9 ft. | 21. 27 yd. 2 ft. |

Change to yards :	Change to feet :	Change to yards :
-------------------	------------------	-------------------

- | | | |
|------------------|-------------------|-------------------|
| 22. 4 rd. | 28. 5 rd. 4 ft. | 34. 2 mi. 100 yd. |
| 23. 12 rd. | 29. 4 rd. 7 ft. | 35. 5 mi. 200 yd. |
| 24. 10 rd. 2 yd. | 30. 12 rd. 8 ft. | 36. 4 mi. 150 yd. |
| 25. 15 rd. 4 yd. | 31. 15 rd. 6 ft. | 37. 3 mi. 27 rd. |
| 26. 24 rd. 1 yd. | 32. 21 rd. 8 ft. | 38. 5 mi. 100 rd. |
| 27. 31 rd. 5 yd. | 33. 32 rd. 12 ft. | 39. 10 mi. 96 rd. |

Changing Decimal Expressions

Illustration :

Change .45 of a yard to feet and inches.

$$.45 \text{ of a yard} = .45 \text{ of 3 feet} = .45 \times 3 \text{ ft.} = 1.35 \text{ ft.}$$

$$.35 \text{ of a foot} = .35 \text{ of 12 inches} = .35 \times 12 \text{ in.} = 4.2 \text{ in.}$$

Therefore, .45 yd. = 1 ft. 4.2 in. *Result.*

BLACKBOARD PRACTICE

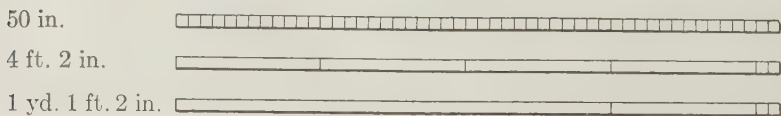
Change to lower denominations :

- | | | |
|-------------|--------------|--------------|
| 1. .25 yd. | 9. .625 yd. | 17. .375 rd. |
| 2. .35 yd. | 10. .875 yd. | 18. .625 rd. |
| 3. .54 yd. | 11. 2.75 yd. | 19. 1.5 mi. |
| 4. .75 yd. | 12. 5.35 yd. | 20. 1.25 mi. |
| 5. .92 yd. | 13. .25 rd. | 21. .125 mi. |
| 6. 1.15 yd. | 14. .125 rd. | 22. .375 mi. |
| 7. .125 yd. | 15. .225 rd. | 23. .625 mi. |
| 8. .375 yd. | 16. .275 rd. | 24. .975 mi. |

Changing a Given Denominate Number to a Number of a Higher Denomination

We may change inches to feet or we may change feet to yards, or to rods, etc.

From the figure it should be clear that changing a denominate number to a number of higher denomination *merely changes the form.*



Clearly, the length, 54 inches, is expressed in different form when named in higher denominations.

The process of changing a given denominate number to a denominate number of a higher denomination is sometimes called "**Reduction Ascending.**"

Change 194 inches to a higher denomination.

There are 12 inches in 1 foot.

Then, $194 \div 12$ = the number of feet in 194 inches $\begin{array}{r} 12 \overline{)194} \\ 24 \\ \hline 16 \end{array}$
 = 16 ft. (with 2 inches remaining). $\begin{array}{r} 3 \overline{)16 \text{ ft.}} \\ 9 \\ \hline 5 \text{ yd.} \end{array}$ 2 in.

There are 3 feet in 1 yard.

Then, $16 \div 3$ = the number of yards in 16 feet
 = 5 yd. (with 1 foot remaining).

Therefore, 194 inches = 5 yd. 1 ft. 2 in. *Result.*

To Change a Given Denominate Number to a Number of a Higher Denomination

Divide the given number of units by the number of units of the given denomination, that make one of the next higher denomination.

Divide the quotient obtained, and each successive quotient in like manner, until the whole given number is changed to the required higher denominations.

The last quotient, with the several remainders annexed in their order, is the required reduction.

ORAL PRACTICE

How many feet are there
in:

How many yards are there
in:

1. 48 in.?

6. 84 in.?

11. 6 ft.?

16. 26 ft.?

2. 60 in.?

7. 120 in.?

12. 9 ft.?

17. 39 ft.?

3. 72 in.?

8. 132 in.?

13. 12 ft.?

18. 75 ft.?

4. 96 in.?

9. 144 in.?

14. 15 ft.?

19. 108 ft.?

5. 108 in.?

10. 288 in.?

15. 18 ft.?

20. 210 ft.?

How many feet and inches are there in :	How many yards and feet are there in :
--	---

- | | | | |
|-------------|--------------|-------------|-------------|
| 21. 30 in.? | 25. 100 in.? | 29. 10 ft.? | 33. 37 ft.? |
| 22. 42 in.? | 26. 125 in.? | 30. 13 ft.? | 34. 55 ft.? |
| 23. 34 in.? | 27. 150 in.? | 31. 16 ft.? | 35. 62 ft.? |
| 24. 65 in.? | 28. 192 in.? | 32. 25 ft.? | 36. 76 ft.? |

BLACKBOARD PRACTICE

Change to higher units :

- | | | | |
|------------|------------|------------|-------------|
| 1. 100 in. | 8. 7 ft. | 15. 15 yd. | 22. 385 rd. |
| 2. 128 in. | 9. 13 ft. | 16. 19 yd. | 23. 450 rd. |
| 3. 140 in. | 10. 20 ft. | 17. 29 yd. | 24. 490 rd. |
| 4. 157 in. | 11. 32 ft. | 18. 37 yd. | 25. 512 rd. |
| 5. 165 in. | 12. 47 ft. | 19. 55 yd. | 26. 641 rd. |
| 6. 184 in. | 13. 63 ft. | 20. 64 yd. | 27. 660 rd. |
| 7. 193 in. | 14. 78 ft. | 21. 78 yd. | 28. 735 rd. |

Change to rods, yards, and feet :

Change to yards and feet :

- | | | | |
|-------------|-------------|-------------|-------------|
| 29. 59 ft. | 33. 97 in. | 37. 66 ft. | 41. 66 yd. |
| 30. 73 ft. | 34. 119 in. | 38. 135 ft. | 42. 165 ft. |
| 31. 400 ft. | 35. 287 in. | 39. 44 yd. | 43. 175 yd. |
| 32. 750 ft. | 36. 358 in. | 40. 198 ft. | 44. 375 ft. |

REVIEW

Change orally to inches :

- | | |
|--------------------|---------------------------------|
| 1. 4 ft. ; 8 ft. | 6. 3 ft. 6 in. ; 2 ft. 3 in. |
| 2. 6 ft. ; 3 ft. | 7. 4 ft. 8 in. ; 5 ft. 7 in. |
| 3. 7 ft. ; 9 ft. | 8. 6 ft. 5 in. ; 7 ft. 9 in. |
| 4. 10 ft. ; 11 ft. | 9. 9 ft. 7 in. ; 10 ft. 7 in. |
| 5. 15 ft. ; 12 ft. | 10. 12 ft. 6 in. ; 15 ft. 6 in. |

Change orally to feet :

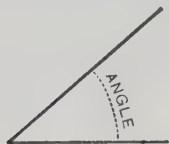
- | | |
|---------------------|---------------------------------|
| 11. 4 yd. ; 5 yd. | 16. 5 yd. 2 ft. ; 6 yd. 1 ft. |
| 12. 6 yd. ; 7 yd. | 17. 9 yd. 1 ft. ; 10 yd. 2 ft. |
| 13. 10 yd. ; 9 yd. | 18. 17 yd. 2 ft. ; 18 yd. 2 ft. |
| 14. 13 yd. ; 15 yd. | 19. 34 yd. 1 ft. ; 40 yd. 1 ft. |
| 15. 24 yd. ; 36 yd. | 20. 57 yd. 2 ft. ; 65 yd. 2 ft. |

Change orally to higher denominations :

- | | | | |
|------------|------------|--------------|-------------|
| 21. 36 in. | 26. 8 ft. | 31. 6 yd. | 36. 33 yd. |
| 22. 54 in. | 27. 11 ft. | 32. 11 yd. | 37. 55 yd. |
| 23. 62 in. | 28. 19 ft. | 33. 13 yd. | 38. 66 yd. |
| 24. 75 in. | 29. 33 ft. | 34. 16.5 yd. | 39. 110 yd. |
| 25. 88 in. | 30. 50 ft. | 35. 27.5 yd. | 40. 121 yd. |
-
41. Change 7624 feet to higher units.
 42. Change 6 rd. 4 yd. 2 ft. to feet.
 43. Change 1000 inches to higher units.
 44. Change .624 of a mile to feet.
 45. Change 3 mi. 110 rd. 40 ft. to feet.
 46. How many inches are there in one mile?
 47. Change .025 of a mile to yards.
 48. Change .54 mi. to yards, feet, and inches.
 49. What part of a mile is 440 yards?
 50. Change 275 ft. 9 in. to inches.

MEASURES OF SURFACE

The difference in the direction of two lines that meet is called an **Angle**.



If a line meets another line so that two equal angles are formed, each angle is called a **Right Angle**.

A figure having four equal sides and four right angles is called a **Square**.

If each side of a square is one inch long the square is called a **Square Inch**.

The Unit of Surface Measure is a square.

The Area of a Figure is the number of square units in its surface.

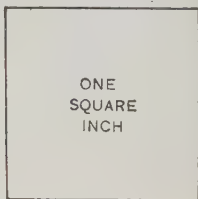


Illustration :

In the figure below it will be observed that a number of square units make a row, and that a number of rows make up the surface.

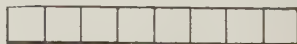
1 square unit :



1 row of 8 square units :

3 rows of 8 square units each :

Area of the figure $3 \times 8 = 24$ square units.

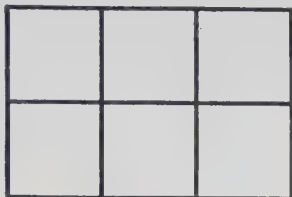


Surface Measure is based upon the common unit, one square inch.



A surface measured in square inches is illustrated at the right.

The area consists of two rows of square units, three units to each row. Hence, the area (2×3) square inches. Or, we may consider that there were three vertical, or upright, rows, with two units in each row. And in this case the area would be (3×2) square units = (3×2) square inches. It is clear, therefore, that the area is the same whether the rows are counted horizontally or vertically.



Measures that are used for measuring only area are called **Surface Measures**.

TABLE OF SURFACE MEASURES

The Primary Unit	= 1 square inch	abbreviated	1 sq. in.
144	square inches	= 1 square foot	" 1 sq. ft.
9	square feet	= 1 square yard	" 1 sq. yd.
$30\frac{1}{4}$	square yards	= 1 square rod	" 1 sq. rd.
160	square rods	= 1 acre	" 1 A.
640	acres	= 1 square m'le	" 1 sq. mi.

ORAL PRACTICE

Change to square inches :

1. 2 sq. ft. ; 4 sq. ft.
2. 3 sq. ft. ; 6 sq. ft.
3. 5 sq. ft. ; 7 sq. ft.
4. 10 sq. ft. ; 12 sq. ft.
5. 20 sq. ft. ; 25 sq. ft.
6. $\frac{1}{2}$ sq. ft. ; $\frac{1}{8}$ sq. ft.
7. $\frac{1}{4}$ sq. ft. ; $\frac{3}{4}$ sq. ft.
8. $\frac{2}{3}$ sq. ft. ; $\frac{5}{8}$ sq. ft.
9. .5 sq. ft. ; .50 sq. ft.
10. .75 sq. ft. ; .25 sq. ft.

Change to square feet :

- | | |
|----------------|-----------------|
| 11. 4 sq. yd. | 16. 15 sq. yd. |
| 12. 5 sq. yd. | 17. 20 sq. yd. |
| 13. 7 sq. yd. | 18. 30 sq. yd. |
| 14. 9 sq. yd. | 19. 50 sq. yd. |
| 15. 10 sq. yd. | 20. 100 sq. yd. |

Change to acres :

- | | | |
|-----------------|-----------------|------------------|
| 21. 320 sq. rd. | 24. 80 sq. rd. | 27. 640 sq. rd. |
| 22. 640 sq. rd. | 25. 40 sq. rd. | 28. 960 sq. rd. |
| 23. 480 sq. rd. | 26. 120 sq. rd. | 29. 1280 sq. rd. |

BLACKBOARD PRACTICE

Change to the next lower units :

- | | | |
|---------------|----------------|-----------------|
| 1. 10 sq. ft. | 8. 24 sq. yd. | 15. 42 A. |
| 2. 24 sq. ft. | 9. 15 sq. rd. | 16. 60 A. |
| 3. 30 sq. ft. | 10. 24 sq. rd. | 17. 3 sq. mi. |
| 4. 45 sq. ft. | 11. 35 sq. rd. | 18. 8 sq. mi. |
| 5. 60 sq. ft. | 12. 5 A. | 19. 10 sq. mi. |
| 6. 10 sq. yd. | 13. 10 A. | 20. 24 sq. mi. |
| 7. 15 sq. yd. | 14. 15 A. | 21. 110 sq. mi. |

Change to square inches :

22. 10 sq. ft. 40 sq. in.
 23. 15 sq. ft. 30 sq. in.
 24. 20 sq. ft. 54 sq. in.
 25. 30 sq. ft. 90 sq. in.
 26. 45 sq. ft. 140 sq. in.
 27. 54 sq. ft. 27 sq. in.

Change to square feet :

28. 9 sq. yd. 5 sq. ft.
 29. 12 sq. yd. 7 sq. ft.
 30. 24 sq. yd. 6 sq. ft.
 31. 35 sq. yd. 1 sq. ft.
 32. 48 sq. yd. 4 sq. ft.
 33. 65 sq. yd. 3 sq. ft.

Change to square inches :

- | | |
|------------------|------------------|
| 34. 2.5 sq. ft. | 37. .3 sq. yd. |
| 35. 4.9 sq. ft. | 38. 4.5 sq. yd. |
| 36. 5.25 sq. ft. | 39. 12.8 sq. yd. |

40. 2 sq. ft. 17 sq. in. ; 4 sq. ft. 15 sq. in.
 41. 13 sq. ft. 90 sq. in. ; 15 sq. ft. 75 sq. in.
 42. 20 sq. ft. 120 sq. in. ; 18 sq. ft. 132 sq. in.

Change to square feet :

- | | | |
|-----------------|------------------|-----------------|
| 43. 340 sq. in. | 46. 12 sq. yd. | 49. 4 sq. rd. |
| 44. 556 sq. in. | 47. 15 sq. yd. | 50. 3.5 sq. rd. |
| 45. 840 sq. in. | 48. 30.5 sq. yd. | 51. 121 sq. rd. |

Change to square yards :

- | | | |
|-----------------|------------------|--------------|
| 52. 14 sq. rd. | 55. 1547 sq. in. | 58. 4 A. |
| 53. 45 sq. rd. | 56. 543 sq. ft. | 59. 2.5 A. |
| 54. 125 sq. rd. | 57. 901 sq. ft. | 60. 0.275 A. |

Change to square rods :

- | | | |
|------------------|-----------------|--------------|
| 61. 100 sq. ft. | 64. 75 sq. yd. | 67. 8 A. |
| 62. 2400 sq. ft. | 65. 340 sq. yd. | 68. 2.25 A. |
| 63. 6500 sq. ft. | 66. 720 sq. yd. | 69. 3.625 A. |

Change to acres :

Change to square miles :

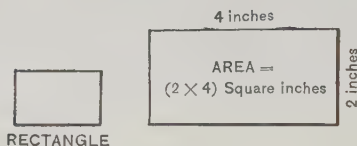
- | | |
|--------------------|---------------|
| 70. 720 sq. rd. | 74. 3420 A. |
| 71. 1200 sq. rd. | 75. 5460 A. |
| 72. 32,400 sq. yd. | 76. 32,000 A. |
| 73. 2.375 sq. mi. | 77. 48,000 A. |

APPLICATIONS OF SURFACE MEASURES

I. The Rectangle

A figure bounded by four straight lines, and having four right angles, is called a **Rectangle**.

We have already learned that the area of a surface is measured by the number of square units in it. Therefore,



The **Area of a Rectangle** is obtained by counting the number of units in a row and the number of rows, and taking the product of the two numbers. Or,

The area of a rectangle may be expressed as the product of

The number of units in the length $\times$ the number of units in the width $\times$ the unit of square measure.

$$\text{Area} = \text{Length} \times \text{Width} \times 1 \text{ sq. unit.}$$

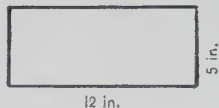
The kind of unit used to measure the length and the width of a rectangle must be the same for both measurements. It follows, then, that

The area of a rectangle is a number of square inches when both its length and its breadth are given in inches.

The area of a rectangle is a number of square feet when both its length and its breadth are given in feet. Etc.

The Perimeter of a figure is the length of the line that bounds it.

Thus, the perimeter of a rectangle in the figure is 2 times (5 in. + 12 in.) = 2×17 in. = 34 in.



Finding One Side of a rectangle when the area and the other side are given.

Since the product of the numbers that express the length and the width of a rectangle equals the number of square units in its area, we may divide the number of square units in the area by either dimension and obtain the other dimension.

Illustration :

The area of a rectangle whose length and breadth are 20 ft. and 9 ft., respectively, is (20×9) sq. ft. = 180 sq. ft.

Then 180 (the area) $\div 20$ (the length) = 9 (the width)
and 180 (the area) $\div 9$ (the width) = 20 (the length).

ORAL PRACTICE

Give, in square units of the same name, the area of a rectangle whose length and width respectively are :

- | | | |
|--------------------|--------------------|--------------------|
| 1. 5 in., 6 in. | 11. 6 ft., 9 ft. | 21. 20 yd., 25 yd. |
| 2. 5 in., 7 in. | 12. 8 ft., 10 ft. | 22. 10 rd., 17 rd. |
| 3. 8 in., 10 in. | 13. 7 ft., 12 ft. | 23. 10 rd., 24 rd. |
| 4. 9 in., 12 in. | 14. 8 ft., 13 ft. | 24. 10 rd., 31 rd. |
| 5. 10 in., 12 in. | 15. 9 ft., 20 ft. | 25. 12 rd., 20 rd. |
| 6. 10 in., 17 in. | 16. 12 ft., 15 ft. | 26. 12 rd., 30 rd. |
| 7. 15 in., 20 in. | 17. 12 ft., 20 ft. | 27. 15 rd., 30 rd. |
| 8. 15 in., 30 in. | 18. 10 yd., 18 yd. | 28. 16 rd., 30 rd. |
| 9. 20 in., 30 in. | 19. 10 yd., 24 yd. | 29. 20 rd., 50 rd. |
| 10. 40 in., 50 in. | 20. 15 yd., 30 yd. | 30. 30 rd., 90 rd. |

BLACKBOARD PRACTICE

Find the area of the following rectangles, whose length and width respectively are :

- | | | |
|---|-------------------------------|---------------------|
| 1. 15 in., 10 in. | 6. 5 ft., 4 ft. | 11. 9 yd., 5 ft. |
| 2. 18 in., 12 in. | 7. 10 ft., 3 ft. | 12. 10 yd., 10 ft. |
| 3. 20 in., 14 in. | 8. 19 ft., 13 ft. | 13. 18 ft., 4 yd. |
| 4. 36 in., 15 in. | 9. 15 yd., 12 yd. | 14. 30 rd., 22 yd. |
| 5. 48 in., 22 in. | 10. 24 yd., 18 yd. | 15. 110 yd., 11 rd. |
| 16. 4 ft. 2 in., 3 ft. 1 in. | 20. 8 ft. 10 in., 6 ft. 7 in. | |
| 17. 3 ft. 4 in., 2 ft. 5 in. | 21. 5 yd. 2 ft., 3 yd. 2 ft. | |
| 18. 4 ft. 5 in., 3 ft. 4 in. | 22. 8 yd. 1 ft., 5 yd. 2 ft. | |
| 19. 6 ft. 9 in., 5 ft. 11 in. | 23. 15 rd. 4 yd., 8 rd. 3 yd. | |
| 24. 20 rd. 4 yd., 10 rd. 2 yd. 1 ft. | | |
| 25. 24 rd. 3 yd. 2 ft., 8 rd. 5 yd. 2 ft. | | |

26. How many square feet are there in a rectangular floor 27.5 ft. long and 14 ft. wide?

27. How many square yards are there in a walk 216 ft. long and 4 ft. 6 in. wide?

28. How many square rods are there in a rectangular field 414 ft. long and 54.45 ft. wide?

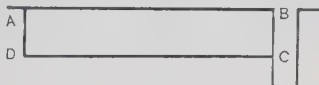
29. How many square yards are there in the ceiling of a rectangular hall 148 feet long and 69 feet wide?

30. A rectangular field is 140 rods long and 24 rods wide. How many acres are there in the area of the field?

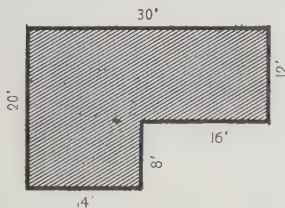
31. A rectangular room is 45 feet long, and the area of the carpet that covers it is 900 square feet. How wide is the room?

32. A street pavement, rectangular in shape and 442 feet long, has an area of 17,680 square feet. How many feet are there in the width of the street?

33. A rectangular field, $ABCD$, is cut up into building lots, each of which is 72.5 feet in width measured along the street from A to B . If the length of AB is 580 feet, and if the area of the field is 58,000 square feet, how many feet are there in the depth of the lots? How many lots are there in the field? How many square feet are there in the area of each lot?

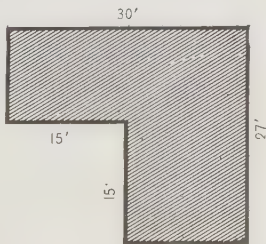


34. A garden 150 feet long and 45 feet wide is surrounded by a grass plot 18 feet wide. Find the area of the garden. Find the area of the grass plot. Find the sum of the areas of both the garden and the grass plot. (Make a neat drawing to illustrate the problem, and explain your solution.)

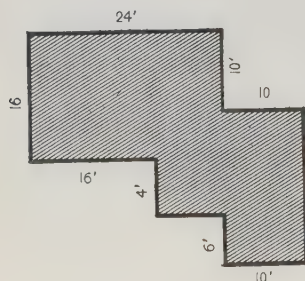
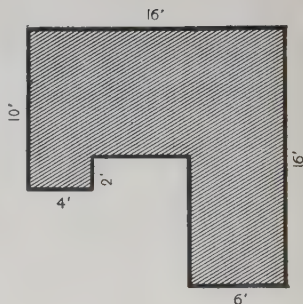


35. The figure at the left represents the shape and the dimensions of the floor of a room. How much will it cost to paint the floor of this room at 20 cents per square foot?

36. A gardener contracts to sod an area shaped as in the figure at the right. If he cuts the new sod one foot square, how many must he cut for the given area?

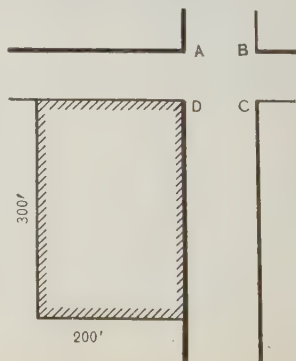


37. The ceiling of a room shaped as in the figure at the right is plastered at a cost of \$.75 per square yard, and is then kalsomined for \$3.68. Find the cost of both plastering and kalsomining per square yard.



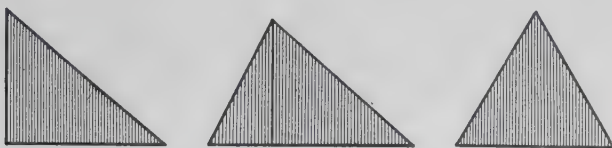
38. The cellar floor of a dwelling-house is shaped as in the figure at the left, and is covered with concrete. If the cost per square yard is \$1.10, find the cost of laying the floor.

39. A man lives on the corner of a block, the street in front of his house is 100 feet wide, and the street at the side of his house is 60 feet wide. Both streets are newly paved and he must pay \$1.65 per square yard for the area of one half the streets opposite his property. If the city pays for the portion in the intersection of the streets (the area $ABCD$), how much must this man pay for his part of the new paving?



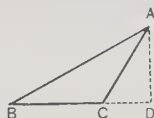
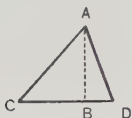
II. The Triangle

A Triangle is a figure bounded by three straight lines.



The meeting-point of any two sides of a triangle is called a **Vertex**.

A straight line drawn from any vertex and making right angles with the side opposite that vertex is called an **Altitude**.



In the triangle at the left AD is an altitude on the side BC . In certain triangles the altitude from one vertex meets the opposite side only when that side is extended. In the triangle at the right the altitude from A meets the side BC when that side is extended to D .

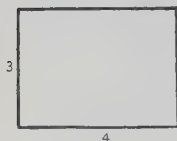
For convenience any side of a triangle to which an altitude is drawn may be considered as a **Base**. In most of our work we assume the side upon which a triangle is supposed to stand as the base.

A triangle having a right angle is called a **Right Triangle**.

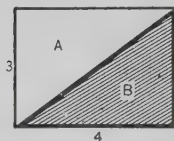
A triangle having three equal sides is called an **Equilateral Triangle**.

In the figures at the top of the page the triangle at the left is a right triangle. The triangle at the right is an equilateral triangle.

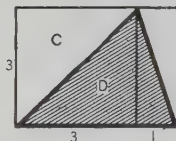
The Area of a Triangle



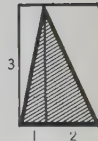
(1)



(2)



(3)



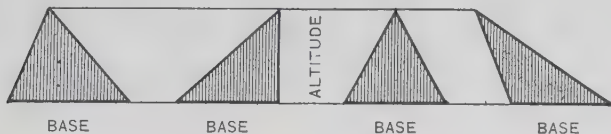
(4)

1. What is the area of the rectangle in (1)?
2. What is the area of the rectangle in (2)?
3. How does the area of the triangle *A* in (2) compare with the area of the triangle *B*?
4. The rectangle in (2) has a base of 4 inches and an altitude of 3 inches. What is the base of the triangle *B*? What is the altitude of the triangle *B*? What is the area of the triangle *B*?
5. What is the area of the triangle *C* in (3)? What is the area of the triangle *D* in (3)? What is the sum of the areas of the triangles *C* and *D*? What is the area of the rectangle in (3)? How does the area of the rectangle compare with the sum of the areas of the triangles in (3)?
6. What is the area of the rectangle in (4)? What is the area of the triangle in the shaded portion of (4)? Can you state a relation between the area of a triangle and the area of a rectangle which has the same base and altitude as the triangle?

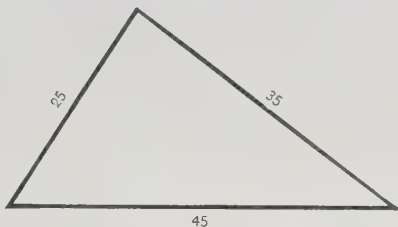
From the relations discovered we may make the following conclusion:

The Area of a Triangle = $\frac{1}{2}$ the product of the length of the base $\times$ the length of the altitude $\times$ the unit of measure.

In measuring the base and the altitude the same unit must be used for both. Triangles may have different shapes and still have the same base and altitude, as shown in the following illustration.



The Perimeter of a triangle is the sum of the lengths of the three sides. In the figure at the right the perimeter is $(25 + 35 + 45)$ units.



ORAL PRACTICE

Give the area of a triangle whose base and altitude respectively are :

- | | | |
|-------------------|--------------------|--------------------|
| 1. 10 in., 8 in. | 8. 12 ft., 10 ft. | 15. 10 yd., 18 yd. |
| 2. 12 in., 10 in. | 9. 14 ft., 6 ft. | 16. 19 yd., 12 yd. |
| 3. 10 in., 6 in. | 10. 20 ft., 8 ft. | 17. 16 yd., 18 yd. |
| 4. 14 in., 12 in. | 11. 24 ft., 10 ft. | 18. 14 yd., 24 yd. |
| 5. 16 in., 14 in. | 12. 16 ft., 20 ft. | 19. 18 rd., 10 rd. |
| 6. 20 in., 12 in. | 13. 15 ft., 14 ft. | 20. 12 rd., 15 rd. |
| 7. 30 in., 18 in. | 14. 20 ft., 19 ft. | 21. 20 rd., 25 rd. |

BLACKBOARD PRACTICE

Find the area of the following triangles, when the base and altitude respectively are :

- | | | |
|-------------------------------|------------------------------------|----------------------------------|
| 1. 24 in., 10 in. | 6. 24 ft., 14 ft. | 11. 15 ft., 45 in. |
| 2. 36 in., 14 in. | 7. 36 ft., 19 ft. | 12. 120 in., 4.5 ft. |
| 3. 54 in., 26 in. | 8. 48 ft., 31 ft. | 13. 30 ft., 2.5 rd. |
| 4. 37 in., 28 in. | 9. 57 ft., 33 ft. | 14. 40 rd., 40 yd. |
| 5. 51 in., 35 in. | 10. 34 yd., 19 yd. | 15. 100 ft., $33\frac{1}{3}$ yd. |
| 16. 4 ft. 6 in., 2 ft. 3 in. | 20. 12 ft. 10 in., 2 yd. | |
| 17. 6 ft. 5 in., 4 ft. 4 in. | 21. 14 ft. 1 in., 2 yd. | |
| 18. 5 ft. 2 in., 4 ft. 10 in. | 22. 3 yd. 5 in., 4 ft. 2 in. | |
| 19. 8 ft. 7 in., 9 ft. 4 in. | 23. 4 yd. 5 in., 6 ft. 10 in. | |
| | 24. 8 rd. 5 yd. 1 ft., 2 yd. 1 ft. | |
| | 25. 4 rd. 4 yd., 2 rd. 2 yd. 2 ft. | |

26. The area of a triangle is 1260 square feet, and its altitude is 30 feet. What is the base?

27. The area of a triangle is 1840 square feet, and its base is 92 feet. What is the altitude?

28. The area of a triangle is 48 square feet, and the altitude is 72 inches. What is the length of the base in feet?

29. How many square feet are there in the area of a triangular pavement whose base is 110 feet 6 inches, and whose altitude is 42 feet 5 inches?

30. What is the altitude of a triangle whose base is .625 feet, and whose area is .625 square feet?

31. The gable of a house is a triangle whose base is 32 feet 6 inches, and whose altitude is 14 feet 10 inches. What is the area of this gable?

32. The sum of two of the sides of a triangle is 54 feet, and the sum of all three sides is 82 feet. What is the length of the third side?

33. How many square inches are there in the surface of a triangular card whose base is $2\frac{3}{4}$ inches, and whose altitude is $2\frac{1}{4}$ inches?

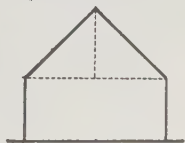
34. The base of a triangular pavement is 4 feet, and the area is 2 square yards. How many feet are there in the altitude?

35. The altitude of a triangular concrete paving block is 36 inches, and the base is $1\frac{1}{3}$ yards long. How many square feet are there in its area?

36. The sides of a triangle are all equal, and their sum is 150 inches. If the altitude of the triangle is 43.3 inches, what is its area?

37. The four faces on the roof of a tower are equal triangles. The tower is 12 feet square, and the altitude of each of the triangles is 16 feet. What is the total area of the roof in square feet?

38. The end of a barn is 50 feet wide, 16 feet high to the eaves, and the triangular gable is 24 feet high from the level of the eaves to the peak. Find the total area of one end of the barn in square feet.



39. A Boy Scout made a tent 8 feet high, and the ends of it were triangular in shape and 6 feet wide at the bottom. How many square yards of material did he need for these ends, not allowing for seams or hems?

40. A path crosses a rectangular park from one corner to the opposite corner. If the park is 840 feet long and 450 feet wide, what is the area of the portion lying on either side of the center-line of the path?

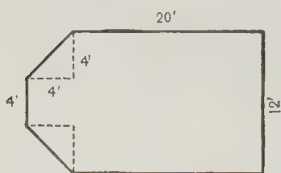
41. The three sides of a triangular grass plot are each 120 rods long. How many rods are there in the distance around the grass plot?

42. The sum of the three sides of a triangular park is 245 feet, one side of it is 30 yards long, and another side is 4 rods long. Find the number of feet in the length of the third side.

43. A patchwork quilt is made up of 388 triangles. Each triangle has equal sides 6 inches long, and the altitude of each is 5.196 inches. Find the area of the quilt in square inches, and also in square feet.

44. A bathroom floor is 6 feet wide and 10 feet long. How many tiles will be needed to cover the floor if each tile is a right triangle whose base and altitude are each 3 inches?

45. The floor of a dining room is shaped as in the figure, and a carpenter contracts to scrape it for \$.05 a square foot.



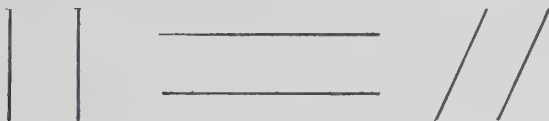
A painter stains and varnishes it for \$.35 a square yard. Find to the nearest cent the total expense of scraping and finishing this floor. (What kind of a figure is made if the two triangles are put together

with the long side of one against the long side of the other?)

46. Two of the three sides of a triangular park are of equal length, and the third side is 400 feet long. If the sum of all three sides of the park is 1000 feet, how many feet are there in each of the equal sides? If the altitude of this park on the longest side is 223.6 feet, how many square feet are there in its area?

III. The Parallelogram

Straight lines that are at all points equally distant from each other are called **Parallel Lines**.



EXAMPLES OF PARALLEL LINES

A figure having four straight sides and its opposite sides parallel is called a **Parallelogram**.

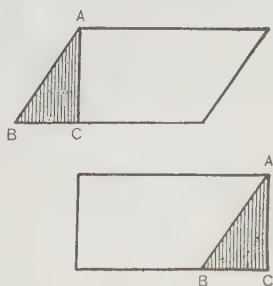


EXAMPLES OF PARALLELOGRAMS

The Area of a Parallelogram

A parallelogram may be changed to a rectangle with the same base and the same altitude if we use the principle shown in the illustration at the right.

We may consider that the triangle ABC is cut from one end and placed upon the other end as in the lower figure. Evidently no change has been made in the amount of area, and the change in form has given us the rectangle with which we are already familiar. The area of a rectangle equals the product of its base by its altitude. Therefore, the



Area of a Parallelogram = Base times Altitude.

As in the case of the rectangle, the area of a parallelogram divided by the base or by the altitude gives, in either case, the other dimension.

ORAL PRACTICE

Give, in square units of the same name, the area of the parallelogram whose base and altitude are :

- | | | |
|-------------------|--------------------|--------------------|
| 1. 12 in., 9 in. | 9. 9 ft., 6 ft. | 17. 8 yd., 5 yd. |
| 2. 14 in., 7 in. | 10. 10 ft., 8 ft. | 18. 12 yd., 7 yd. |
| 3. 15 in., 6 in. | 11. 12 ft., 9 ft. | 19. 16 yd., 10 yd. |
| 4. 16 in., 10 in. | 12. 13 ft., 10 ft. | 20. 15 rd., 9 rd. |
| 5. 18 in., 10 in. | 13. 16 ft., 9 ft. | 21. 18 rd., 10 rd. |
| 6. 20 in., 12 in. | 14. 18 ft., 10 ft. | 22. 20 rd., 15 rd. |
| 7. 30 in., 15 in. | 15. 20 ft., 12 ft. | 23. 30 rd., 20 rd. |
| 8. 20 in., 16 in. | 16. 30 ft., 14 ft. | 24. 50 rd., 24 rd. |

BLACKBOARD PRACTICE

Find the areas of the following parallelograms, if the base and altitude are respectively :

- | | |
|-------------------|-------------------------------|
| 1. 15 in., 9 in. | 9. 54 rd., 31 rd. |
| 2. 25 in., 16 in. | 10. 75 rd., 50 yd. |
| 3. 36 in., 24 in. | 11. 2 ft. 3 in., 1 ft. 7 in. |
| 4. 20 ft., 15 ft. | 12. 8 ft. 4 in., 5 ft. 2 in. |
| 5. 28 ft., 17 ft. | 13. 10 ft. 3 in., 4 ft. 7 in. |
| 6. 42 yd., 13 yd. | 14. 4 yd. 2 ft., 3 yd. 1 ft. |
| 7. 50 rd., 14 rd. | 15. 7 yd. 1 ft., 2 yd. 2 ft. |
| 8. 63 rd., 20 rd. | 16. 12 rd. 3 yd., 5 rd. 2 yd. |

17. 15 rd. 2 yd., 10 rd. 4 yd.

18. 12 rd. 3 yd. 1 ft., 2 rd. 5 yd.

19. 10 rd. 1 yd. 1 ft., 3 rd. 1 yd. 2 ft.

20. 25 rd. 4 yd. 2 ft., 10 rd. 2 yd. 1 ft.

21. The area of a parallelogram is 1350 square feet, and the length of its base is 45 feet. What is its altitude?

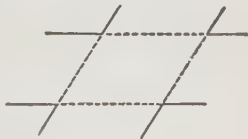
22. Find the width of a parallelogram whose area is 450 square feet, and whose length is 10 yards.

23. What is the area of a parallelogram whose length is 175 feet and whose width is $33\frac{1}{3}$ yards?

24. A parallelogram is 30 yards long and 50 feet wide, and another parallelogram is 50 yards long and 30 feet wide. How much difference is there in their areas?

25. A city boulevard has a grass plot containing 1 acre in the form of a parallelogram. If the distance between two of its parallel sides is 45 rods, what is the width of the parallelogram in rods?

26. Two city streets intersect so that the pavement forms a parallelogram. The length of one side of the parallelogram is 90 feet, and its area is 600 square yards. How many feet are there in the width of the parallelogram?



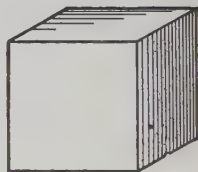
27. A rectangular grass plot is surrounded by a concrete walk 5 feet wide. The grass plot is 100 feet long and 30 feet wide. Draw a sketch to show the plot and the walk, and find the number of square feet in the area of each.

MEASURES OF VOLUME

A **Solid** has three dimensions, length, breadth, and thickness.

A **Cube** is a solid having six equal square sides or faces.

A **Cubic Inch** is a cube, each of whose six faces is a square inch.



I. The Volume of a Cube

The **Volume of a Cube** is the number of cubic units that the cube contains.

The figure indicates the manner in which cubic units make up the volume of a cube.



1 cubic unit.



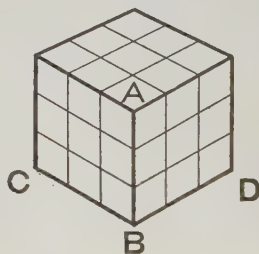
1 row of 3 cubic units.



3 rows of 3 cubic units each.



3 layers of (3×3) cubic units each.



The volume of the solid is, therefore, $3 \times 3 \times 3 = 27$ cubic units.

It will be observed that the number of units which give the three factors we have multiplied is the number of units in each of the three edges AB , BC , and BD , of the solid. Therefore, we may conclude that

The volume of a cube is equal to the product of the three equal dimensions of the cube, all expressed in the same unit of measure.

We have learned that we may indicate the "square of 5" by writing 5^2 , so we may indicate the "cube of 5" in the form 5^3 . Here, also, the small figure "3" indicates that we are to multiply $5 \times 5 \times 5$.

TABLE OF CUBIC MEASURE

The Primary Unit = 1 cubic inch	abbreviated 1 cu. in.
1728 cubic inches = 1 cubic foot	" 1 cu. ft.
27 cubic feet = 1 cubic yard	" 1 cu. yd.

BLACKBOARD PRACTICE

Change to lower units:

- | | | |
|----------------------------|-----------------------------|------------------------------|
| 1. 3 cu. ft. | 8. $42\frac{1}{2}$ cu. ft. | 15. $133\frac{1}{3}$ cu. yd. |
| 2. 12 cu. ft. | 9. $16\frac{2}{3}$ cu. ft. | 16. 8.5 cu. ft. |
| 3. 18 cu. ft. | 10. $54\frac{1}{3}$ cu. ft. | 17. 3.15 cu. ft. |
| 4. 27 cu. ft. | 11. 20 cu. yd. | 18. 4.25 cu. yd. |
| 5. 30 cu. ft. | 12. 27 cu. yd. | 19. 15.65 cu. yd. |
| 6. 35 cu. ft. | 13. 35 cu. yd. | 20. 45.75 cu. yd. |
| 7. $66\frac{2}{3}$ cu. ft. | 14. 125 cu. yd. | |

Change to higher units:

- | | | |
|------------------|--------------------|---------------------|
| 21. 2592 cu. in. | 25. 3888 cu. in. | 29. 1242 cu. ft. |
| 22. 3600 cu. in. | 26. 10,368 cu. in. | 30. 45.12 cu. ft. |
| 23. 6912 cu. in. | 27. 243 cu. ft. | 31. 431.1 cu. ft. |
| 24. 2160 cu. in. | 28. 756 cu. ft. | 32. 2760.25 cu. ft. |

II. The Volume of a Rectangular Solid

For the volume of any rectangular solid we have :

1 cubic unit.



1 row of 4 cubic units.



3 rows of 4 cubic units each.



3 layers of (3×4) cubic units each.



Therefore, the volume $= 3 \times 3 \times 4$ cubic units.

The volume of a rectangular solid is equal to the product of its length, breadth, and thickness, all expressed in the same units of measure.

BLACKBOARD PRACTICE

Find the volume of a rectangular solid whose dimensions are :

1. $12'' \times 15'' \times 10''$.

6. $36'' \times 42'' \times 11''$.

2. $16'' \times 10'' \times 20''$.

7. $54' \times 42' \times 11'$.

3. $14'' \times 25'' \times 30''$.

8. $62' \times 37' \times 15'$.

4. $12'' \times 19'' \times 8''$.

9. $10' 6'' \times 5' 3'' \times 2' 4''$.

5. $18'' \times 24'' \times 20''$.

10. $14' 3'' \times 10' 7'' \times 5' 1''$.

11. $16' 4'' \times 8' 2'' \times 4' 1''$.

12. $2 \text{ yd. } 1 \text{ ft.} \times 3 \text{ yd.} \times 1 \text{ yd. } 1 \text{ ft.}$



13. One of the boxes in this express truck is 18 inches long, 14 inches wide, and 10 inches thick. How many cubic inches are there in its volume? How many cubic feet?

14. The body of this express truck is 12 feet long, 3 feet 8 inches wide, and 1 foot 6 inches deep. How many cubic feet are there in its volume?

15. The largest box in this truck is 4 feet long, 3 feet wide, and 3 feet deep. How many cubic feet are there in its volume?

16. The box on the sidewalk that Betty is looking at is 2 feet 6 inches long, 2 feet 3 inches wide, and 2 feet deep. How many cubic inches are there in its volume?

17. A rectangular box is 4 feet long, 3 feet wide, and 6 inches deep, inside dimensions. How many cubic feet of cement will it hold?

18. All of the inside edges of a rectangular box are 19 inches in length. How many cubic feet of sand will fill this box level full?

MEASURES OF CAPACITY

I. Liquid Measure

Liquid Measure is used for measuring liquids, and in estimating the capacity of tanks, reservoirs, etc.

TABLE OF LIQUID MEASURE

The Primary Unit = 1 gill	abbreviated 1 gi.
4 gills = 1 pint	" 1 pt.
2 pints = 1 quart	" 1 qt.
4 quarts = 1 gallon	" 1 gal.

For many calculations it is convenient to remember that

$31\frac{1}{2}$ gallons = 1 barrel	abbreviated 1 bbl.
63 gallons = 1 hogshead	" 1 hhd.
1 gallon = 231 cubic inches.	

$31\frac{1}{2}$ gallons is a legal standard for a barrel, but it will be found that barrels vary considerably in size, and each is usually marked with its known capacity. Hogsheads also vary in size and are individually marked.

A gallon of water weighs approximately $8\frac{1}{3}$ pounds.

A cubic foot of water contains 7.480 gallons.

BLACKBOARD PRACTICE

Change to the next lower unit :

- | | | | |
|-----------|-----------|------------------------|---------------|
| 1. 24 pt. | 4. 30 qt. | 7. $54\frac{1}{2}$ qt. | 10. 54 gal. |
| 2. 38 pt. | 5. 45 qt. | 8. 82.5 qt. | 11. 62 gal. |
| 3. 54 pt. | 6. 70 qt. | 9. 100 qt. | 12. 72.5 gal. |

Change to the next higher unit :

- | | | | |
|------------|------------|-------------|--------------------------|
| 13. 40 gi. | 16. 57 gi. | 19. 240 qt. | 22. $124\frac{1}{2}$ qt. |
| 14. 40 pt. | 17. 84 pt. | 20. 320 qt. | 23. 142.5 qt. |
| 15. 56 pt. | 18. 62 qt. | 21. 454 qt. | 24. 15.75 qt. |

Change to pints:

25. 58 gi. 30. $140\frac{1}{2}$ qt. 35. 156 gi. 40. 15 gal. 2 qt.
26. 65 gi. 31. $156\frac{3}{4}$ qt. 36. 240 gi. 41. 42 gal. 1 qt.
27. 88 gi. 32. 28.5 gal. 37. 324 qt. 42. 156 gal. 1 qt.
28. 36 qt. 33. $64\frac{1}{2}$ gal. 38. $240\frac{1}{2}$ gal. 43. 241 gal. 2 qt.
29. 89 qt. 34. 105 gal. 39. $354\frac{3}{4}$ gal. 44. 351 gal. 1 qt.

45. How many quart bottles are needed to hold 76 gallons of milk?

46. How many gallons of molasses will be needed to fill 272 tin cans, each containing 1 pint?

47. The capacity of a certain cistern is 125 cubic feet. What is its capacity in gallons?

48. 320 school ink-wells hold $\frac{1}{2}$ gill each. How many times can the whole number be filled from a 10-gallon can of ink?

49. A wholesale dealer in groceries buys 640 gallons of olive oil, and sells it all in bottles each holding $\frac{1}{2}$ pint. How many bottles of this oil does he sell?

50. A tank 3 feet long, 2 feet wide, and 1 foot 6 inches deep is filled with water. How many gallons of water are there in the tank?

51. A gasoline tank on an automobile is 25 inches long, 14 inches wide, and 10 inches deep. How many gallons of gasoline will this tank hold?

52. If milk costing \$.45 a gallon is retailed at \$.15 a quart, and if the delivery expense is \$.015 a quart, what is the profit on the sale and delivery of 400 quarts?

II. Dry Measure

Dry measure is used principally for measuring grains, fruits, and vegetables.

TABLE OF DRY MEASURE

The Primary Unit = 1 pint abbreviated 1 pt.			
2 pints	= 1 quart	"	1 qt.
8 quarts	= 1 peck	"	1 pk.
4 pecks	= 1 bushel	"	1 bu.

It is convenient to remember that

$$1 \text{ bushel} = 2150.42 \text{ cu. in.}$$

BLACKBOARD PRACTICE

Change to the next lower units :

- | | | | |
|-----------|------------------------|--------------------------|--------------------------|
| 1. 24 qt. | 5. $12\frac{1}{2}$ qt. | 9. 54 pk. | 13. 32 bu. |
| 2. 35 qt. | 6. $31\frac{1}{2}$ qt. | 10. 28 pk. | 14. $45\frac{1}{4}$ bu. |
| 3. 46 qt. | 7. 42.5 qt. | 11. $72\frac{3}{4}$ pk. | 15. $176\frac{1}{2}$ bu. |
| 4. 91 qt. | 8. 128.5 qt. | 12. $110\frac{1}{2}$ pk. | 16. $214\frac{3}{4}$ bu. |

Change to the next higher units :

- | | | | |
|------------|-------------|-------------|-------------|
| 17. 54 pt. | 20. 104 pt. | 23. 78 qt. | 26. 145 pk. |
| 18. 86 pt. | 21. 56 qt. | 24. 59 pk. | 27. 241 pk. |
| 19. 92 pt. | 22. 63 qt. | 25. 100 pk. | 28. 342 pk. |

Change to pecks :

Change to bushels :

- | | | | |
|------------|--------------------------|-------------|---------------|
| 29. 28 bu. | 33. 112 bu. | 37. 171 pk. | 41. 156 pt. |
| 30. 46 bu. | 34. $154\frac{1}{2}$ bu. | 38. 234 pk. | 42. 384 pt. |
| 31. 56 bu. | 35. 171.5 bu. | 39. 352 pk. | 43. 230.4 pt. |
| 32. 79 bu. | 36. 50.75 bu. | 40. 128 qt. | 44. 25.8 pt. |

45. How many cubic inches are there in a dry quart?

46. A half-peck of new potatoes sells for \$.65. At this rate how much will a bushel of new potatoes sell for?

47. Using the table at the end of this book, find how many pounds a bushel of potatoes should weigh in your state.

48. How many quart boxes will be required to hold 3 bushels, 3 pecks, and 3 quarts of strawberries?

49. How many bushels of wheat can be placed in a bin 10 feet 6 inches long, 5 feet 4 inches wide, and 3 feet 6 inches deep?

50. Suppose you should buy a peck of potatoes and find that they weigh 14 lb. In your state how much loss does this weight represent?

51. A box holds 3 bushels 2 pecks of grain when level full. How many times can this box be filled from a larger box which holds 15 bushels?

52. A gardener sold two quarts of berries to each of 21 customers, and on the following day he sold 3 quarts to each of 19 customers. How many bushels of berries did he sell in the two days?

53. A wagon-box is 6 feet long, 4 feet 6 inches wide, and 3 feet deep, inside measurements, and is filled with oats. If the oats are sold for \$.65 a bushel, find to the nearest cent the amount received for them.

54. A carload of sweet potatoes cost a dealer \$1.35 a bushel, and he sold them at \$1.00 per $\frac{5}{8}$ bushel basket. How much did he make per bushel, if the cost of freight, hauling, and selling was \$.05 per bushel?

MEASURES OF WEIGHT

Avoirdupois Weight is the common measure of weight in everyday use and is the standard used in weighing heavy articles such as coal, grain, etc.

TABLE OF AVOIRDUPOIS WEIGHT

The Primary Unit	= 1 ounce	abbreviated	1 oz.
16 ounces	= 1 pound	"	1 lb.
100 pounds	= 1 hundredweight	"	1 cwt.
20 hundredweight	= 1 ton	"	1 T.

BLACKBOARD PRACTICE

Change to the next lower denominations:

- | | | |
|-----------|------------------|------------------------|
| 1. 15 lb. | 4. 10 lb. 10 oz. | 7. 2 T. 30 cwt. 50 lb. |
| 2. 24 lb. | 5. 5 cwt. 70 lb. | 8. 3 T. 50 cwt. 30 lb. |
| 3. 35 lb. | 6. 2 cwt. 40 lb. | 9. 5 T. 40 cwt. 65 lb. |

Change to the next higher denominations:

- | | | | |
|--------------|-------------|--------------|----------------|
| 10. 200 oz. | 13. 295 lb. | 16. 3540 lb. | 19. 54,240 lb. |
| 11. 500 oz. | 14. 367 lb. | 17. 8754 lb. | 20. 67,185 lb. |
| 12. 1800 oz. | 15. 674 lb. | 18. 7595 lb. | 21. 87,650 lb. |

Change to ounces:

Change to pounds:

- | | | | |
|------------|-------------|-------------|--------------------|
| 22. 15 lb. | 25. 65 lb. | 28. 154 oz. | 31. 12 cwt. 54 lb. |
| 23. 32 lb. | 26. 81 lb. | 29. 336 oz. | 32. 35 cwt. 70 lb. |
| 24. 54 lb. | 27. 100 lb. | 30. 589 oz. | 33. 1 T. 3 cwt. |
34. Change .275 T. to pounds.
35. Change 3.375 T. to tons, hundredweight, and pounds.
36. Change .625 T. to lower denominations.
37. Change 64,875 pounds to tons, and a decimal of a ton.

Standards of Grain Measure

The Bushel. Many states have established the legal weight of a bushel of the various agricultural products, but it is impracticable to give here a complete table of the different standards. Familiarize yourself with the standards of your own state. (See p. 286.)

For practice in the following exercises use the weights in the table below.

STANDARD OF WEIGHT OF GRAINS, ETC., IN POUNDS

Barley	48	Oats	32
Beans	60	Potatoes	60
Buckwheat	48	Rye	56
Corn (shelled)	56	Wheat	60

BLACKBOARD PRACTICE

What is the weight of :

- 50 bushels of potatoes?
- 100 bushels of corn?
- 100 bushels of potatoes?
- 120 bushels of oats?
- 275 bushels of potatoes?
- 200 bushels of wheat?
- 340 bushels of corn?
- 500 bushels of oats?
- 450 bushels of barley?
- 460 bushels of buckwheat?
- 300 bushels of potatoes and 50 bushels of oats?
- 45 bushels of wheat and 50 bushels of corn?
- 60 bushels of potatoes and 85 bushels of barley?
- 1000 bushels of corn, 1500 bushels of potatoes, and 2000 bushels of oats?

15. How many tons of potatoes are there in 1000 bushels?

16. How many tons of wheat are there in 1200 bushels?

17. How many tons of oats are there in 3000 bushels?

How many bushels are there in a ton of

18. Potatoes? 20. Oats? 22. Barley?

19. Corn? 21. Wheat? 23. Buckwheat?

24. A farmer's wagon-box will hold just sixty bushels of potatoes. How many tons of potatoes will this wagon-box hold?

25. From a field of thirty acres a farmer produces an average of 130 bushels of potatoes to the acre. How much will he get for them at \$3 a hundredweight?

26. A grain dealer ships 35 tons of oats, and 1200 bushels of wheat, and pays a freight charge of \$.0775 per hundredweight. What is the amount of the freight bill?

Miscellaneous Standards in Everyday Use

Certain standard units of weight occur occasionally in business and it is well to memorize the following:

1 bbl. flour = 196 pounds. 1 bbl. salt = 280 pounds.

1 keg nails = 100 pounds. 1 bbl. beef = 200 pounds.

1 bbl. lime = 240 pounds. 1 bale cotton = 500 pounds.

1 cubic foot of water = $62\frac{1}{2}$ pounds.

1 long ton = 2240 pounds.

1 ton of new hay in a mow occupies about 512 cubic feet.

BLACKBOARD PRACTICE

What is the weight of:

1. 5 barrels of flour?

3. 12 bales of cotton?

2. 9 kegs of nails?

4. 15 barrels of salt?

- | | |
|------------------------|------------------------|
| 5. 16 barrels of beef? | 7. 35 bales of cotton? |
| 6. 20 barrels of lime? | 8. 45 barrels of salt? |

9. A tank 5 feet long, 3 feet wide, and 2 feet deep will hold how many pounds of water?

10. A tank 8' 6" long, 3' 4" wide, and 2' 6" deep will hold how many pounds of water?

11. What is the weight of a block of stone 6 feet long, 2 feet wide, and 9 inches thick, the stone being 2.6 times as heavy as water?

12. An auto-truck delivers in one load 6 barrels of flour, 3 kegs of nails, 3 barrels of lime, and 2 barrels of salt. What is the total weight of this load?

13. A hay mow that is 27 feet long, 16 feet wide, and 12 feet deep will hold how many tons of hay? What is the hay worth at \$12.50 a ton?

14. A dealer buys a carload of coal containing 100,000 pounds, and pays \$3.50 per ton of 2000 pounds. How much does he pay for the carload of coal?

15. A coal-dealer delivers five loads of coal, the loads weighing respectively 2470 lb., 2700 lb., 2680 lb., 2850 lb., and 2690 lb. How many long tons of coal does he deliver in the five loads?

16. A field of 32 acres yields an average of 120 bushels of potatoes to the acre. If the freight rate is \$.0825 per 100 pounds, find the freight charge for shipping the potatoes; and also find how much the freight is per bushel.

MEASURES OF TIME

TABLE OF TIME MEASURE

The Primary Unit = 1 second		abbreviated	1 sec.
60 seconds	= 1 minute	"	1 min.
60 minutes	= 1 hour	"	1 hr.
24 hours	= 1 day	"	1 da.
7 days	= 1 week	"	1 wk.
365 days	= 1 calendar year	"	1 yr.

You should memorize also the number of days in the

CALENDAR MONTHS

January (Jan.)	31	July	31
February (Feb.)	28	August (Aug.)	31
March (Mar.)	31	September (Sept.)	30
April (Apr.)	30	October (Oct.)	31
May	31	November (Nov.)	30
June	30	December (Dec.)	31

In a Leap Year February has 29 days.

The Leap Year results from the fact that the actual time required by the earth for a complete revolution around the sun is somewhat in excess of the 365 days called a Calendar Year.

BLACKBOARD PRACTICE

Change to the next lower unit :

- | | | | |
|-------------|-----------|-----------|------------|
| 1. 240 min. | 4. 15 hr. | 7. 15 da. | 10. 56 wk. |
| 2. 320 min. | 5. 24 hr. | 8. 40 da. | 11. 9 yr. |
| 3. 100 min. | 6. 36 hr. | 9. 26 wk. | 12. 27 yr. |

Change to the next higher units :

- | | | | |
|--------------|--------------|-------------|--------------|
| 13. 240 sec. | 16. 420 min. | 19. 452 hr. | 22. 136 mo. |
| 14. 420 sec. | 17. 208 hr. | 20. 67 mo. | 23. 760 da. |
| 15. 300 min. | 18. 325 hr. | 21. 128 da. | 24. 1000 da. |

25. Change 12 hr. 10 min. to minutes.

26. Change 3 wk. 4 da. 5 hr. to hours.

27. Change 5 yr. 8 wk. 18 da. to days.

How many days from

28. April 1 to May 27 ?

29. August 9 to October 5 ?

30. September 17 to December 30 ?

31. March 15 to May 29 ?

32. April 1 to December 31 ?

33. July 9 to November 12 ?

34. January 5 to April 17 ?

35. October 1 to February 1 ?

36. February 1 to October 1 ?

37. May 19 to April 12 ?

38. October 15 to May 27 ?

39. March 11 to January 7 ?

40. November 12 to August 7 ?

41. June 23 to February 10 ?

42. Express 3 hours as a decimal part of a day.

43. Express 8 hours as a decimal part of 1 week.

44. How many minutes are there in one day? How many seconds?

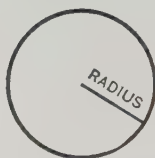
45. July 15, 1917, fell on a Tuesday. On what day of the week did July 15, 1918, fall?

46. Change 15 hr. 10 min. 5 sec. to seconds, and express the result as a fractional part of 1 day.

CIRCULAR MEASURE

A **Circle** is a figure bounded by a curved line, all points of which are equally distant from a point within called the **Center**.

Because of the needs of later mathematics, the circle may be defined as a *curved line*, all points of which are equally distant from a point within called the center.



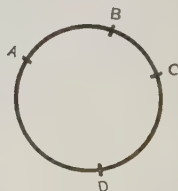
The bounding line of a circle is called the **Circumference**.

The distance from the center of a circle to any point on the circumference is called a **Radius**. (Plural: Radii.)

The distance from any point on the circumference, measured through the center, and ending in the circumference, is called a **Diameter**.

An **Arc** is a portion of a circumference.

In the figure at the right *AB*, *BC*, *CD*, and *DA* are arcs of the given circumference.



For the **Measurement of Arcs** of circles the circumference is divided into 360 equal parts called **Degrees**. For greater accuracy in measurements the degrees are subdivided as in the following:

TABLE OF CIRCULAR MEASURE

The Primary Unit	= 1 second	abbreviated	1''
60 seconds	= 1 minute	"	1'
60 minutes	= 1 degree	"	1°
360 degrees	= 1 circumference		

THE MEASUREMENT OF ANGLES

If a circumference is divided into four equal parts, and diameters are drawn as in the figure at the right, each of the four angles formed at the center of the circle lies opposite the arc of 90° .



The two straight lines meet at right angles, for any one of the four radii meets a diameter and makes equal angles with it.

Angular Measure, therefore, is the measure of an angle by means of a circumference that has the vertex of the angle for its center.

The measure of angles is made by the use of the units of circular measurement.

For convenience the *Right Angle*, or 90° , is made a common unit of measure in angular measurements.

BLACKBOARD PRACTICE

Change to minutes :

- | | | | | |
|-----------------|-----------------|--------------|-----------------------------|----------------------|
| 1. 24° . | 4. 75° . | 7. $540''$. | 10. 10.5° . | 13. $10^\circ 10'$. |
| 2. 35° . | 5. 86° . | 8. $780''$. | 11. 24.25° . | 14. $24^\circ 15'$. |
| 3. 56° . | 6. 94° . | 9. $840''$. | 12. $75\frac{1}{3}^\circ$. | 15. $72^\circ 36'$. |

Change to the lowest given unit :

- | | | |
|----------------------|---------------------------|---------------------------|
| 16. $10' 15''$. | 20. $35^\circ 35'$. | 24. $67^\circ 58' 10''$. |
| 17. $24' 20''$. | 21. $40^\circ 39'$. | 25. $35^\circ 20' 15''$. |
| 18. $54' 11''$. | 22. $10^\circ 15' 20''$. | 26. $45^\circ 45' 45''$. |
| 19. $15^\circ 20'$. | 23. $45^\circ 20' 50''$. | 27. $59^\circ 59' 59''$. |
28. How many degrees are there in $\frac{3}{4}$ of a circumference ?
29. How many degrees are there in $\frac{5}{8}$ of a circumference ?
30. How many degrees are there in $\frac{5}{4}$ of a circumference ?

PERCENTAGE

Per Cent means "by the hundred."

The expression is an abbreviation of the Latin *per centum*.

If a merchant made a profit of \$10 on every sale of \$100 worth of goods, he made "10 on the 100," or "10 per cent."

The sign for per cent is %.

A per cent of a number may be written in three ways.

Thus: Seven per cent of five hundred may be expressed:

(1) As a per cent: 7% of 500.

(2) As a fraction: $\frac{7}{100}$ of 500.

(3) As a decimal: .07 of 500.

When a Per Cent is Written as a Fraction:

The Numerator is the number of hundredths wanted.

The Denominator is 100.

Thus: $3\% = \frac{3}{100}$.

$25\% = \frac{25}{100}$.

$75\% = \frac{75}{100}$.

$8\% = \frac{8}{100}$.

$37\% = \frac{37}{100}$.

$93\% = \frac{93}{100}$.

ORAL PRACTICE

Give the equivalent fraction for:

1. 15%.

5. 40%.

9. 19%.

13. 72%.

2. 20%.

6. 31%.

10. 51%.

14. 79%.

3. 25%.

7. 55%.

11. 63%.

15. 84%.

4. 35%.

8. 62%.

12. 38%.

16. 98%.

When a Per Cent is Written as a Decimal:

The whole number indicating per cent is the number of hundredths in the decimal.

Thus: 15% = .15.

45% = .45.

75% = .75.

25% = .25.

62% = .62.

90% = .90.

A whole number with a fraction indicating per cent gives a decimal in the thousandths, or one which may extend indefinitely.

$$12\frac{1}{2}\% = .125.$$

$$18\frac{1}{4}\% = .1825.$$

$$33\frac{1}{3}\% = .3333^+.$$

ORAL PRACTICE

Give the equivalent decimal for :

1. 12% .

4. 30% .

7. 32% .

10. 75% .

2. 15% .

5. 45% .

8. 57% .

11. 87% .

3. 25% .

6. 50% .

9. 65% .

12. 96% .

Give, to three or four places, the equivalent decimal for :

13. $12\frac{1}{2}\%$.

16. $18\frac{3}{4}\%$.

19. $37\frac{1}{2}\%$.

22. $83\frac{1}{3}\%$.

14. $16\frac{1}{4}\%$.

17. $25\frac{3}{4}\%$.

20. $62\frac{1}{2}\%$.

23. $87\frac{1}{2}\%$.

15. $20\frac{1}{2}\%$.

18. $33\frac{1}{4}\%$.

21. $66\frac{2}{3}\%$.

24. $92\frac{3}{4}\%$.

When Fractions are Written as Per Cents :

If the given denominator is 100, the numerator is the number in the given per cent.

Illustrations :

$$\frac{7}{100} = 7\%.$$

$$\frac{15}{100} = 15\%.$$

$$\frac{67}{100} = 67\%.$$

If the denominator is not 100, change the fraction to a decimal. Move the point two places to the right, and the whole number is the per cent.

Illustrations :

$$\frac{15}{16} = .9375 = 93.75\% \text{ or } 93\frac{3}{4}\%. \quad \frac{2}{3} = .6666^+ = 66.66^+ \% \text{ or } 66\frac{2}{3}\%.$$

A decimal remainder is usually written as a fraction.

BLACKBOARD PRACTICE

Change to an equivalent per cent :

1. $\frac{12}{100}$.

4. $\frac{35}{100}$.

7. $\frac{3}{4}$.

10. $\frac{7}{8}$.

13. $\frac{6}{7}$.

2. $\frac{31}{100}$.

5. $\frac{87}{100}$.

8. $\frac{5}{8}$.

11. $\frac{9}{10}$.

14. $\frac{7}{9}$.

3. $\frac{48}{100}$.

6. $\frac{99}{100}$.

9. $\frac{5}{12}$.

12. $\frac{13}{15}$.

15. $\frac{19}{20}$.

When Decimals are Written as Per Cents :

The point in the given decimal is moved two places to the right, and the remainder is written as a fraction. The sign % is annexed.

Illustrations :

$$.15 = 15\%$$

$$.375 = 37\frac{1}{2}\%$$

$$.2475 = 24.75\% \text{ or } 24\frac{3}{4}\%$$

ORAL PRACTICE

Give the equivalent per cent for :

$$1. .25. \quad 4. .375. \quad 7. .875. \quad 10. .33\frac{1}{3}.$$

$$2. .45. \quad 5. .625. \quad 8. .0875. \quad 11. .83\frac{1}{3}.$$

$$3. .125. \quad 6. .05. \quad 9. .0025. \quad 12. .66\frac{2}{3}.$$

The following table should be memorized :

TABLE OF EQUIVALENTS

$\frac{1}{4} = 25\%$	$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{1}{5} = 20\%$
$\frac{1}{2} = 50\%$	$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{2}{5} = 40\%$
$\frac{3}{4} = 75\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{3}{5} = 60\%$
$1 = 100\%$	$\frac{7}{8} = 87\frac{1}{2}\%$	$\frac{4}{5} = 80\%$
$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{1}{6} = 16\frac{2}{3}\%$	
$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{5}{6} = 83\frac{1}{3}\%$	

Using the fractional equivalents, give orally :

$$25\% \text{ of } 300.$$

$$40\% \text{ of } 500.$$

$$37\frac{1}{2}\% \text{ of } 800.$$

$$50\% \text{ of } 450.$$

$$60\% \text{ of } 500.$$

$$16\frac{2}{3}\% \text{ of } 1200.$$

$$75\% \text{ of } 600.$$

$$80\% \text{ of } 600.$$

$$33\frac{1}{3}\% \text{ of } 1200.$$

$$20\% \text{ of } 600.$$

$$12\frac{1}{2}\% \text{ of } 800.$$

$$66\frac{2}{3}\% \text{ of } 1500.$$

To Find a Per Cent of a Given Number

Illustrations :

1. Find 5% of 720.

The Decimal Method

$$\begin{array}{r} 5\% = .05 \quad 720 \\ \quad \quad \quad .05 \\ \hline \quad \quad 36.00 \end{array}$$

36. *Result.*

The Fractional Method

$$\begin{aligned} 5\% &= \frac{5}{100} = \frac{1}{20} \\ \frac{1}{20} \text{ of } 720 &= 720 \div 20 \\ &= 36. \quad \text{Result.} \end{aligned}$$

2. Find $12\frac{1}{2}\%$ of 340.

The Decimal Method

$$\begin{aligned} 12\frac{1}{2}\% &= .125. \\ .125 \times 340 &= 42.5. \quad \text{Result.} \end{aligned}$$

The Fractional Method

$$\begin{aligned} 12\frac{1}{2}\% &= \frac{1}{8}. \\ \frac{1}{8} \times 340 &= 340 \div 8 = 42.5. \quad \text{Result.} \end{aligned}$$

BLACKBOARD PRACTICE

Using the Decimal Method, find :

- | | | |
|----------------|----------------|-----------------|
| 1. 8% of 150. | 5. 13% of 450. | 9. 45% of 890. |
| 2. 12% of 150. | 6. 18% of 521. | 10. 60% of 900. |
| 3. 25% of 225. | 7. 21% of 700. | 11. 51% of 785. |
| 4. 30% of 340. | 8. 45% of 516. | 12. 76% of 105. |

Using the Fractional Method, and the Table of Equivalents, find :

- | | | |
|-----------------|-------------------------------|-------------------------------|
| 13. 25% of 160. | 17. $12\frac{1}{2}\%$ of 400. | 21. 60% of 480. |
| 14. 25% of 280. | 18. 16% of 420. | 22. $62\frac{1}{2}\%$ of 728. |
| 15. 50% of 360. | 19. $37\frac{1}{2}\%$ of 560. | 23. $83\frac{1}{3}\%$ of 846. |
| 16. 40% of 450. | 20. $33\frac{1}{3}\%$ of 750. | 24. $87\frac{1}{2}\%$ of 960. |

Find, by the shortest method :

- | | | |
|------------------|------------------|--------------------------------|
| 25. 15% of 1250. | 29. 17% of 1300. | 33. 16% of 4500. |
| 26. 20% of 3450. | 30. 24% of 2400. | 34. $23\frac{1}{2}\%$ of 3100. |
| 27. 25% of 3600. | 31. 37% of 3251. | 35. $62\frac{1}{2}\%$ of 4840. |
| 28. 40% of 3750. | 32. 45% of 4000. | 36. $87\frac{1}{2}\%$ of 6000. |

37. What is the decimal equivalent for $\frac{1}{2}\%$? $\frac{1}{2}\%$ of 150?
 38. What is the decimal equivalent for $\frac{1}{4}\%$? $\frac{1}{4}\%$ of 250?
 39. Find $\frac{1}{2}\%$ of 165. $\frac{1}{2}\%$ of 315. $\frac{1}{4}\%$ of 1200.
 40. What is the meaning of 125%? 150%? 200%?

The number of which a per cent is required is called the **Base**. (In the illustration 240 is the base.)

The number of hundredths in the required per cent is the **Rate**. (In the illustration .05 is the rate.)

240	<i>Base.</i>
.05	<i>Rate.</i>
12.00	<i>Percentage.</i>

The product of the base by the rate is called the **Percentage**. (In the illustration 12 is the percentage.) The process may be stated briefly in the form:

$$\text{Rate} \times \text{Base} = \text{Percentage.}$$

In finding a per cent of a given number of dollars care must be taken in placing the decimal point in the result. Study the following illustrations.

\$75	\$125	\$5.75	\$12.50	\$135.75
.08	.15	.25	.125	.35
<u>\$6.00</u>	<u>625</u>	<u>2875</u>	<u>6250</u>	<u>67875</u>
	125	1150	2500	40725
	<u>\$18.75</u>	<u>\$1.4375</u>	<u>1250</u>	<u>\$47.5125</u>
			\$1.56250	

WRITTEN APPLICATIONS

1. A young man earns \$1200 a year, and spends 75% of it for living expenses. How much does he pay for living expenses?

2. A young man earns \$1200 a year, and saves 20% of it. How much does he save? How much does he spend?

3. A family has been paying \$25 a month rent, but the owner has increased the rent 30%. How many dollars per month has the rent increased?

4. A man purchased an automobile for \$1500, and in a year the running expenses amounted to 16% of the purchase price. How much did he spend for running expenses?

5. A young man bought a house for \$4500, and borrowed $62\frac{1}{2}\%$ of the purchase price from a trust company. How much money did he borrow?

6. A young man inherited \$5000, and spent $12\frac{1}{2}\%$ of it for a special business course in college. How much did the college course cost him?

7. A gentleman bought an automobile for \$1650 and after using it one year sold it at a loss of $33\frac{1}{3}\%$ of its original cost. How much did he lose on the car?

8. Taxes at the rate of $2\frac{1}{2}\%$ were levied on property assessed at \$3500. How much was the tax for one year? What was the amount of taxes paid in five years?

9. A merchant has \$7500 invested in his business, and his annual income during 5 years averages $37\frac{1}{2}\%$ of his investment. Find his average annual income.

10. A young man insured his furniture for \$2000, and the annual cost for the insurance was $\frac{4}{5}\%$. How much did his insurance cost for three years?

11. A farmer planted untested seed and raised 45 bushels of corn to the acre. The next year he planted seed which had been tested, and his yield increased $33\frac{1}{3}\%$. How much corn did he raise per acre the second year?

12. An agent for an apartment house received a commission of 2% for collecting the rent. In 1917 he collected \$5400, and in 1918, \$4500. What was the total amount he received in commissions in these two years?

COMMISSION

A **Commission Merchant** is an agent who buys and sells merchandise, or transacts *other* business, for another person.

The **Principal** in the commission business is the person for whom the business is transacted.

Commission is the charge made by the agent for transacting the business. Commission is usually paid at a certain rate per cent of the amount in the transaction.

Illustration :

A fruit-grower sends 2000 baskets of peaches to a commission agent, who sells them for \$0.50 per basket and charges a commission of 2%.

The principal is the fruit-grower.

The amount received for the consignment is $2000 \times \$0.50 = \1000.00 .

The commission paid to the agent is $2\% \text{ of } \$1000 = \20.00 .

Commission is sometimes charged at a stated price per unit of quantity, as per bale, barrel, bushel, ton, etc.

The **Gross Proceeds** of a sale is the entire amount received from the sale of the goods.

The **Net Proceeds** of a sale is the amount the principal receives after the agent's commission and any other expenses of the sale are deducted from the gross proceeds.

In the illustration :

The gross proceeds are $2000 \times \$0.50 = \1000.00 .

The net proceeds are $\$1000 - \$20 = \$980.00$.

The same general principles govern the ordinary transactions in real estate, collections, etc. The real estate agent is sometimes called a **Broker**, and the agent who makes a business of collecting bills, accounts, rents, etc., is called a **Collector**.

Nearly all problems under commission follow the principles of percentage, and

The Amount of the Transaction = The Base.

The Rate of Commission charged = The Rate.

The Agent's Commission = The Percentage.

BLACKBOARD PRACTICE

Find the commission on :

- | | |
|---------------------------------|----------------------------------|
| 1. \$300 at 2%. | 10. \$5000 at $3\frac{1}{4}\%$. |
| 2. \$350 at 2%. | 11. \$6500 at $4\frac{1}{4}\%$. |
| 3. \$400 at 3%. | 12. \$7500 at $3\frac{3}{4}\%$. |
| 4. \$525 at $2\frac{1}{2}\%$. | 13. \$4000 at $\frac{1}{2}\%$. |
| 5. \$630 at $2\frac{1}{2}\%$. | 14. \$5000 at $\frac{3}{4}\%$. |
| 6. \$750 at $3\frac{1}{2}\%$. | 15. \$6000 at $\frac{1}{4}\%$. |
| 7. \$1200 at $1\frac{1}{2}\%$. | 16. \$7500 at $1\frac{1}{2}\%$. |
| 8. \$1500 at 2%. | 17. \$8000 at $1\frac{1}{4}\%$. |
| 9. \$2000 at $2\frac{1}{2}\%$. | 18. \$9500 at $1\frac{3}{4}\%$. |

Find the net proceeds on the following sales :

- | | |
|----------------------------------|--------------------------------------|
| 19. \$300 at $1\frac{1}{4}\%$. | 30. \$4250 at $3\frac{1}{4}\%$. |
| 20. \$375 at $1\frac{1}{2}\%$. | 31. \$5600 at $3\frac{1}{2}\%$. |
| 21. \$450 at $1\frac{3}{4}\%$. | 32. \$7800 at $4\frac{1}{4}\%$. |
| 22. \$600 at $2\frac{1}{4}\%$. | 33. \$275.50 at $\frac{1}{4}\%$. |
| 23. \$750 at $2\frac{1}{2}\%$. | 34. \$580.25 at $\frac{1}{2}\%$. |
| 24. \$860 at $2\frac{3}{4}\%$. | 35. \$787.50 at $\frac{3}{4}\%$. |
| 25. \$975 at $3\frac{1}{4}\%$. | 36. \$1127.50 at $11\frac{1}{3}\%$. |
| 26. \$3000 at $2\frac{1}{4}\%$. | 37. \$1567.10 at $4\frac{3}{4}\%$. |
| 27. \$2500 at $2\frac{1}{2}\%$. | 38. \$2356.60 at $6\frac{1}{2}\%$. |
| 28. \$3200 at $2\frac{1}{2}\%$. | 39. \$3500.00 at $7\frac{1}{2}\%$. |
| 29. \$3750 at $2\frac{3}{4}\%$. | 40. \$4250 at $5\frac{3}{4}\%$. |

WRITTEN APPLICATIONS

1. An agent charges a commission of 2% on the \$2500 received from the sale of a house. What amount does he receive for selling it?

2. A commission merchant sold 50 barrels of Baldwin apples at \$2.50 a barrel. If his commission was 4%, how much should he receive?

3. A farmer sold 100 acres of wood land at \$145 an acre, and paid an agent 20% commission for making the sale. How much did he pay the agent?

4. A lawyer charged 6% for collecting a note of \$150. How much should he pay to the owner of the note?

5. 1000 bushels of wheat were sold at \$2.10 a bushel, and the commission agent who sold it, received $\frac{1}{2}\%$ per bushel. How much did he receive?

6. A dealer paid \$9.50 a barrel for 75 barrels of flour, and the commission agent who sold the flour received $1\frac{1}{2}\%$ of the purchase price. Find the amount of the agent's commission.

7. 450 bushels of potatoes were marketed at \$1.10 per bushel. The producer paid the selling agent 5% of the gross proceeds. What was the net amount received by the producer?

8. A real estate dealer sold a row of small city houses for \$62,500, and his commission on the sale was $2\frac{1}{2}\%$. Find the amount of his commission, and also the net proceeds he should pay to the owner.

9. 30,000 feet of lumber were sold on board the cars for \$42.50 per thousand, and the freight charges to the buyer's yard were \$45. If the owner paid the commission agent $2\frac{1}{2}\%$, and the *buyer* paid the freight, find the actual cost of the lumber to the buyer, and the actual amount received by the owner.

INTEREST

Interest is money paid for the use of money.

Illustration :

Suppose your father borrows \$100, and repays the loan at the end of 1 year.

If he pays \$6 for the use of the money, the \$6 is called *interest*.

The Principal is the money on which interest is paid.

In the illustration the principal is \$100.

The Time is the period for which the money is loaned.

In the illustration the time is 1 year.

The Rate of interest is the number of hundredths of the principal paid as interest for one year.

In the illustration \$6 is paid for the use of \$100.

What decimal part of \$100 is \$6?

$$\frac{\$6}{\$100} = \frac{6}{100} = .06.$$

But, expressed as a per cent, $.06 = 6\%$.

That is, the rate in the illustration is six per cent.

Simple Interest is interest paid only on the principal.

In this book the word "interest" means simple interest.

Problems in simple interest are applications of Percentage in which

The Principal = The Base.

The Rate = The Rate Per Cent.

The Interest = The Percentage.

In addition to these three familiar terms we find a new term, "*time*," in all of the problems in interest.

If the time during which a sum of money bears interest is one year or a number of full years, the multiplier is a whole number.

Thus: The interest for 3 years at 6% on \$100 is the product of

$$3 \times .06 \times \$100.$$

If the time is a number of years and months, the multiplier is a mixed number representing the whole number of years and the fractional part of a year in the time.

Thus: The interest for 3 years and 8 months at 6% on \$100 is the product of

$$3\frac{2}{3} \times .06 \times \$100.$$

METHODS OF FINDING INTEREST

I. When the Time is Given in Years

Illustration:

Find the interest on \$500 for 3 years at 6%.

$$.06 \times \$500 = \$30, \text{ the interest for 1 year at 6\%.}$$

$$3 \times \$30 = \$90, \text{ the interest for 3 years at 6\%.} \quad \textit{Result.}$$

BLACKBOARD PRACTICE

With the principal, the time, and the rate as follows, find the interest on:

- | | |
|---------------------------|--|
| 1. \$200 for 4 yr. at 6%. | 8. \$600 for 5 yr. at 4%. |
| 2. \$300 for 5 yr. at 5%. | 9. \$700 for 6 yr. at 6%. |
| 3. \$350 for 5 yr. at 6%. | 10. \$800 for 4 yr. at $4\frac{1}{2}\%$. |
| 4. \$400 for 4 yr. at 6%. | 11. \$1000 for 6 yr. at $5\frac{1}{4}\%$. |
| 5. \$500 for 6 yr. at 5%. | 12. \$2000 for 10 yr. at 5.4% . |
| 6. \$600 for 5 yr. at 6%. | 13. \$2100 for 8 yr. at $4\frac{1}{2}\%$. |
| 7. \$700 for 6 yr. at 5%. | 14. \$2500 for 4 yr. at $6\frac{1}{2}\%$. |

II. When the Time is Given in Years and Months

Illustration :

Find the interest on \$150 for 4 yr. 6 mo. at 5%.

$.05 \times \$150 = \7.50 , the interest for 1 year at 5%.

4 yr. 6 mo. = $4\frac{6}{12}$ yr. = $4\frac{1}{2}$ yr. or 4.5 yr.

$4.5 \times \$7.50 = \33.75 , the interest for 4 yr. 6 mo. at 5%.

In many cases the fractional part of a year will give a result in which a fraction of a cent appears. In such cases a fraction of one half cent or more is called one cent.

Illustration :

Find the interest on \$255 for 3 yr. 7 mo. at 6%.

$.06 \times \$255 = \15.30 , the interest for 1 year at 6%.

3 yr. 7 mo. = $3\frac{7}{12}$ yr.

$3\frac{7}{12} \times \$15.30 = \54.825 , the interest for 3 yr. 7 mo. at 6%

In this case the result would be called \$54.83.

BLACKBOARD PRACTICE

With the principal, the time, and the rate as follows, find the interest on :

1. \$100 for 2 yr. 6 mo. at 6%.
2. \$100 for 3 yr. 6 mo. at 5%.
3. \$150 for 2 yr. 6 mo. at 6%.
4. \$175 for 3 yr. 6 mo. at 5%.
5. \$200 for 4 yr. 3 mo. at 5%.
6. \$350 for 3 yr. 4 mo. at $5\frac{1}{2}$ %.
7. \$400 for 2 yr. 9 mo. at 6%.
8. \$750 for 3 yr. 8 mo. at $5\frac{1}{2}$ %.
9. \$150 for 2 yr. 1 mo. at 6%.
10. \$200 for 2 yr. 2 mo. at 5%.
11. \$240 for 3 yr. 5 mo. at 6%.
12. \$300 for 4 yr. 5 mo. at 5.4%.

13. \$360 for 3 yr. 7 mo. at 6%.
14. \$540 for 3 yr. 11 mo. at 5.4%.
15. \$680 for 5 yr. 10 mo. at 6%.
16. \$875 for 4 yr. 11 mo. at 5½%.

III. When the Time is Given in Years, Months, and Days

Illustration :

Find the interest on \$200 for 2 yr. 3 mo. 10 da. at 6%.

$$.06 \times \$200 = \$12.00, \text{ the interest on } \$200 \text{ for 1 year at } 6\%.$$

2 years, 3 months, 10 days = 2 years, $3\frac{1}{3}$ months.

$$\text{But } 3\frac{1}{3} \text{ mo.} = \frac{3\frac{1}{3}}{12} = \frac{\frac{10}{3}}{12} = \frac{10}{3} \times \frac{1}{12} = \frac{10}{36} = \frac{5}{18} \text{ of 1 year.}$$

Therefore, 2 yr., 3 mo. 10 da. = $2\frac{5}{18}$ yr.

$$\text{Then } 2\frac{5}{18} \times \$12.00 = \$27.33\frac{1}{3}, \text{ the interest on } \$200 \text{ for } 2 \text{ yr. } 3 \text{ mo. } 10 \text{ da.}$$

Or, \$27.33. Result.

BLACKBOARD PRACTICE

With the principal, the rate, and the time as follows, find the interest on :

1. \$200 for 1 yr. 2 mo. 12 da. at 6%.
2. \$200 for 2 yr. 3 mo. 20 da. at 5%.
3. \$225.50 for 1 yr. 2 mo. 12 da. at 6%.
4. \$240.75 for 2 yr. 3 mo. 15 da. at 4½%.
5. \$300.54 for 3 yr. 4 mo. 21 da. at 5%.
6. \$325.90 for 2 yr. 3 mo. 24 da. at 5½%.
7. \$410.52 for 3 yr. 5 mo. 13 da. at 6%.
8. \$450.67 for 1 yr. 6 mo. 11 da. at 4.5%.
9. \$575.42 for 3 yr. 5 mo. 17 da. at 6%.
10. \$890.47 for 2 yr. 8 mo. 28 da. at 5.4%.

SAVING AND INVESTING MONEY

The following tables show the rapidity with which small savings invested at regular intervals accumulate interest.

MONTHLY DEPOSIT OF ONE TO TEN DOLLARS

DEP.	1 YR.	2 YR.	3 YR.	4 YR.	5 YR.
\$ 1	\$ 12.19	\$ 24.74	\$ 37.66	\$ 50.97	\$ 64.67
2	34.38	49.48	75.32	101.94	129.34
3	36.57	74.22	112.98	152.91	194.01
4	43.76	98.96	150.64	203.88	258.68
5	60.95	123.70	188.30	254.85	323.72
6	73.14	148.44	225.96	305.82	386.02
7	85.33	173.18	263.62	356.79	452.69
8	97.52	197.92	301.28	407.76	517.35
9	109.71	222.66	338.94	458.73	582.03
10	121.90	247.40	376.60	509.70	647.53

WEEKLY DEPOSIT OF \$1 TO \$5

DEP.	1-YR.	2 YR.	3 YR.
\$1	\$ 52.40	\$106.84	\$280.24
2	104.81	213.69	325.86
3	157.22	320.54	488.79
4	209.62	427.39	651.73
5	262.03	534.24	814.66

How to Read the Table. Suppose you deposit \$5 each month for 3 years. Opposite \$5 and under 3 years you find \$188.30, which is the total of your deposits and interest for three years.

ORAL PRACTICE

1. How much will your savings and interest amount to if you deposit two dollars each week during the years 1920 and 1921?

2. Compare the result you found in problem 1 with the amount of your savings and interest if you should save two dollars each month during the same years.

3. Find how much a weekly saving of two dollars will amount to in 3 years, and then find the monthly saving and the corresponding period of time that will accumulate approximately the same sum.

THE POSTAL SAVINGS PLAN

The Postal Savings Plan was established by the United States Government "for the purpose of providing facilities for depositing savings at interest, with the security of the Government for repayment." To become a depositor:

1. Buy a Postal Savings Card.

This card, with one Postal Savings Stamp attached, costs 10 cents.

Fill the card by purchasing nine additional stamps at any time.

These ten stamps represent the saving of One Dollar.

2. Exchange the Postal Savings Card, with its ten stamps, for a Postal Savings Certificate.

The Government pays 2% interest on this deposit, or \$.02.

Other details will be supplied by the Post Office selling the stamps.

It should be remembered that:

1. Interest is paid only on deposits left for one year.
2. Any part, or the whole, of your deposits may be withdrawn at any time.
3. Postal Savings Certificates may be bought in denominations of \$1, \$2, \$5, \$10, \$20, and \$50, without the slower process of saving through the purchase of Postal Savings Stamps.

4. No person may deposit further sums after the total of his deposits has reached \$1000.
5. Interest on Postal Savings Certificates begins on the first day of the month following the month in which the certificate is issued. Interest is due and payable at the end of each full year.

WRITTEN APPLICATIONS

Find the interest on postal savings deposits when the deposit and the time are :

- | | | |
|-------------------|--------------------|--------------------|
| 1. \$20 for 3 yr. | 4. \$50 for 5 yr. | 7. \$125 for 4 yr. |
| 2. \$45 for 4 yr. | 5. \$96 for 4 yr. | 8. \$240 for 6 yr. |
| 3. \$60 for 5 yr. | 6. \$100 for 6 yr. | 9. \$500 for 5 yr. |

10. Postal Savings Certificates were purchased as follows : \$10 on Mar. 1, \$20 on May 10, and \$50 on Aug. 25. Find the interest due on August 25 of the following year.

SCHOOL SAVINGS BANKS

The school savings bank originated in France in 1834, but very few were established until 1874, when several were organized by a Belgian named Francis Laurent. The first school savings bank in the United States appears to have been started at Beloit, Wisconsin, in 1876; but the first permanent and effective efforts were made in Long Island City, New York, in 1885 by Mr. John H. Thiry.

STATISTICS OF SCHOOL SAVINGS BANKS IN THE UNITED STATES (1915)

Number of cities and towns reporting	280
Number of schools reported	1925
Total enrollment in these schools	928,784
Number of depositors in these schools	398,540
Amount on deposit to the credit of these pupils . . .	\$1,792,640.10

WRITTEN APPLICATIONS

1. Find the average amount on deposit by each of the 398,540 depositors in the school savings banks of the United States.

2. In 1915 the number of school children enrolled in the public schools of the United States was 19,990,316. What amount would these children have on deposit in savings banks, if their average deposit was equal to that of the children who had savings accounts that year?

3. In 1915 Pennsylvania led in the number of school depositors with a total of 108,832, and these children had \$304,963.34 on deposit. In the same year California led in the total amount on deposit with \$316,302.36, and had 24,238 depositors. Find the average amount to the credit of a depositor in each of these states, and compare the results with the average deposit in the United States as determined in example 1.

4. Grouped by sections the number of towns and cities with organized school savings banks, and the amount on deposit in 1915 were :

New England States	75 cities	\$202,962.20
Eastern States	74 cities	485,426.65
Southern States	15 cities	33,328.59
Middle Western States	80 cities	647,698.49
Western States	16 cities	29,847.47
Pacific States	20 cities	393,376.70

Find the total number of cities having school savings banks; the total amount on deposit in those cities; and the average amount per city on deposit in each section.

THE LIBERTY LOAN BONDS

A Bond is a formal written promise to pay a specified sum at a given time.

The Liberty Loan Bonds are United States Government Bonds, and include four issues.

The First Liberty Loan bears interest at $3\frac{1}{2}\%$ and is due June 15, 1947.

The Second Liberty Loan bears interest at 4% and is due November 15, 1942.

The Third Liberty Loan bears interest at $4\frac{1}{4}\%$ and is due September 15, 1928.

The Fourth Liberty Loan bears interest at $4\frac{1}{4}\%$ and is due October 15, 1938.

The Victory Loan bears interest at $4\frac{3}{4}\%$ and is due May 20, 1923.

Since the third and fourth loans bear a rate of interest higher than that of the second, the government offered to exchange 4% bonds for $4\frac{1}{4}\%$ bonds, provided the change was made before November 9, 1918.

Investment Value of the Liberty Loan Bonds

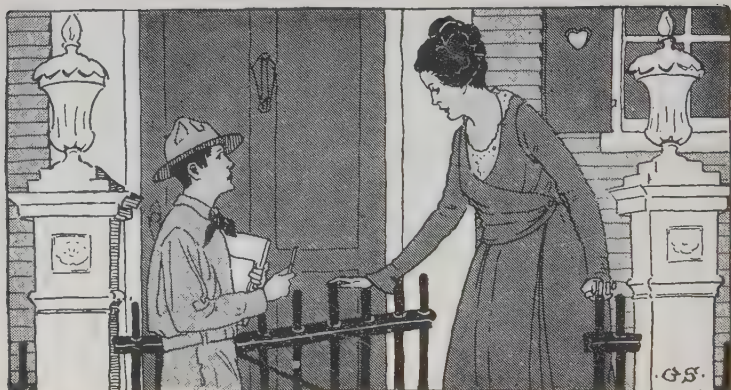
Illustration :

Suppose your father owns \$2500 worth of the Third Liberty Loan bonds. On this investment, the government pays him annually $4\frac{1}{4}\%$. He receives annually, therefore,

$$4\frac{1}{4}\% \text{ of } \$2500 = .0425 \times \$2500 = \$106.25.$$

Each year while he holds these bonds the government will pay him \$106.25. In 1928 the government will pay him \$2500 in gold.

The interest on the Liberty Loan Bonds is payable semi-annually, or every six months. The bonds are issued in denominations of \$50, \$100, \$500, \$1000, \$5000, \$10,000, \$50,000, and \$100,000.



WRITTEN APPLICATIONS

1. A Boy Scout sold seventy-five Liberty Loan Bonds of the \$50 denomination, and 14 of the \$100 denomination. What was the total amount of these sales?

2. A man received a net profit of \$2250 a year from the rent of a house, but he sold the house for \$60,000 and invested the money in $4\frac{1}{4}\%$ Liberty Bonds. How much was his net income increased by the change?

3. A widow received an annual income of $2\frac{1}{2}\%$ on \$16,000. How much would her annual income be increased if she should invest this amount in Second Liberty Loan Bonds at 4% interest? How much would it be increased if she had invested this amount in Third Liberty Loan Bonds at $4\frac{1}{4}\%$ interest?

4. A man invested \$20,000 in the First Liberty Loan Bonds, receiving his interest at $3\frac{1}{2}\%$, six months after the purchase. If he invested all of this interest payment in \$100 Second Liberty Loan 4% Bonds, how many bonds did he buy?

GENERAL REVIEW

Everyday Uses for Elementary Arithmetic

1. Find the cost of $\frac{7}{8}$ of a mile of fence wire at 6 cents a foot.
2. How many boxes, each $6'' \times 3'' \times 2''$, can be packed in a box $4' \times 2' \times 1'$?
3. Find the freight charge on 8250 lb. of merchandise if the rate is \$.32 per cwt.
4. How many acres of land are there in a field which is $\frac{1}{4}$ mile long and $\frac{3}{16}$ mile wide?
5. How many cubic feet are there in the volume of a box which is 1 yd. long, 2 ft. wide, and 18 in. deep?
6. A floor is 30 ft. long and 25 ft. wide. How much will the area of the floor be increased if each dimension is increased 10%?
7. A bin is 10 ft. long, 5 ft. wide, and 4 ft. deep. How much will its volume be increased if each dimension is increased 10%?
8. Only 80% of the cream is obtained when milk is skimmed by hand. How many pounds of cream are lost at this rate when 280 lb. are obtained?
9. Louis bought 5 bushels of chestnuts for \$2.25 a bushel, and sold them all at 5¢ a half pint. How much profit did he make on them?
10. In a recent year Wisconsin had 4,338,000 sheep, and produced 29,040,000 pounds of wool. What was the average production of wool per sheep?
11. A dealer bought football suits at \$33 per dozen, and sold them for \$4 each. What was the per cent of profit that he made on them?

12. $\frac{2}{3}$ of a share of a mine sold for \$24,000, and the share was worth 60% of the value of the whole mine. Find the value of the whole mine.

13. In a certain city 1245 pupils were promoted from the eighth grade to high school, but 498 of them did not enter high school. What per cent of those promoted did enter high school?

14. A cubic foot of water approximately equals 7.5 gallons. If a tank 15 ft. long and 8 ft. wide contains 10,500 gallons of water, how deep is the water in the tank?

15. Divide a family income of \$2000 so that 20% of it is spent for rent, 40% of it for food and clothing, 18% of it for miscellaneous expenses, and the balance is saved. How much of the income is saved?

16. Two automobiles leave Chicago at the same time, one of them going north at the rate of 20.5 miles an hour, and the other going south at the rate of 24.3 miles an hour. At this rate how far apart are they at the end of 7.5 hours?

17. 5% of the amount of a gas bill is deducted by a certain company if the bill is paid within ten days. At that rate, how much should I pay if the gas bill calls for \$6.40, but I pay the bill on the fourth day after it is presented?

18. In the air sound travels 1087 ft. a second. About how many seconds must pass before the sound of a gun reaches me if the gun is located 4 mi. distant from me?

19. A train running at the rate of 25 miles in 30 minutes meets another train which is running at the rate of 40 miles in 50 minutes. Which of the two trains is traveling at the faster speed?

20. .47 of a farm is planted to grains, .35 of it to grass, and .25 of .4 of it is used for the buildings. If the remainder, or 8 acres, is woodland, how many acres are there in the whole farm?

21. Find the value of $3.6 + 4.8 \div 3 - .25$; $3.6 \div 4.8 + 3 - .25$; and $3.6 + 4.8 \times 3 \div .25$. What important principles have you used in finding the three results?

22. A boy weighed 45 pounds on his 5th birthday. If in each of the next five years he gained 10% in weight over the preceding year, how much did he weigh, to the nearest .1 of 1 pound, on his 10th anniversary?

23. A man working with a cornplanter can do the work of 3 men who work without the aid of a machine and who are paid \$2.25 each per day. If a man with a team is paid \$4.25 a day, how much can this farmer save by working 1 man and a machine for 8 days of planting?

24. A man spent 25% of his income for rent, 24% of it for food, 6% of it for fuel and light, 21% of it for clothing, and 12% of it for other expenses. If his income was \$1600, and he saved the balance, find how much he saved.

25. A ton of tomatoes produced when canned 65 dozen quarts, and these were all sold at 95 cents a dozen. Find the approximate price per pound received for the tomatoes.

26. After a fire a merchant sold his damaged stock at 30% of the original price. At that rate how much did he receive for 25 yd. of gingham which had been sold at 65¢ a yard, and a barrel of flour which had been sold for \$10.50?

27. A lot measuring 24 ft. along a street, and 75 ft. deep sold for \$2 a square foot. Another lot on the same street and of the same dimensions sold for \$150 a "front foot," that is, per foot of length on the street. How much did each lot sell for?

28. A farmer seeds a field with a mixture of timothy, red top, and clover. If he uses 3 parts of timothy, 2 parts of red top, and 1 part of clover, how many pounds of seed of each kind must he use to mix 150 pounds?

29. A man lived for one year in a house for which he paid \$40 per month rent, and his fuel bill amounted to \$75. The following year he lived in an apartment costing \$35 a month, in which he was furnished with heat; but the cost of moving was \$25, and some new furniture cost \$100. Find his actual cash outlay in each year.

30. Henry earned \$10 in each of five consecutive weeks. He spent 10% of his first week's earnings, 20% of the second week's earnings, 30% of the third week's earnings, none of the fourth week's earnings, and all of the fifth week's. If he saved the balance in each case, how much did he save in the five weeks?

31. An owner of oil land often receives $12\frac{1}{2}\%$ of the production of it, and the crude petroleum produced is usually worth \$1 a barrel. At these rates what should be received each day by a landowner on whose property 1250 barrels of crude oil are produced daily?

32. Frank's War garden produced vegetables which he sold for \$42.75. His expenses were: plowing, \$3.50; seed, \$1.05; fertilizer, \$2.10; and spraying material, \$1.10. If he worked 140 hours in the garden, how much did his profits on his sales pay him for each hour of his work?

33. Dick's father gave him an allowance of \$10 a month, but insisted that Dick must save at least 40% of it, and that he should not spend more than $12\frac{1}{2}\%$ of it on amusements. How much must Dick save each month? How much may he spend each month on amusements?

34. Three boys bought a motor-cycle costing \$160. Tom paid $\frac{1}{4}$ of the purchase price, Jerry paid $37\frac{1}{2}\%$ of it, and Dick paid $\frac{9}{40}$ of the cost. Did these three shares together pay for the motor-cycle? What was the amount of cash that each boy contributed?

35. A sheet of tin is made into a rectangular box by cutting into the tin at the corners and turning up portions of it for the sides. The sheet is 18 inches long and 12 inches wide, and 2 inches are turned up on all four sides. How many cubic inches are there in the volume of the box? How many square inches are there in the four squares that are cut from the corners of the sheet?

36. 250 pounds of early potatoes were sold for 6¢ a pound, and 350 pounds of a later crop were sold for 4¢ a pound. The cost of digging and hauling the potatoes to market was 20% of the total amount received for both crops. How much did it cost the producer per bushel to dig and haul the potatoes?

37. 2% of the eggs laid by a farmer's 500 hens were used by his family, 72% of them were sold at an average price of 50¢ a dozen, and the remainder were sold at an average price of 40¢ a dozen. If the average number of eggs laid was 150 per hen for the whole year, how much did the producer receive for the eggs he sold? How many eggs were used by his family?

ADDITIONAL TOPICS FOR STUDY AND REFERENCE

ADDITION OF DENOMINATE NUMBERS

Illustration :

1. Add 10 ft. 9 in. and 7 ft. 6 in.

ft.	in.
10	9

The units of like denomination are written in vertical columns.

Adding the column of lowest denomination : 9 in. + 6 in. = 15 in.

7	6
18	3

15 in. = 1 ft. + 3 in. Write 3 in. in the result, and add 1 ft. to

the next denomination.

1 ft. + 7 ft. + 10 ft. = 18 ft. Write 18 ft. in the result.

Therefore, the complete result is 18 ft. 3 in.

BLACKBOARD PRACTICE

Add :

ft.	in.	ft.	in.	ft.	in.	ft.	in.
1. 16	10	2. 19	7	3. 26	7	4. 35	11
8	6	10	11	14	10	14	1

lb.	oz.	lb.	oz.	bu.	pk.	bu.	pk.
5. 15	11	6. 26	15	7. 8	3	8. 17	1
10	9	11	13	12	2	21	3

qt.	pt.	bu.	pk.	mi.	rd.	hr.	min.
9. 54	1	10. 37	1	11. 45	50	12. 15	25
9		11	3	19	75	17	55

ft.	in.	ft.	in.	T.	lb.	wk.	da.
13. 35	11	14. 54	10	15. 7	157	16. 15	3
18	10	67	11	10	876	19	6

yd.	ft.	in.	hr.	min.	sec.	bu.	pk.	qt.
17. 35	1	9	18. 11	50	19	19. 10	3	1
8	2	10	10	19	55	17	1	
16		8		17	15		2	6

	sq. yd.	sq. ft.	sq. in.		A.	sq. rd.	sq. yd.	sq. ft.
20.	19	8	90	21.	8	110	4	6
	12	7	50		7	90	5	8
	10	1	10		3	121	1	3

22. Add 10 yd. 2 ft. 10. in., 3 yd. 1 ft. 11 in., and 2 ft. 7 in.

23. Add 15 hr. 36 min., 7 hr. 11 min., 5 hr. 25 sec., and 19 min. 35 sec.

24. Add 10 T. 4 cwt. 54 lb., 5 T. 1 cwt. 90 lb., and 11 T. 17 cwt. 66 lb.

25. Add 4 cu. yd. 5 cu. ft., 8 cu. yd. 7 cu. ft., and 25 cu. yd. 13 cu. ft.

26. Add 5 yr. 3 mo. 15 da., 4 yr. 10 mo. 13 da., and 6 yr. 8 mo. 17 da.

27. Add 11 gal. 3 qt. 1 pt. 3 gi., 18 gal. 2 qt. 1 gi., and 1 qt. 1 pt. 1 gi.

28. Add 10 hr. 11 min. 19 sec., 8 hr. 41 min. 54 sec., 16 hr. 7 min. 47 sec.

29. Add 19 cu. yd. 10 cu. ft., 17 cu. yd. 18 cu. ft., 32 cu. yd. 13 cu. ft.

SUBTRACTION OF DENOMINATE NUMBERS

Illustration :

1. From 15 ft. 5 in. subtract 8 ft. 9 in.

Place like units in the same column and begin with the lowest denomination.

We cannot subtract 9 in. from 5 in.

Consider 15 ft. 5 in. as 14 ft. + 12 in. + 5 in., or, 14 ft. 17 in. Then subtract as in ordinary subtraction.

$$\begin{array}{r}
 15 \text{ ft.} \quad 5 \text{ in.} \\
 \underline{8 \text{ ft.} \quad 9 \text{ in.}} \\
 14 \text{ ft.} \quad 17 \text{ in.} \\
 \underline{8 \text{ ft.} \quad 9 \text{ in.}} \\
 6 \text{ ft.} \quad 8 \text{ in.}
 \end{array}$$

Result.

You will soon learn to make this change mentally.

2. How long a time elapsed between March 10, 1914, and October 7, 1918?

Both dates are written in the numbers representing the year, the month, and the day, respectively.

Allowing 30 days to the month, the subtraction is made as in illustration No. 1.

1918	10	7	
1914	3	10	
1918	9	37	
1914	3	10	
4	6	27	<i>Result.</i>

BLACKBOARD PRACTICE

Subtract :

	ft.	in.		ft.	in.		lb.	oz.		T.	cwt.
1.	15	11	2.	34	9	3.	43	9	4.	5	7
	<u>7</u>	<u>9</u>		<u>11</u>	<u>10</u>		<u>18</u>	<u>14</u>		<u>3</u>	<u>10</u>
	lb.	oz.		bu.	pk.		hr.	min.		wk.	da.
5.	76	7	6.	21	1	7.	31	19	8.	15	2
	<u>47</u>	<u>15</u>		<u>11</u>	<u>2</u>		<u>16</u>	<u>49</u>		<u>10</u>	<u>6</u>
	bu.	pk.	qt.		yd.	ft.	in.		hr.	min.	sec.
9.	15	3	5	10.	11	2	9	11.	18	50	31
	<u>9</u>	<u>3</u>	<u>7</u>		<u>8</u>		<u>11</u>		<u>17</u>	<u>54</u>	<u>42</u>
	T.	cwt.	lb.		sq. yd.	sq. ft.		hr.	min.	sec.	
12.	17	14	55	13.	154	7	14.	21	12	15	
	<u>7</u>	<u>19</u>	<u>90</u>		<u>101</u>	<u>8</u>		<u>10</u>	<u>15</u>	<u>20</u>	

How many years, months, and days elapsed between :

15. October 10, 1918, and January 5, 1919?

16. November 15, 1917, and March 10, 1919?

17. November 13, 1916, and May 7, 1918?

18. October 29, 1901, and February 6, 1919?

Find how many years, months, and days have passed since :

19. December 27, 1867.

22. December 5, 1870.

20. March 1, 1885.

23. December 23, 1843.

21. April 7, 1872.

24. January 18, 1910.

TABLES OF DENOMINATE NUMBERS

LENGTH MEASURE

12 inches (in.)	=1 foot (ft.)
3 feet	=1 yard (yd.)
$5\frac{1}{2}$ yards (or $16\frac{1}{2}$ ft.)	=1 rod (rd.)
40 rods	=1 furlong (fur.)
320 rods (or 5280 ft.)	=1 mile (mi.)
1 mi. = 320 rd. = 1760 yd. = 5280 ft.	

LIQUID MEASURE

4 gills (gi.)	=1 pint (pt.)
2 pints	=1 quart (qt.)
4 quarts	=1 gallon (gal.)
$31\frac{1}{2}$ gallons	=1 barrel (bbl.)
63 gallons (or 2 bbl.)	=1 hogshead (hhd.)
1 bbl. = $31\frac{1}{2}$ gal. = 126 qt. = 252 pt.	

SURFACE MEASURE

144 square inches (sq. in.)	=1 square foot (sq. ft.)
9 square feet	=1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	=1 square rod (sq. rd.)
160 square rods	=1 acre (A.)
640 acres	=1 square mile (sq. mi.)
1 A. = 160 sq. rd. = 4840 sq. yd. = 43,560 sq. ft.	

DRY MEASURE

2 pints (pt.)	=1 quart (qt.)
8 quarts	=1 peck (pk.)
4 pecks	=1 bushel (bu.)
1 bu. = 4 pk. = 32 qt. = 64 pt.	

VOLUME MEASURE

1728 cubic inches (cu. in.)	=1 cubic foot (cu. ft.)
27 cubic feet	=1 cubic yard (cu. yd.)
128 cubic feet	=1 cord (cd.)
16 cubic feet	=1 cord foot (cd. ft.)
8 cord feet	=1 cord (cd.)

TIME MEASURE

60 seconds (sec.)	=1 minute (min.)
60 minutes	=1 hour (hr.)
24 hours	=1 day (da.)
7 days	=1 week (wk.)
365 days	=1 common year
366 days	=1 leap year

WEIGHT MEASURE

16 ounces (oz.)	=1 pound (lb.)
100 pounds	=1 hundredweight (cwt.)
2000 pounds (or 20 cwt.)	=1 ton (T.)

CIRCULAR MEASURE

60 seconds (")	=1 minute (')
60 minutes	=1 degree (°)
360 degrees	=1 circumference

LEGAL WEIGHTS OF VARIOUS PRODUCTS IN POUNDS TO THE BUSHEL

STATES	WHEAT	RYE	BUCKWHEAT	SHELLED CORN	OATS	BARLEY	APPLES	POTATOES	SWEET POTATOES	BEANS	ONIONS	FLAXSEED
Alabama	60	56		56	32	47			55	60		
Arizona	60	56			32	45				55		
Arkansas	60	56	52	56	32	48	50	60	50	60	57	56
California	60	54	40		32	50						
Colorado	60	56	52		32	48		60		60	57	
Connecticut	60	56	48		32	48	48	60	54	60	52	55
Delaware	60											
Florida	60	56		56	32	48	48		60	60	56	
Georgia	60	56	52	56	32	47			55	60	57	56
Idaho	60	56	42		36	48	45	60				56
Illinois	60	56	52	56	32	48			50	60	57	56
Indiana	60	56	50	56	32	48		60	55	60	48	
Iowa	60	56	52	56	32	48	48	60	46	60	57	56
Kansas	60	56	50		32	48	48	60	50	60	57	56
Kentucky	60	56	56	56	32	47		60	55	60	57	56
Louisiana	60	56										
Maine	60	50	48		32	48	44	60		60	52	
Maryland					26			56				
Massachusetts	60	56	48	50	32	48	48	60	54	60	52	55
Michigan	60	56	48	56	32	48	48		56	60	54	56
Minnesota	60	56	50	56	32	48	50		55	60	52	
Mississippi	60	56	48	56	32	48			60	60	57	56
Missouri	60	56	52	56	32	48	48		56	60	57	56
Montana	60	56	52	56	32	48	45	60		60	57	56
Nebraska	60	56	52	56	32	48			50	60	57	56
Nevada												
New Hampshire	60	56			32			60		62		
New Jersey	60	56	50		30	48	50		54	60	57	55
New Mexico												
New York	60	56	48		32	48	48		54	60	57	55
North Carolina	60	56	50		32	48						55
North Dakota	60	56	42	56	32	48	50		46	60	52	56
Ohio	60	56	50	56	32	48	50		50	60	55	56
Oklahoma	60	56	42	56	32	48			46	60	52	56
Oregon	60	56	42		32	46	45	60				
Pennsylvania	66	56	48		32	47		56			50	
Rhode Island	60	56	48	56	32	48	48		54	60	50	56
South Carolina												
South Dakota	60	56	42	56	32	48			46	60	52	56
Tennessee	60	56	50	56	32	48	50		50	60	56	56
Texas	60	56	42	56	32	48	45		55	60	57	56
Utah												
Vermont	60	56	48		32	48	46	60		62	52	
Virginia	60	56	52	56	30	48			56	60	57	56
Washington	60	56	42		32	48	45	60				56
West Virginia	60	56	52		32	48		60		60		56
Wisconsin	60	56	50		32	48	50		54	60	57	56
Wyoming												
U. S. Customs	60	56	42		32	48	60	60				56
Dist. of Col.								60				

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13. 1562. 14. 2876. 15. 2180. 16. 2829. 17. 2950. 18. 3368.
19. 2723. 20. 3763. 21. 3462. 22. 4616. 23. 6100. 24. 6017.
25. 4896. 26. 9559. 27. 19,919. 28. 29,140. 29. 25,554.

Page 20.—30. 165,873. 31. 295,355. 32. 346,273. 33. 370,099.
34. \$555.67. 35. \$487.79. 36. \$1638.17. 37. \$2847.22. 38. \$2788.71.
39. \$2945.69. 40. \$3805.84. 41. \$5460.02.

Page 21.—42. \$6119.24. 43. \$10,519.35. 44. \$24,957.95.
45. \$135,120.91.

Page 24.—1. 90. 2. 91. 3. 104. 4. 51. 5. 76. 6. 111.
7. 116. 8. 188. 9. 265. 10. 262. 11. 1099. 12. 1127. 13. 2156.
14. 3959. 15. 6601. 16. 440. 17. 2084. 18. 3584. 19. 3339.
20. 3754. 21. 22,939. 22. 21,465. 23. 33,708. 24. 28,025. 25. 85,187.
26. \$3.25. 27. \$2.85. 28. \$1.85. 29. \$1.51. 30. \$.65. 31. \$6.43.
32. \$5.51. 33. \$4.02. 34. \$2.25. 35. \$1.29. 36. \$12.75. 37. \$22.93.
38. \$13.64. 39. \$12.52. 40. \$11.69. 41. \$48.05. 42. \$1,892.
43. \$171.25. 44. \$285.25. 45. \$88.20.

Page 28.—1. 3484. 2. 11,392. 3. 21,510. 4. 25,143. 5. 33,002.
6. 36,180. 7. 48,692. 8. 61,418. 9. 67,907. 10. 79,872. 11. 85,410.
12. 82,414. 13. 214,255. 14. 304,647. 15. 442,232. 16. 654,357.
17. 537,425. 18. 629,706. 19. 382,075. 20. 728,622. 21. 198,792.
22. 523,565. 23. 944,303. 24. 1,895,270. 25. 2,923,592. 26. 3,193,255.
27. 4,446,848. 28. 4,407,828. 29. 6,844,965. 30. 9,267,692. 31. 28,600.
32. 72,540. 33. 112,868. 34. 196,200. 35. 353,829. 36. 365,820.

37. 573,344. 38. 428,248. 39. 579,123. 40. 710,838. 41. 357,048.
 42. 561,150. 43. 477,462. 44. 729,872. 45. 26,624. 46. 55,200.
 47. 118,440. 48. 161,789. 49. 178,648. 50. 411,480. 51. 274,439.
 52. 486,200. 53. 645,400. 54. 514,368. 55. 338,465. 56. 602,330.
 57. 700,492.

- Page 29. — 58. \$119.60. 59. \$176.32. 60. \$208. 61. \$229.50.
 62. \$474.32. 63. \$735. 64. \$1366.40. 65. \$4069.66. 66. \$5147.22.
 67. \$7640. 68. \$8032. 69. \$12,045.60. 70. \$16,272.75. 71. \$27,946.50.
 72. \$80,676. 73. \$169,722. 74. \$337,612.50. 75. \$922,684.50.

- Page 31. — 1. 65. 2. 38. 3. 118. 4. 56. 5. 212. 6. 109.
 7. 156. 8. 107. 9. 120. 10. 215. 11. 58; Rem., 138. 12. 54.
 13. 45; Rem., 145. 14. 47. 15. 47; Rem., 1. 16. 42. 17. 66; Rem.
 120. 18. 54. 19. 86; Rem., 20. 20. 75. 21. 913; Rem., 89. 22. 634.
 23. 526; Rem., 79. 24. 1024. 25. 943. 26. 1024. 27. 1009.
 28. 1024. 29. 1024. 30. 1024.

- Page 32. — 1. 45. 2. 58. 3. 76. 4. 90. 5. 125. 6. 127.
 7. 453. 8. $765\frac{50}{100}$. 9. $915\frac{40}{100}$. 10. $937\frac{25}{100}$. 11. $98\frac{876}{1000}$. 12. 14.
 13. $15\frac{105}{1000}$. 14. $25\frac{560}{1000}$. 15. $356\frac{126}{1000}$.

- Page 33. — 1. 12. 2. 12. 3. 16. 4. 12. 5. 13. 6. 28. 7. 90.
 8. 9. 9. 80. 10. 8. 11. 90. 12. 900. 13. $70\frac{5}{10}$. 14. $60\frac{2}{10}$.
 15. $50\frac{1}{10}$. 16. $28\frac{20}{100}$. 17. $26\frac{46}{100}$. 18. $33\frac{21}{100}$.

- Page 38. — 1. $\frac{2}{3}$. 2. $\frac{5}{8}$. 3. $\frac{3}{5}$. 4. $\frac{7}{12}$. 5. $\frac{4}{7}$. 6. $\frac{3}{8}$. 7. $\frac{5}{8}$.
 8. $11\frac{1}{8}$. 9. $12\frac{1}{2}$. 10. $10\frac{1}{2}$. 11. $1\frac{3}{8}$. 12. $1\frac{1}{10}$. 13. $1\frac{1}{4}$. 14. $1\frac{1}{2}$. 15. $1\frac{1}{8}$.
 16. $\frac{7}{8}$. 17. $1\frac{1}{2}$. 18. $\frac{10}{7}$. 19. $1\frac{1}{4}$. 20. $1\frac{3}{4}$. 21. $1\frac{1}{8}$.
 1. $5\frac{1}{2}$. 2. $8\frac{1}{2}$. 3. $11\frac{1}{2}$. 4. $9\frac{1}{2}$. 5. $12\frac{3}{4}$. 6. $12\frac{3}{4}$. 7. $11\frac{1}{2}$. 8. $12\frac{3}{10}$.
 9. $10\frac{1}{8}$. 10. $10\frac{1}{2}$. 11. $16\frac{1}{2}$. 12. $18\frac{3}{4}$. 13. $5\frac{1}{4}$. 14. $11\frac{7}{12}$. 15. $9\frac{7}{60}$.
 16. $11\frac{23}{60}$. 17. $14\frac{33}{40}$. 18. $19\frac{1}{4}$. 19. $22\frac{11}{24}$. 20. $22\frac{3}{8}$. 21. $27\frac{3}{8}$.
 22. $26\frac{1}{12}$. 23. $36\frac{1}{2}$. 24. $38\frac{3}{8}$.

- Page 39. — 1. $\frac{3}{8}$. 2. $\frac{1}{3}$. 3. $\frac{7}{16}$. 4. $\frac{1}{6}$. 5. $\frac{5}{16}$. 6. $\frac{1}{4}$. 7. $\frac{3}{8}$.
 8. $\frac{3}{8}$. 9. $\frac{9}{16}$. 10. $\frac{5}{8}$. 11. $\frac{7}{16}$. 12. $\frac{7}{16}$. 13. $\frac{1}{3}$. 14. $\frac{1}{5}$. 15. $\frac{7}{12}$.

- Page 40. — 1. $2\frac{1}{2}$. 2. $2\frac{3}{8}$. 3. $4\frac{1}{8}$. 4. $2\frac{1}{4}$. 5. $1\frac{1}{2}$. 6. $5\frac{5}{8}$.
 7. $1\frac{1}{2}$. 8. $3\frac{1}{4}$. 9. $6\frac{3}{8}$. 10. $3\frac{3}{8}$. 11. $5\frac{1}{4}$. 12. $5\frac{3}{8}$. 13. $3\frac{1}{2}$. 14. $2\frac{3}{8}$.
 15. $3\frac{5}{16}$. 16. $7\frac{3}{4}$. 17. $4\frac{4}{15}$. 18. $19\frac{1}{12}$.
 1. $\frac{3}{4}$ yd. 2. $\frac{1}{4}$ yd. 3. $\frac{7}{8}$ yd. 4. $\frac{3}{8}$ yd. 5. $\frac{5}{8}$ yd. 6. $\frac{5}{8}$ mile.
 7. $\frac{1}{16}$ mile. 8. $\frac{3}{16}$ rod. 9. $\frac{1}{8}$ ton. 10. $1\frac{1}{8}$ pounds. 11. $1\frac{1}{2}$ ft.
 12. $18\frac{3}{4}$ gals. 13. $25\frac{1}{12}$ ft. 14. $1\frac{1}{8}$ ft. 15. $5\frac{3}{4}$ hrs.; $\frac{3}{4}$ hr. 16. $8\frac{7}{12}$ yds.;
 $2\frac{1}{12}$ yds.

- Page 41. — 17. $1\frac{3}{8}$ mi. 18. $\frac{1}{2}$ mi. 19. $4\frac{3}{4}$ pounds. 20. $\frac{5}{12}$ days.
 21. 89. 22. $8\frac{11}{12}$ yds.

- Page 42. — 23. $1\frac{5}{8}$ cans; $6\frac{3}{8}$ cans. 24. $1\frac{1}{2}$ doz. 25. $4\frac{9}{10}$ hrs. 26. $7\frac{1}{4}$ miles.
 27. 17 gallons. 28. 67 pounds. 29. \$7.00.

- Page 46. — 1. $\frac{1}{4}$. 2. $\frac{1}{4}$. 3. $\frac{2}{3}$. 4. $\frac{4}{7}$. 5. $\frac{1}{2}$. 6. $\frac{5}{7}$. 7. $\frac{3}{8}$. 8. $\frac{1}{2}$.
 9. $\frac{1}{8}$. 10. $\frac{1}{2}$. 11. $\frac{1}{8}$. 12. $\frac{1}{5}$. 13. $\frac{2}{3}$. 14. $\frac{3}{5}$. 15. $\frac{3}{4}$. 16. $\frac{1}{2}$. 17. $\frac{5}{8}$.
 18. $\frac{1}{4}$. 19. $\frac{1}{3}$. 20. $\frac{5}{6}$. 21. $\frac{7}{8}$. 22. $\frac{1}{4}$. 23. $\frac{3}{4}$. 24. $\frac{3}{8}$. 25. $\frac{8}{9}$. 26. $\frac{5}{6}$.
 27. $\frac{1}{2}$. 28. $\frac{9}{10}$. 29. $1\frac{1}{2}$. 30. $\frac{3}{4}$.

Page 47.—1. $\frac{6}{10}$. 2. $\frac{8}{10}$. 3. $\frac{9}{12}$. 4. $\frac{10}{16}$. 5. $\frac{6}{14}$. 6. $\frac{9}{11}$. 7. $\frac{15}{27}$.
8. $\frac{24}{28}$. 9. $\frac{21}{24}$. 10. $\frac{15}{27}$. 11. $\frac{6}{22}$. 12. $\frac{9}{32}$. 13. $\frac{23}{40}$. 14. $\frac{14}{20}$. 15. $\frac{30}{50}$.

Page 49.—1. $\frac{5}{10}$. 2. $\frac{13}{16}$. 3. $\frac{17}{20}$. 4. $\frac{10}{10}$. 5. $\frac{23}{36}$. 6. $\frac{35}{54}$. 7. $\frac{27}{39}$.
8. $\frac{55}{110}$. 9. $\frac{21}{30}$. 10. $\frac{19}{26}$. 11. $\frac{28}{35}$. 12. $\frac{7}{10}$. 13. $\frac{53}{84}$. 14. $\frac{45}{54}$. 15. $\frac{89}{115}$.
16. $\frac{58}{111}$. 17. $\frac{56}{99}$. 18. $\frac{53}{84}$. 19. $\frac{13}{16}$. 20. $\frac{47}{100}$. 21. $\frac{24}{54}$. 22. $\frac{129}{200}$.
23. $\frac{103}{124}$. 24. $\frac{83}{155}$. 25. $\frac{97}{124}$. 26. $\frac{133}{144}$. 27. $\frac{69}{111}$. 28. $\frac{67}{125}$. 29. $\frac{53}{78}$.
30. $\frac{95}{118}$. 31. $\frac{46}{116}$. 32. $\frac{103}{244}$. 33. $\frac{65}{114}$. 34. $\frac{143}{244}$. 35. $\frac{111}{116}$. 36. $\frac{229}{352}$.
37. $\frac{207}{255}$. 38. $\frac{119}{200}$. 39. $\frac{203}{352}$. 40. $\frac{287}{352}$.

Page 51.—1. $\frac{21}{21}$. 2. 2. 3. 3. 4. $\frac{21}{20}$. 5. $4\frac{1}{2}$. 6. $3\frac{1}{6}$. 7. $2\frac{1}{3}$.
8. 4. 9. $3\frac{1}{4}$. 10. $2\frac{1}{12}$. 11. 6. 12. $1\frac{1}{2}$. 13. 3. 14. 6. 15. 21.
16. $5\frac{5}{8}$. 17. $2\frac{1}{3}$. 18. 9. 19. 4. 20. $3\frac{1}{6}$. 21. $2\frac{2}{3}$. 22. $3\frac{1}{5}$. 23. $5\frac{1}{3}$.
24. $2\frac{5}{16}$. 25. $4\frac{1}{10}$. 26. $9\frac{9}{25}$. 27. $6\frac{1}{2}$. 28. $4\frac{2}{3}$. 29. $5\frac{1}{3}$. 30. $3\frac{1}{12}$.
31. 3. 32. 3. 33. $3\frac{1}{2}$. 34. $4\frac{1}{3}$. 35. $6\frac{9}{10}$. 36. $7\frac{3}{5}$. 37. $5\frac{5}{8}$. 38. 6.
39. $9\frac{9}{10}$. 40. $5\frac{1}{2}$.

Page 54.—1. 3, 3, 5. 2. 2, 2, 2, 3, 3. 3. 2, 41. 4. 2, 3, 3, 5.
5. 2, 2, 2, 2, 3. 6. 2, 2, 2, 3, 5. 7. 5, 5, 5. 8. 2, 2, 3, 13.
9. 2, 2, 3, 3, 5. 10. 2, 2, 7, 7. 11. 3, 3, 5, 5. 12. 2, 2, 2, 2, 3, 3, 3.
13. 3, 3, 3, 5, 5. 14. 2, 2, 2, 2, 53. 15. 2, 2, 3, 3, 3, 3. 16. 2, 2, 2, 3, 43.
17. 2, 2, 2, 3, 7, 7. 18. 11, 11, 11. 19. 2, 2, 2, 2, 3, 3, 3. 20. 3, 3, 5, 7, 7.
21. 2, 2, 7, 107. 22. 3, 7, 13, 13. 23. Prime. 24. 2, 2, 2, 953.
25. 2, 2, 11, 13, 17.

Page 55.—1. 6. 2. 9. 3. 24. 4. 32. 5. 16. 6. 11.
7. 48. 8. 15. 9. 60. 10. 15. 11. 6. 12. 12. 13. 32.
14. 14. 15. 20.

Page 56.—1. 48. 2. 180. 3. 60. 4. 96. 5. 84. 6. 30.
7. 48. 8. 36. 9. 48. 10. 144. 11. 60. 12. 240.
13. 540. 14. 600. 15. 4200. 16. 7560. 17. 11,000. 18. 2400.
19. 800. 20. 2400. 21. 1440. 22. 1800. 23. 720. 24. 6048.
25. 1800.

Page 57.—1. $\frac{3}{8}$; $\frac{2}{6}$. 2. $\frac{8}{12}$; $\frac{2}{12}$. 3. $\frac{8}{12}$; $\frac{9}{12}$. 4. $\frac{16}{40}$; $\frac{15}{40}$. 5. $\frac{15}{20}$; $\frac{16}{20}$.
6. $\frac{10}{12}$; $\frac{10}{12}$. 7. $\frac{18}{30}$; $\frac{10}{30}$. 8. $\frac{28}{42}$; $\frac{18}{42}$. 9. $\frac{20}{24}$; $\frac{12}{24}$. 10. $\frac{27}{45}$; $\frac{20}{45}$. 11. $\frac{15}{18}$; $\frac{13}{18}$.
12. $\frac{10}{10}$; $\frac{18}{24}$. 13. $\frac{4}{8}$; $\frac{8}{24}$. 14. $\frac{8}{24}$; $\frac{14}{24}$. 15. $\frac{12}{18}$; $\frac{15}{18}$.
16. $\frac{14}{24}$; $\frac{16}{24}$. 17. $\frac{24}{24}$; $\frac{15}{24}$. 18. $\frac{24}{60}$; $\frac{18}{60}$. 19. $\frac{72}{120}$; $\frac{45}{120}$.
20. $\frac{48}{120}$; $\frac{100}{120}$. 21. $\frac{36}{36}$; $\frac{16}{36}$. 22. $\frac{45}{120}$; $\frac{80}{120}$. 23. $\frac{20}{20}$; $\frac{20}{20}$.
24. $\frac{48}{48}$; $\frac{27}{48}$.

Page 58.—1. $1\frac{1}{12}$. 2. $1\frac{7}{12}$. 3. $1\frac{1}{6}$. 4. $11\frac{1}{20}$. 5. $1\frac{5}{24}$. 6. $\frac{19}{24}$.
7. $1\frac{7}{12}$. 8. $1\frac{19}{24}$. 9. $1\frac{5}{8}$. 10. $1\frac{1}{3}$. 11. $1\frac{1}{3}$. 12. $1\frac{2}{3}$. 13. $1\frac{5}{8}$.
14. $1\frac{9}{10}$. 15. $1\frac{4}{6}$. 16. $1\frac{13}{18}$. 17. $1\frac{7}{5}$. 18. $1\frac{1}{2}$. 19. $1\frac{13}{8}$. 20. $1\frac{1}{2}$.
21. $1\frac{35}{24}$. 22. $1\frac{3}{10}$. 23. $2\frac{3}{10}$. 24. $1\frac{1}{3}$. 25. $1\frac{1}{3}$. 26. $1\frac{1}{5}$. 27. $2\frac{1}{4}$.
28. $2\frac{2}{10}$. 29. $1\frac{3}{24}$. 30. $\frac{29}{30}$. 31. $1\frac{1}{3}$. 32. $2\frac{1}{3}$. 33. $1\frac{5}{12}$. 34. $1\frac{7}{18}$.
35. $2\frac{1}{4}$. 36. $1\frac{1}{10}$. 37. $2\frac{7}{8}$. 38. $1\frac{3}{2}$. 39. $1\frac{1}{5}$. 40. $2\frac{3}{4}$. 41. $2\frac{7}{16}$.
42. $1\frac{3}{4}$. 43. $3\frac{9}{40}$. 44. $2\frac{5}{12}$. 45. $5\frac{5}{21}$.

Page 59.—1. $5\frac{3}{4}$. 2. $8\frac{7}{12}$. 3. $7\frac{5}{8}$. 4. $7\frac{7}{10}$. 5. $8\frac{5}{8}$. 6. $10\frac{3}{8}$.
7. $7\frac{8}{15}$. 8. $7\frac{3}{8}$. 9. $15\frac{1}{12}$. 10. $12\frac{1}{12}$. 11. $8\frac{3}{8}$. 12. $17\frac{1}{2}$. 13. $8\frac{1}{2}$.
14. $16\frac{7}{20}$. 15. $20\frac{7}{24}$. 16. $14\frac{1}{16}$. 17. $14\frac{1}{8}$. 18. $15\frac{3}{4}$. 19. $11\frac{1}{2}$.
20. $12\frac{3}{5}$. 21. $22\frac{1}{20}$. 22. $10\frac{1}{3}$. 23. $20\frac{1}{32}$. 24. $17\frac{5}{8}$. 25. $56\frac{1}{2}$.
26. $55\frac{1}{2}$. 27. $60\frac{1}{6}$. 28. $77\frac{1}{8}$. 29. $81\frac{5}{8}$. 30. $71\frac{1}{12}$. 31. $92\frac{1}{8}$. 32. $100\frac{1}{12}$.

33. $102\frac{7}{5}$. 34. $128\frac{9}{10}$. 35. $102\frac{4}{15}$. 36. $142\frac{7}{10}$. 37. $161\frac{1}{11}$. 38. $180\frac{4}{5}$.
39. $187\frac{3}{8}$.

Page 60.—1. $3\frac{7}{12}$ hrs. 2. 4 hrs. 3. $3\frac{3}{4}$ hrs. 4. 42 yards. 5. $\frac{1}{2}$ hr.

- Page 61.—6. $1\frac{1}{4}$ inches. 7. $19\frac{3}{4}$ yards. 8. Spends $\frac{1}{16}$. 9. 75 mi.
10. \$3.40. 11. 28 yards. 12. Walks $\frac{2}{15}$. 13. $\frac{9}{16}$ inch. 14. $19\frac{1}{2}$ hours.
15. $36\frac{1}{4}$ ft. 16. $40\frac{1}{4}$ in.

Page 62.—17. $21\frac{9}{120}$ A. 18. \$3.60. 19. \$210.75.

1. $\frac{5}{12}$. 2. $\frac{5}{12}$. 3. $\frac{3}{8}$. 4. $\frac{1}{3}$. 5. $\frac{7}{12}$. 6. $\frac{7}{24}$. 7. $\frac{1}{8}$. 8. $\frac{17}{24}$.
9. $\frac{1}{2}$. 10. $\frac{1}{4}$. 11. $\frac{3}{4}$. 12. $\frac{3}{6}$. 13. $\frac{1}{5}$. 14. $\frac{1}{10}$. 15. $\frac{3}{10}$.
16. $\frac{3}{16}$. 17. $\frac{3}{8}$. 18. $\frac{1}{16}$. 19. $\frac{1}{30}$. 20. $\frac{7}{60}$. 21. $\frac{1}{30}$. 22. $\frac{1}{10}$.
23. $\frac{3}{48}$. 24. $\frac{3}{8}$.

- Page 63.—1. $4\frac{1}{2}$. 2. $5\frac{1}{3}$. 3. $7\frac{1}{2}$. 4. $5\frac{3}{8}$. 5. $6\frac{5}{8}$. 6. $8\frac{1}{5}$.
7. $7\frac{1}{2}$. 8. $8\frac{1}{3}$. 9. $9\frac{3}{8}$. 10. $9\frac{1}{5}$. 11. $11\frac{1}{5}$. 12. $10\frac{5}{8}$. 13. $14\frac{1}{2}$.
14. $13\frac{1}{3}$. 15. $16\frac{1}{3}$. 16. $17\frac{1}{8}$. 17. $18\frac{1}{2}$. 18. $20\frac{4}{15}$. 19. $24\frac{5}{16}$.
20. $39\frac{1}{16}$. 21. $51\frac{9}{20}$. 22. $59\frac{1}{15}$. 23. $71\frac{1}{20}$. 24. $84\frac{1}{25}$.

- Page 64.—1. $1\frac{1}{6}$. 2. $2\frac{1}{4}$. 3. $1\frac{5}{12}$. 4. $2\frac{1}{6}$. 5. $2\frac{1}{8}$. 6. $4\frac{1}{20}$. 7. $4\frac{1}{12}$.
8. $3\frac{1}{3}$. 9. $2\frac{7}{24}$. 10. $4\frac{5}{14}$. 11. $2\frac{3}{8}$. 12. $3\frac{1}{12}$. 13. $2\frac{3}{10}$. 14. $3\frac{1}{2}$.
15. $2\frac{7}{12}$. 16. $2\frac{1}{2}$. 17. $2\frac{3}{10}$. 18. $3\frac{7}{12}$. 19. $1\frac{3}{4}$. 20. $2\frac{5}{14}$. 21. $2\frac{1}{6}$.
22. $2\frac{1}{15}$. 23. $2\frac{7}{24}$. 24. $2\frac{2}{30}$. 25. $2\frac{1}{18}$. 26. $3\frac{7}{40}$. 27. $4\frac{1}{2}$. 28. $3\frac{3}{8}$.
29. $10\frac{1}{16}$. 30. $9\frac{1}{20}$.
1. $\frac{5}{12}$ yard. 2. $3\frac{1}{4}$ miles. 3. \$6 $\frac{1}{2}$. 4. \$2.75. 5. $5\frac{1}{2}$ pounds.
6. \$2.50.

- Page 65.—7. $1\frac{1}{2}$ ft. 8. $5\frac{3}{8}$ ft. 9. $4\frac{3}{8}$ ft. 10. $2\frac{7}{8}$ miles. 11. $18\frac{1}{2}$ pounds.
12. $\frac{1}{24}$. 14. $\frac{1}{3}$ sec., $2\frac{2}{3}$ sec. 15. 12 yd., $10\frac{1}{4}$ yd., $13\frac{1}{4}$ yd.

- Page 66.—1. $1\frac{1}{2}$. 2. $1\frac{1}{12}$. 3. $1\frac{1}{4}$. 4. $\frac{1}{8}$. 5. $\frac{5}{8}$. 6. $\frac{7}{16}$. 7. $1\frac{1}{12}$.
8. $1\frac{7}{8}$. 9. $2\frac{5}{16}$. 10. $\frac{3}{2}$. 11. 1. 12. $\frac{1}{12}$. 13. $\frac{1}{2}$. 14. $\frac{7}{8}$. 15. $\frac{2}{3}$.
16. $\frac{1}{3}$. 17. $\frac{1}{16}$. 18. $\frac{1}{9}$. 19. $1\frac{1}{8}$. 20. $4\frac{1}{4}$. 21. $2\frac{1}{2}$. 22. $3\frac{1}{8}$.
23. $4\frac{1}{2}$. 24. $6\frac{1}{12}$.
1. $\frac{1}{24}$ barrel. 2. $\frac{1}{30}$. 3. $\frac{1}{30}$.

- Page 67.—4. $10\frac{7}{8}$ A. 5. $4\frac{2}{3}$ ft. 6. \$5 $\frac{1}{2}$. 7. $8\frac{1}{3}$ yds. 8. $2\frac{2}{3}$ ft.
9. 15 yrs. 10. $94\frac{1}{4}$ pounds. 11. $\frac{7}{60}$. 12. $\frac{1}{18}$.

Page 68. 13. $\frac{1}{6}$.

- Page 69.—1. 6. 2. 6. 3. 1. 4. 1. 5. $1\frac{1}{12}$. 6. 1. 7. $\frac{4}{5}$.
8. $1\frac{3}{8}$. 9. $2\frac{1}{4}$. 10. $\frac{3}{8}$. 11. 1. 12. 1. 13. $\frac{3}{8}$. 14. $\frac{7}{45}$. 15. 4.
16. $1\frac{1}{14}$. 17. $\frac{2}{3}$.
1. \$420. 2. \$23.75. 3. \$5.25. 4. $37\frac{1}{2}$ yards. 5. 12 books.
6. \$.14.

Page 70.—7. $43\frac{1}{5}$ bu. 8. $10\frac{1}{3}$ loads.

- Page 71.—1. $1\frac{3}{8}$. 2. $1\frac{3}{4}$. 3. $1\frac{1}{8}$. 4. $3\frac{3}{8}$. 5. $2\frac{3}{4}$. 6. $4\frac{1}{16}$. 7. $3\frac{3}{8}$.
8. $3\frac{3}{8}$. 9. $3\frac{1}{4}$. 10. $2\frac{3}{4}$. 11. $2\frac{3}{8}$. 12. $6\frac{3}{8}$. 13. $4\frac{1}{8}$. 14. $1\frac{4}{5}$. 15. $10\frac{1}{2}$.
16. $2\frac{3}{4}$. 17. $8\frac{3}{8}$. 18. $3\frac{3}{8}$. 19. $20\frac{2}{3}$. 20. $13\frac{1}{4}$. 21. $12\frac{3}{4}$. 22. 5. 23. $5\frac{1}{4}$.
24. $7\frac{1}{2}$.

- Page 72. 1. 14. 2. 17. 3. 29. 4. $39\frac{3}{8}$. 5. 50. 6. 46.
7. 66. 8. 57. 9. 96. 10. $115\frac{1}{2}$. 11. 123. 12. 140. 13. $178\frac{1}{2}$.
14. 170. 15. $202\frac{1}{2}$. 16. 220. 17. $294\frac{1}{2}$. 18. 404. 19. $205\frac{1}{2}$. 20. 279.
21. $375\frac{1}{2}$. 22. $377\frac{1}{2}$. 23. $502\frac{3}{8}$. 24. 980.

Page 73.—1. 14. 2. 21. 3. 15. 4. 15. 5. $25\frac{1}{2}$. 6. $26\frac{1}{4}$.
 7. $52\frac{1}{2}$. 8. $66\frac{3}{4}$. 9. 44. 10. 48. 11. 35. 12. $46\frac{1}{2}$. 13. $16\frac{3}{4}$.
 14. $39\frac{1}{2}$. 15. 56. 16. 64. 17. $35\frac{3}{4}$. 18. $46\frac{1}{4}$. 19. $32\frac{3}{4}$. 20. 66.
 21. $48\frac{3}{4}$. 22. 40. 23. $45\frac{5}{8}$. 24. 40. 25. $60\frac{1}{2}$. 26. 40. 27. 50.
 28. 60. 29. $79\frac{1}{2}$. 30. $71\frac{1}{4}$. 31. $64\frac{3}{4}$. 32. 69. 33. 90. 34. $75\frac{1}{3}$.
 35. 56. 36. 75. 37. $74\frac{2}{5}$. 38. $41\frac{2}{5}$. 39. 58.

Page 75.—1. 30. 2. 36. 3. 77. 4. 175. 5. 329. 6. 165. 7. 156.
 8. $129\frac{1}{2}$. 9. $87\frac{3}{4}$. 10. 86. 11. 165. 12. 182. 13. 352. 14. $612\frac{1}{2}$.
 15. 556. 16. 894. 17. $411\frac{3}{4}$. 18. $318\frac{3}{4}$. 19. $334\frac{1}{2}$. 20. 585. 21. 665.
 22. 749. 23. 865. 24. 1145. 25. 350. 26. 520. 27. 630. 28. 595.
 29. 891. 30. 675. 31. 861. 32. 936. 33. 1178. 34. 1416.

Page 76.—1. $\frac{1}{20}$. 2. $\frac{1}{4}$. 3. $\frac{2}{5}$. 4. $\frac{1}{3}$. 5. $\frac{2}{3}$. 6. $\frac{2}{5}$. 7. $\frac{7}{20}$. 8. $\frac{2}{7}$.
 9. $\frac{17}{20}$. 10. $\frac{2}{5}$. 11. $\frac{2}{3}$. 12. $\frac{4}{15}$.

Page 77.—13. $\frac{2}{3}$. 14. $\frac{2}{5}$. 15. $\frac{2}{7}$. 16. $\frac{2}{7}$. 17. $\frac{3}{7}$. 18. $\frac{5}{7}$. 19. $\frac{6}{7}$.
 20. $\frac{7}{11}$. 21. $\frac{7}{9}$. 22. $\frac{5}{11}$. 23. $\frac{8}{9}$. 24. $1\frac{2}{7}$. 25. $\frac{3}{4}$. 26. $\frac{2}{3}$. 27. $\frac{2}{50}$.
 28. $\frac{3}{8}$. 29. $\frac{42}{43}$. 30. $\frac{3}{4}$. 31. $\frac{9}{44}$. 32. $\frac{5}{6}$. 33. $\frac{13}{30}$. 34. $\frac{2}{35}$. 35. $\frac{1}{3}$.
 36. $\frac{7}{12}$. 37. $\frac{20}{143}$. 38. $\frac{5}{6}$. 39. $\frac{2}{3}$. 40. $\frac{8}{5}$.
 1. $21\frac{1}{2}$ hrs. 2. \$1.65. 3. 21 runs. 4. 18 seconds.

Page 78.—5. 24,000 ft. 6. 5625 pounds. 7. $\frac{9}{10}$. 8. $\frac{2}{20}$ hour.
 9. $1\frac{1}{4}$ yd. 10. $\frac{1}{4}$ mile. 11. \$3300. 12. \$2.04; \$11.22. 13. \$81.90.

Page 79.—14. \$3654. 15. \$2626 $\frac{4}{5}$.

Page 80.—1. $\frac{1}{12}$. 2. $\frac{2}{15}$. 3. $\frac{3}{20}$. 4. $\frac{3}{20}$. 5. $\frac{3}{80}$. 6. $\frac{1}{30}$. 7. $\frac{7}{25}$.
 8. $\frac{9}{80}$. 9. $\frac{1}{48}$. 10. $\frac{3}{32}$. 11. $\frac{2}{11}$. 12. $\frac{3}{16}$. 13. $\frac{5}{17}$. 14. $\frac{1}{10}$. 15. $1\frac{3}{5}$.
 16. $\frac{8}{11}$. 17. $100\frac{5}{8}$. 18. $12\frac{5}{9}$. 19. $2\frac{5}{25}$. 20. $\frac{5}{96}$. 21. $\frac{3}{35}$. 22. $2\frac{7}{8}$. 23. $7\frac{5}{8}$.
 24. $\frac{1}{55}$. 25. $\frac{1}{65}$. 26. $\frac{2}{38}$. 27. $\frac{1}{10}$. 28. $\frac{5}{38}$. 29. $1\frac{5}{6}$. 30. $2\frac{3}{80}$.

Page 81.—1. 2. 2. $\frac{9}{10}$. 3. $1\frac{1}{5}$. 4. $\frac{5}{6}$. 5. $1\frac{1}{2}$. 6. 3.

Page 82.—7. $\frac{7}{8}$. 8. $\frac{3}{8}$. 9. $1\frac{1}{3}$. 10. $1\frac{1}{20}$. 11. $1\frac{1}{3}$. 12. $1\frac{1}{3}$. 13. $1\frac{1}{6}$.
 14. $1\frac{1}{3}$. 15. $1\frac{1}{9}$. 16. $2\frac{1}{3}$. 17. $\frac{2}{3}$. 18. $\frac{3}{8}$. 19. $\frac{1}{4}$. 20. $1\frac{1}{9}$. 21. $\frac{3}{8}$.
 22. $\frac{5}{8}$. 23. $\frac{5}{9}$. 24. $1\frac{1}{10}$. 25. $1\frac{1}{2}$. 26. $1\frac{1}{10}$. 27. $1\frac{1}{5}$. 28. $\frac{3}{4}$. 29. $1\frac{1}{5}$.
 30. $1\frac{3}{8}$. 31. 4. 32. 4. 33. $2\frac{5}{8}$. 34. 4. 35. $12\frac{1}{4}$. 36. $5\frac{1}{4}$. 37. 6.
 38. 17. 39. $6\frac{2}{3}$. 40. 6. 41. 10. 42. 10. 43. $1\frac{1}{9}$. 44. $2\frac{2}{3}$. 45. $1\frac{7}{10}$.
 46. $1\frac{1}{3}$. 47. 4. 48. 4.

Page 83.—1. 6. 2. $6\frac{3}{4}$. 3. 10. 4. $8\frac{3}{4}$. 5. $9\frac{1}{7}$. 6. $16\frac{1}{5}$. 7. $11\frac{3}{4}$.
 8. $14\frac{1}{2}$. 9. $13\frac{1}{2}$. 10. 24. 11. 20. 12. $31\frac{1}{2}$. 13. $28\frac{1}{2}$. 14. 32.
 15. $37\frac{1}{3}$. 16. $7\frac{1}{2}$. 17. $73\frac{1}{2}$. 18. 81. 19. 108. 20. 108. 21. 120.
 22. 96. 23. 200. 24. $106\frac{2}{3}$. 25. $1\frac{1}{4}$. 26. $1\frac{3}{13}$. 27. $1\frac{1}{3}$. 28. $1\frac{1}{3}$.
 29. $1\frac{1}{8}$. 30. $2\frac{4}{5}$. 31. $1\frac{1}{5}$. 32. $1\frac{1}{11}$. 33. $1\frac{1}{11}$. 34. $2\frac{1}{2}$. 35. $3\frac{1}{8}$. 36. $1\frac{1}{2}$.
 37. 6. 38. $5\frac{1}{4}$. 39. 3. 40. $2\frac{1}{2}$. 41. 14. 42. 7. 43. 5. 44. 4. 45. 6.
 46. $7\frac{1}{2}$. 47. 7. 48. $9\frac{1}{2}$.
 1. 60 bags. 2. 36 loads. 3. 26 loads. 4. $73\frac{1}{2}$ gal.

Page 84.—5. 120 bu. 6. $7\frac{3}{4}$ barrels. 7. 12 hours. 8. $51\frac{1}{2}$.
 9. $23\frac{1}{2}$ bu. 10. \$12. 11. 20 barrels. 12. 115 yards. 13. $139\frac{1}{2}$ times.
 14. 220 sweaters.

Page 86.—1. $\frac{4}{10}$. 2. $\frac{1}{2}$. 3. $\frac{2}{3}$. 4. $\frac{5}{8}$. 5. $\frac{1}{12}$. 6. $\frac{1}{3}$. 7. $\frac{1}{3}$.
 8. $\frac{3}{10}$. 9. $\frac{2}{3}$. 10. $\frac{2}{3}$. 11. $\frac{2}{3}$. 12. $\frac{2}{3}$. 13. $\frac{3}{8}$. 14. $\frac{1}{15}$. 15. $\frac{2}{3}$.
 16. $7\frac{7}{8}$. 17. $1\frac{5}{8}$. 18. $\frac{3}{5}$. 19. $\frac{3}{8}$. 20. $\frac{1}{5}$ of crop. 21. $\frac{3}{10}$. 22. $\frac{5}{6}$ of
 street; $\frac{3}{8}$ of street. 23. $\frac{1}{5}$ of weight. 24. $\frac{1}{16}$ of cost. 25. $\frac{3}{32}$ of distance.

Page 87.—1. 30. 2. 35. 3. 35. 4. 36. 5. 56. 6. 108. 7. 72.
8. 55. 9. 65. 10. 105. 11. 91. 12. 100. 13. 45. 14. 240.
15. 140. 16. 135. 17. 140. 18. 175. 19. 155. 20. 162. 21. 115.
22. 100. 23. 87. 24. 120. 25. 117. 26. 144. 27. 155. 28. 212.
29. 250. 30. 240.

Page 88.—1. $\frac{3}{10}$. 2. $\frac{1}{9}$. 3. $\frac{2}{3}$. 4. $\frac{5}{8}$. 5. $\frac{2}{3}, \frac{7}{8}$. 6. $\frac{5}{8}$.
7. $\frac{1}{10}$. 8. $\frac{1}{6}$.

Page 89.—9. 24 gaw es. 10. 120 miles. 11. 105 boys. 12. 150 trees.
13. \$3000. 14. \$22. 15. \$1440. 16. \$2100. 17. \$1440.
18. \$24,000.

Page 91.—1. $\frac{4}{9}$. 2. $\frac{5}{9}$. 3. $\frac{5}{9}$. 4. $\frac{8}{9}$. 5. $\frac{2}{3}$. 6. $\frac{2}{3}$. 7. $\frac{2}{3}$. 8. $\frac{8}{9}$.
9. $\frac{1}{21}$. 10. $\frac{4}{9}$. 11. $\frac{1}{22}$. 12. $\frac{4}{8}$. 13. $\frac{2}{25}$. 14. $\frac{9}{9}$. 15. $\frac{1}{34}$. 16. $\frac{1}{25}$.
17. $\frac{1}{17}$. 18. $\frac{2}{9}$. 19. $\frac{4}{7}$. 20. $\frac{6}{9}$. 21. $\frac{2}{8}$. 22. $\frac{3}{4}$. 23. $\frac{5}{4}$.
24. $\frac{8}{8}$. 25. $\frac{1}{27}$. 26. $\frac{1}{97}$. 27. $\frac{2}{36}$. 28. $\frac{3}{35}$. 29. $\frac{1}{10}$. 30. $\frac{1}{17}$.
31. $\frac{2}{77}$. 32. $\frac{2}{28}$. 33. $\frac{4}{5}$. 34. $\frac{3}{4}$. 35. $\frac{4}{8}$. 36. $\frac{4}{8}$. 37. $\frac{3}{12}$.
38. $\frac{3}{15}$. 39. $\frac{4}{17}$. 40. $\frac{3}{27}$. 41. $\frac{6}{15}$. 42. $\frac{8}{13}$. 43. $\frac{8}{18}$. 44. $\frac{7}{25}$.
45. $\frac{8}{25}$. 46. $\frac{6}{36}$. 47. $\frac{5}{10}$. 48. $\frac{4}{5}$. 49. $\frac{1}{12}$. 50. $\frac{1}{24}$. 51. $\frac{1}{18}$.
52. $\frac{5}{60}$. 53. $\frac{1}{12}$. 54. $\frac{2}{24}$. 55. $\frac{1}{17}$. 56. $\frac{1}{16}$. 57. $\frac{1}{16}$. 58. $\frac{2}{24}$.
59. $\frac{2}{61}$. 60. $\frac{2}{23}$. 61. $\frac{2}{52}$. 62. $\frac{3}{35}$. 63. $\frac{11}{36}$. 64. $\frac{8}{16}$. 65. $\frac{7}{30}$.
66. $\frac{14}{18}$. 67. $\frac{9}{24}$. 68. $\frac{16}{16}$. 69. $\frac{1}{40}$. 70. $\frac{5}{24}$. 71. $\frac{5}{36}$. 72. $\frac{1}{30}$.
73. $\frac{7}{16}$. 74. $\frac{11}{18}$. 75. $\frac{20}{24}$. 76. $\frac{31}{10}$. 77. $\frac{4}{8}$. 78. $\frac{8}{24}$. 79. $\frac{12}{36}$.
80. $\frac{14}{15}$. 81. $\frac{1}{1}$. 82. $\frac{2}{24}$. 83. $\frac{6}{12}$. 84. $\frac{6}{24}$.

Page 92.—85. $\frac{4}{12}$. 86. $\frac{1}{17}$. 87. $\frac{1}{15}$. 88. $\frac{9}{8}$. 89. 46. 90. $\frac{701}{9}$.
91. 160. 92. $\frac{187}{12}$. 93. $\frac{131}{8}$. 94. $\frac{7}{12}$. 95. 21. 96. $\frac{16}{12}$. 97. 93.
98. $\frac{161}{4}$. 99. $\frac{32}{4}$. 100. 672. 101. $\frac{1}{45}$. 102. $\frac{9}{16}$. 103. $\frac{1}{6}$. 104. $\frac{7}{40}$.
105. $\frac{1}{8}$. 106. $\frac{1}{12}$. 107. $\frac{3}{5}$. 108. $\frac{1}{8}$. 109. $\frac{1}{15}$. 110. $\frac{7}{8}$. 111. $\frac{1}{17}$.
112. $\frac{5}{6}$. 113. $\frac{9}{10}$. 114. $\frac{2}{5}$. 115. $\frac{2}{3}$. 116. 2. 117. 4. 118. $\frac{1}{13}$.
119. $\frac{5}{16}$. 120. $\frac{3}{8}$. 121. $\frac{3}{10}$. 122. $\frac{1}{25}$. 123. $\frac{7}{36}$. 124. $\frac{1}{30}$. 125. $\frac{1}{12}$.
126. $\frac{1}{12}$. 127. $\frac{1}{15}$. 128. $\frac{1}{18}$. 129. $\frac{1}{18}$. 130. $\frac{1}{18}$. 131. $\frac{1}{18}$. 132. $\frac{1}{20}$.
133. $\frac{1}{12}$. 134. $\frac{1}{12}$. 135. $\frac{5}{12}$. 136. $\frac{1}{15}$. 137. $\frac{1}{12}$. 138. $\frac{1}{11}$. 139. 21.
140. $\frac{1}{10}$. 141. $\frac{1}{9}$. 142. $\frac{1}{61}$. 143. $\frac{1}{12}$. 144. $\frac{7}{109}$. 145. 6. 146. 12.
147. 11. 148. 5. 149. 10. 150. $\frac{4}{3}$. 151. $\frac{19}{40}$. 152. $\frac{9}{2}$.

Page 93.—1. \$3.15, \$5.25, \$4.20. 2. 168 consumed, 72 sold.
3. 800 sq. ft. potatoes, 200 sq. ft. beets. 4. \$17.75.

Page 94.—5. 30 lbs., $\frac{1}{2}$ bu.

1. \$8.53. 2. $50\frac{1}{2}$ ¢. 3. $23\frac{1}{2}$ ¢. 4. 49¢. 5. 72¢, \$65.52.

Page 95.—1. \$33 $\frac{1}{2}$. 2. \$1225. 3. \$14.03. 4. \$2 $\frac{2}{3}$. 5. \$29 $\frac{2}{3}$.
6. $\frac{9}{50}$, $\frac{8}{125}$. 7. $\frac{1}{28}$ rent, $\frac{1}{460}$ gas, $\frac{1}{560}$ electricity.

Page 96.—1. \$1 $\frac{1}{2}$. 2. \$14 $\frac{7}{8}$. 3. \$10 $\frac{10}{17}$, \$.34 $\frac{3}{4}$. 4. \$12.60 $\frac{5}{12}$,
\$.41 $\frac{2}{3}$. 5. \$32.56.

Page 97.—1. \$360. 2. \$1200.

Page 98.—3. \$2500. 4. \$30. 5. \$250. 6. $\frac{5}{8}$. 7. $\frac{1}{12}$. 8. \$270.
9. \$281 $\frac{1}{4}$. 10. \$350.

Page 101.—1. .14, .29, .69, .122. 2. .61, .079, .0016, .0201. 3. .211,
.0211, .2000.0011. 4. .1137, .1100.0037, .3711. 5. .059. 6. .092. 7. .0116.

8. .0215. 9. .265. 10. .0712. 11. .0819. 12. .08002. 13. 25.37.
14. 85.0125. 15. 175.0175. 16. 1201.0301. 17. 3570.115. 18. 1079.10307.

Page 102. 1. .250, .125. 2. .154, .750. 3. .0800, .1125. 4. .70000, .12675. 5. .7000, .0700, .0070, .0007. 6. .1350, 1.5000, .1500, 15.0005. 7. .0010, .0100, .0001, .1000. 8. .00350, .03500, .50000, .00175. 9. .1250, .2000, .1500, .0125, .5250. 10. .5000, .1500, .0150, .2500, .0005.

Page 103. — 1. $\frac{2}{5}$. 2. $\frac{4}{5}$. 3. $\frac{3}{25}$. 4. $\frac{41}{100}$. 5. $\frac{1}{2}$. 6. $\frac{7}{20}$. 7. $\frac{11}{30}$.
8. $\frac{3}{4}$. 9. $\frac{83}{100}$. 10. $\frac{1}{10}$. 11. $\frac{1}{8}$. 12. $\frac{9}{200}$. 13. $\frac{27}{200}$. 14. $\frac{63}{125}$.
15. $\frac{31}{40}$. 16. $\frac{29}{200}$. 17. $\frac{125}{125}$. 18. $\frac{21}{250}$. 19. $\frac{12}{125}$. 20. $\frac{99}{1000}$. 21. $\frac{8}{80}$.
22. $\frac{8}{400}$. 23. $\frac{125}{1250}$. 24. $\frac{21}{5000}$. 25. $\frac{19}{2000}$.
1. \$10.50. 2. 12.01. 3. 99.656. 4. 456. 5. 46. 6. 29.28.
7. 29.666. 8. 125.206.

Page 104. — 9. 87.57. 10. 18.14. 11. 19.471. 12. 108.69.
13. 244.51. 14. 15.443. 15. 1056.6. 16. 2.1. 17. 71.424.
18. 170.677. 19. 168.9672. 20. 1345.8426.

Page 105. — 1. \$9.19. 2. \$8.87. 3. 25.148. 4. 13.997. 5. 41.955.
6. 109.788. 7. 61.709. 8. 83.9285. 9. 58.875. 10. 167.625.
11. 937.375. 12. 1981.125. 13. 6.88. 14. 41.284. 15. 54.081.
16. 15.968. 17. 62.97. 18. 38.64. 19. 18.08. 20. 1.97. 21. 16.41.
22. 19.81. 23. 468.58. 24. 1.91. 25. .81. 26. 1003.999.
27. 1003.07. 28. 892.85. 29. 46.8. 30. 64.52.
1. 226.9 miles. 2. 3505.14 feet.

Page 106. — 3. 987.50 pounds. 4. 1060.3 miles. 5. 91.50 feet.
6. 131.1 miles. 7. \$7.40.

Page 107. — 1. 6.5. 2. 32.4. 3. 42.7. 4. 74.4. 5. 100.
6. 184.5. 7. 302.04. 8. 313.80. 9. 386.88. 10. 633.64. 11. 907.36.
12. 1578.15. 13. 28.125. 14. 57.024. 15. 248,124. 16. 579.968.
17. 1510.375. 18. 3178.084.

Page 108. — 1. 6.72. 2. 13.94. 3. 34.02. 4. 44.28. 5. 125.24.
6. 220.05. 7. 7.383. 8. 40.248. 9. 68.355. 10. 127.575. 11. 219.375.
12. 315.208. 13. 8.8395. 14. 21.6974. 15. 75.4091. 16. 102.2364.
17. 83.5737. 18. 1.25725. 19. .525. 20. .0525. 21. .0525.
22. .0525. 23. .0525. 24. .05025. 25. .30012. 26. .671875.
27. 1.90875. 28. 359.375. 29. .032224. 30. .032224.

Page 109. — 1. \$2.40. 2. \$1.57.

Page 110. — 3. \$2.11.

1. \$6.25. 2. \$1.92. 3. \$.45. 4. \$4.20. 5. \$4.50. 6. \$2.00.
7. \$2.50. 8. \$3.75. 9. \$1.15. 10. \$.60.

Page 111. — 1. \$.55. 2. \$5.00; \$2.55. 3. \$6.50. 4. \$9.10. 5. \$.65.

Page 112. — 1. \$1.95. 2. \$2.75. 3. \$1.20. 4. \$.90. 5. \$.80.
6. \$.70. 7. \$.50. 8. \$3.08. 9. \$1.50. 10. \$2.28. 11. \$1.34.
12. \$.80. 13. \$6.43. 14. \$5.32. 15. \$18.65. 16. \$6.25. 17. \$11.85.
18. \$11.04.

Page 114. — 1. \$10.25. 2. \$24.50. 3. \$29. 4. \$116.25. 5. \$138.85.
6. \$12.84.

Page 115.—1. 2.45. 2. 4.29. 3. 5.54. 4. 6.43. 5. 64.3. 6. .466.
7. .775. 8. .0652. 9. 65.2. 10. 14.01. 11. .0022. 12. .00089.
13. .00024. 14. .0002. 15. .1005.

Page 116.—1. 1.22. 2. $.58\frac{7}{8}$. 3. .56. 4. 10.2. 5. .064. 6. 1.21.
7. $.059\frac{2}{3}$. 8. .0102. 9. $7.9\frac{25}{32}$. 10. $.088\frac{6}{45}$. 11. 3.56. 12. .333. 13. .0916.
14. 8.61. 15. .899. 16. $1.666\frac{2}{3}$. 17. 1.750. 18. $.833\frac{1}{3}$. 19. $2.666\frac{2}{3}$.
20. $.857\frac{1}{7}$. 21. 1.125. 22. .134. 23. $.003\frac{3}{4}$. 24. $.000\frac{5}{7}$. 25. $.583\frac{1}{3}$.
26. $.154\frac{1}{11}$. 27. .048.

Page 117.—1. 5760. 2. 3760. 3. 3632. 4. 7020. 5. 4110. 6. 71,600.
7. 1,504,000. 8. 145,700. 9. 1,607,000. 10. 363,200. 11. 576,000.
12. 576,000. 13. 128,800. 14. 180,000. 15. 362,000.
1. 1200. 2. 6900. 3. 750. 4. $951\frac{7}{8}$. 5. 6100. 6. 780. 7. 9400.
8. 12,900. 9. $908\frac{2}{3}$. 10. 13,000. 11. 3380. 12. 42,800. 13. 9190.
14. 87,000. 15. 86,000.

Page 118.—1. 69. 2. 9.4. 3. $.908\frac{2}{3}$. 4. 1290. 5. 130,000. 6. $7.56\frac{1}{2}$.
7. 76.2. 8. 7620. 9. 872. 10. 875. 11. 14.26. 12. $1275.352\frac{1}{2}$. 13. 3.455.
14. $5833.988\frac{4}{7}$. 15. 742.7. 16. $4.693\frac{11}{16}$. 17. $17.091\frac{3}{4}$. 18. $.561\frac{1}{2}$.
19. $.858\frac{4}{5}$. 20. $6319.191\frac{5}{8}$. 21. $14.658\frac{1}{2}$. 22. $.0083\frac{3}{4}$. 23. $2.034\frac{3}{4}$.

Page 119.—1. \$5. 2. \$15. 3. \$8. 4. \$15. 5. \$25. 6. \$5.50.
7. \$10.50. 8. \$20.75. 9. \$59.60. 10. \$105.
1. 11 yd. 2. 50 hats. 3. 5 books.

Page 120.—4. 18 tons. 5. 20 rugs. 6. 40 weeks. 7. 30 weeks.
8. 60 weeks. 9. 11 weeks. 10. 60 weeks. 11. \$1.57. 12. \$241.80.
13. 184 weeks.

Page 122.—1. 63 in. 2. 77 in. 3. 14 ft. 4. 16 ft. 5. 209 in.
6. 130 in. 7. $28\frac{1}{2}$ yd. 8. 35 yd. 9. 640 rods. 10. 8800 yd.
1. 3 ft. 3 in. 2. 6 ft. 5 in. 3. 6 yd. 1 ft. 4. 18 yd. 2 ft. 5. 19 rd.
 $\frac{1}{2}$ yd. 6. 44 rd. 3 yd. 7. 2 mi. 8. 2 mi. 160 rd. 9. 2 mi.
10. 3 mi. 4160 ft.

Page 123.—1. 50 pt. 2. 74 pt. 3. 100 qt. 4. 148 qt. 5. 120 pt.
6. 192 pt. 7. 240 pt. 8. 360 pt. 9. 15 qt. 10. 21 qt. 11. 23 pt.
12. 31 pt. 13. 19 pt. 14. 37 pt. 15. 47 pt. 16. 67 pt.

Page 124.—1. 17 qt. 2. $22\frac{1}{2}$ qt. 3. 30 qt. 4. 25 gal. 5. 30 gal.
6. $43\frac{3}{4}$ gal. 7. 4 barrels. 8. 6 barrels. 9. 25 hogsheads.
10. 4 hogsheads.

Page 125.—1. 240 oz. 2. 88 oz. 3. 10,000 lb. 4. 6500 lb.
5. 575 oz. 6. 24,135 lb.
1. 3 lb. 2. 4 lb. 3. 5 lb. 4. 7 lb. 5. 2 tons. 6. 3 tons.
7. 5 tons. 8. 7 tons. 9. 3 lb. 6 oz. 10. 4 lb. 1 oz. 11. 6 lb. 4 oz.
12. 15 lb. 10 oz. 13. 1 ton 500 lb. 14. 3 tons 100 lb. 15. 1 ton, 17 cwt.,
50 lb. 16. 2 tons, 18 cwt., 75 lb.

Page 126.—1. 600 sec. 2. 445 sec. 3. 725 sec. 4. 900 min.
5. 444 min. 6. 311 min. 7. 48 hr. 8. 83 hr.

Page 127.—1. $2\frac{1}{4}$ min. 2. $3\frac{1}{2}$ min. 3. $2\frac{5}{12}$ min. 4. $4\frac{1}{8}$ hr.
5. $1\frac{2}{3}$ days. 6. $2\frac{1}{2}$ days. 7. $4\frac{1}{8}$ days. 8. $1\frac{2}{3}$ days; 2400 min.

Page 128.—1. 30 pt. 2. 150 pt. 3. 120 qt. 4. 360 qt. 5. 80 pk.
6. 140 pk. 7. 800 qt. 8. 1024 pt.

1. 9 qt. 2. 5 pk. 3. 6 bu. 4. 12 qt. 1 pt. 5. 6 pk. 2 qt.
6. 7 bu. 2 pk. 7. 2 bu. 8. 3 bu. 3 pk.

Page 129.—1. 810 in., $67\frac{1}{2}$ ft. 3. 106 ft. 4. 660 ft. 5. 10,561 ties.
6. 528 times. 7. 180 knots. 8. 21,600 shades.

Page 130.—1. 2 times, 8 times. 2. 11 gal. 1 qt. 3. 8 qt. 4. 25 pt.
5. 8 times. 6. 12 qt. 24 pt.

Page 131.—7. 240 bottles. 8. 25 times, 10 gallons remaining. 9. 1000
pint cans, 500 quart cans. 10. \$135.05. 11. \$10.90.

1. \$22.50. 2. 3550 lb. 3. 56 baskets.

Page 132.—4. 240 bags. 5. 63 trips. 6. 6350 lb. 7. \$6, \$3 profit.
1. \$13. 2. \$68.25. 3. \$3.78. 4. 48 hr.

Page 133.—5. 62 min. 6. $1\frac{1}{2}$ min. 7. 11:20 A.M. 8. 68 cts.
9. \$4.40. 10. 18 mi. 11. $121\frac{2}{3}$ hr., $57\frac{5}{2}$ da. 12. 3600 sec., 1440 min.,
168 hr., 8760 hr. 13. 88 ft.

Page 134.—1. 16 times, 64 times. 2. 50 times. 3. 6 pk., 2 qt.
4. 64 times. 5. 41 quarts. 6. No. 7. 45 quarts.

Page 135.—8. \$10,400. 9. \$9.75. 10. \$240. 11. \$15.
1. 61 ft. 2. 800 baskets. 3. 1200 cans. 4. 2375 ft.

Page 136.—5. 10 miles. 6. 3200 lb. 7. 720 hr. 8. $9\frac{1}{2}$ cents.
9. \$3.20, \$96. 10. \$3.15. 11. \$30.

Page 140.—1. 50 in. 2. 78 ft. 3. $52\frac{1}{3}$ ft. 4. 60 in. 5. 360 ft.
6. 100 yd. 7. $36\frac{4}{11}$ rods. 8. 100 ft., \$15. 9. \$112.50. 10. \$2.60.
11. 110, 80, 60, 77.

Page 141.—1. 15 sq. in. 2. 32 sq. in. 3. 30 sq. in. 4. 40 sq. in.
5. 104 sq. in. 6. 216 sq. in.

Page 142.—7. 15 sq. ft. 8. 70 sq. ft. 9. 180 sq. ft. 10. 1215 sq. ft.
11. 3150 sq. ft. 12. 18,900 sq. ft. 13. \$24. 14. \$8.67. 15. 12,000 sq. rd.,
75 A. 16. \$1260. 17. 60 bu. 18. \$84,480. 19. \$26.54. 20. \$34.75.

Page 143.—1. 380 sq. ft. 2. 242 sq. ft. 3. 10 A.

Page 144.—4. \$45.34. 5. 915 bu.

Page 145.—1. 140 cu. yd. 2. 216 cu. ft. 3. 800 cu. yd. \$480.
4. \$648.

Page 146.—5. 18 tons. 6. 7812.5 lb.

Page 148.—1. $10\frac{1}{2}$. 2. 400. 3. 543. 4. $\frac{25}{32}$, $1\frac{1}{16}$. 5. 1819.
6. 469. 7. 4 ribbons. 8. $\frac{1}{10}$ mile. 9. 12 ft. wide. 10. 112. 11. 88 ft.
12. $8442\frac{1}{2}$ mi. 13. \$65 $\frac{1}{2}$. 14. $7\frac{7}{10000}$, 7.073.

Page 149.—16. 72 hr. 17. \$.75. 18. 3 in. 19. 23 bbl.
20. 20, 40. 21. 90 mi., $46\frac{2}{3}$ mi. 22. 12 lb. corn, 28 lb. hay.

Page 150.—24. \$3 K. of C., \$3.75 Red Cross, \$11.25. 25. 112 sq. in.,
 $\frac{7}{8}$ of a square foot. 27. $\frac{1}{20}$, .05. 28. 8 hr., 360 mi. 29. \$.35. 30. \$1877.39.
31. \$.67.

Page 151.—32. 24 houses, 32 houses. 33. 257,400,000 bu. 34. 54 boxes. 35. \$20, clothing; \$20, vacation; \$8, saved. 36. 359.52 lb. 38. $206\frac{1}{4}$ miles. 39. 171,000 tons.

Page 152.—40. 20¢. 41. 10 times. 42. \$76. 43. 4.7¢, .3¢ less. 44. Mon. 91.8, Tues. 89, Wed. 89.2, Thurs. 90.4, Fri. 84.6, Total average 89.

Page 155.—1. 200, 150, 209, 200, 222, 282, 212. 2. 215, 228, 254, 202, 300, 262, 260. 3. 120, 149, 182, 195, 285, 301, 305. 4. 256, 237, 275, 308, 300, 326, 352. 5. 316, 387, 296, 400, 424, 450, 443.

Page 156.—6. 343, 450, 501, 439, 488, 528. 7. 1911, 1776, 2466, 3086, 3332, 3709. 8. 1894, 2905, 2476, 4132, 3793, 4318. 9. 24,259. 10. 24,143. 11. 32,429. 12. 33,493. 13. 342,194. 14. 400,459. 15. 332,318. 16. 356,196.

Page 157.—17. 21,943,529. 18. 10,838,533. 19. 18,460,000. 20. 29,995,570.

Page 158.—1. 29, 19, 19, 14, 18, 23, 26, 28, 19. 2. 32, 18, 18, 28, 10, 28, 29, 18, 18. 3. 40, 331, 198, 68, 201, 65, 196, 59, 68. 4. 1748, 976, 1890, 1103, 903, 1231, 2383. 5. 3701, 4872, 10,034, 21,011, 9946, 23,712. 6. 31,092, 14,671, 17,870, 25,001, 23,915, 48,655. 7. 33,127, 99,996, 65,889, 68,890, 89,702.

Page 159.—1. \$26.50. 2. \$18.26. 3. \$16.85. 4. \$7.03. 5. \$53.99. 6. \$31.99. 7. \$28.68. 8. \$46.15. 9. \$82.43. 10. \$101.90. 11. \$87.24. 12. \$137.83. 13. \$483.49. 14. \$524.51. 15. \$317.03. 16. \$154.81. 17. \$285.25. 18. \$432.64. 19. \$1099.99. 20. \$1010.03. 21. \$1390.89. 22. \$662.80. 23. \$2126.84. 24. \$349.24. 25. \$336.26. 26. \$5543.22. 27. \$334,211.89. 28. \$117,711.00. 29. \$218,533.32. 30. \$410,889.58. 31. \$1,291,646.89. 32. \$3,538,034.53. 33. \$160,646.87.

Page 161.—1. 462, 700, 1078, 1296, 918, 763, 935, 1344. 2. 936, 1045, 1308, 1060, 1883, 1668, 1400, 2280. 3. 1645, 2004, 1071, 2163, 2184, 3000, 2322, 3040. 4. 1832, 2365, 2856, 3367, 3880, 3928, 3444, 4491. 5. 1524, 2076, 2615, 2655, 3252, 3342, 3948, 3997. 6. 1869, 3150, 4529, 3930, 5836, 4711, 6183, 4816. 7. 3670, 4458, 5306, 4614, 5439, 6264, 5516, 6320.

1. 8190. 2. 14,720. 3. 23,640. 4. 28,030. 5. 32,700. 6. 90,000. 7. 399,000. 8. 632,800. 9. 342,000. 10. \$622.50. 11. \$2883. 12. \$5202. 13. 21,432. 14. 33,948. 15. 49,131. 16. 59,961. 17. \$11,010. 18. \$280,000. 19. \$1,683,000. 20. \$38,000,000.

Page 162.—21. 49,572. 22. 68,334. 23. 102,025. 24. 224,862. 25. 348,435. 26. 561,568. 27. 1,813,735. 28. 3,570,801. 29. 3,516,240. 30. 5,453,162. 31. \$67,200. 32. \$297,635. 33. \$757,596. 34. \$1,638,566. 35. \$4,520,000. 36. \$301.14. 37. \$184. 38. \$4,796,288. 39. \$637.1992. 40. \$495.1920.

Page 163.—1. 1690 lb. 2. \$78. 3. \$637.50. 4. \$269.10. 5. \$3120. 6. \$5118.75. 7. \$428.40. 8. 21,840 qt. 9. 22,500 lb. 10. \$196.

Page 164.—11. \$27.25. 12. 594 bu., 7020 bu. 13. \$101.25. 14. \$338. 15. \$9.72. 16. \$9.60. 17. \$137.28. 18. \$44.40. 19. \$13. 20. \$16. 21. \$11.65.

Page 165.—1. 22; Rem., 6. 2. 41. 3. 31. 4. 17. 5. 16. 6. 14. 7. 13. 8. 12. 9. 347. 10. 233. 11. 289. 12. 317. 13. 414.

14. 229. 15. 307. 16. 267. 17. 226. 18. 190; Rem., 8. 19. 157.
 20. 414. 21. 727. 22. 675. 23. 824. 24. 729. 25. 1765. 26. 1246.
 27. 1532. 28. 1421. 29. 1215. 30. 1877. 31. 2958. 32. 2939.
 33. 24,141. 34. 22,274. 35. 30,303. 36. 29,293. 37. 2500; Rem., 91.
 1. 631. 2. 403. 3. 2966; Rem., 76. 4. 4133. 5. 1346. 6. 2416.
 7. 845. 8. 2342. 9. 1281. 10. 1834. 11. 4446. 12. 3061. 13. 3240.
 14. 2004. 15. 6234. 16. 3107. 17. 12,076; Rem., 160. 18. 8532.
 19. 12,843; Rem., 238. 20. 21,894. 21. 17,537. 22. 14,672. 23. 16,373.
 24. 25,003. 25. 4051; Rem., 226. 26. 48,348.

- Page 166.—27. 34,567. 28. 67,853. 29. 58,203; Rem., 90. 30. 38,492.
 31. 32,685. 32. 41,856. 33. 468,527; Rem., 282. 34. 61,854. 35. 61,004.
 36. 58,194.
 1. 386; Rem., 59. 2. 778; Rem., 26. 3. 1079; Rem., 31. 4. 2471; Rem.,
 53. 5. 3071; Rem., 25. 6. 5761; Rem., 28. 7. 3155; Rem., 40.
 8. 4255; Rem., 76. 9. 1042; Rem., 58. 10. 1206; Rem., 449. 11. 3241;
 Rem., 11. 12. 2886; Rem., 264. 13. 4217; Rem., 336. 14. 6118; Rem.,
 109. 15. 5370; Rem., 530. 16. 6655; Rem., 605.
 1. \$45.00. 2. \$459.33 $\frac{1}{2}$. 3. \$4.80 a bbl. 4. 80 sacks. 5. 160
 boxes.

- Page 167.—6. \$12.50. 7. \$15.50. 8. \$3.73. 9. \$17.71 $\frac{1}{2}$.
 10. \$173.10. 11. \$45. 12. \$12.50. 13. \$35. 14. \$3. 15. 1750 tons.
 16. \$1.54. 17. \$310.

- Page 168.—18. \$125. 19. 234 $\frac{6}{15}$ bu. 20. 37 $\frac{13}{11}$ lb. 21. 24 $\frac{6}{3}$ lb.
 22. \$75. 23. 100 days. 24. 154 $\frac{3}{4}$ lengths. 25. 28 $\frac{3}{4}$ bundles. 26. \$105.60.
 27. \$4.22 $\frac{3}{4}$. 28. 43 head.

- Page 169.—29. \$31.25. 30. 1 $\frac{23}{100}$ tons. 31. \$.15. 32. 75 boxes.
 33. 8.72. 34. \$1.15 per bu. 35. 21 cows. 36. \$5. 37. 8 cents.
 38. \$.80. 39. \$.03 $\frac{1}{2}$.

- Page 170.—40. \$3.50. 41. \$9. 42. \$1.12. 43. \$1.40. 44. \$.90.
 45. \$2.20. 46. \$.45. 47. \$6.12. 48. \$.30. 49. \$.40. 50. \$1.42 $\frac{3}{4}$.

- Page 172.—1. $\frac{5}{8}$. 2. $\frac{13}{15}$. 3. $\frac{9}{20}$. 4. $\frac{11}{30}$. 5. $\frac{7}{24}$. 6. $\frac{17}{72}$. 7. $\frac{11}{12}$.
 8. $1\frac{5}{12}$. 9. $1\frac{1}{20}$. 10. $1\frac{11}{30}$. 11. $\frac{39}{80}$. 12. $\frac{31}{40}$. 13. $1\frac{11}{16}$. 14. $1\frac{13}{24}$.
 15. $2\frac{4}{5}$. 16. $\frac{9}{20}$. 17. $\frac{16}{83}$. 18. $\frac{19}{90}$. 19. $1\frac{11}{60}$. 20. $\frac{43}{72}$. 21. $\frac{5}{6}$.
 22. $1\frac{13}{16}$. 23. $1\frac{13}{10}$. 24. $2\frac{29}{40}$. 25. $4\frac{9}{42}$. 26. $1\frac{5}{12}$. 27. $1\frac{13}{18}$. 28. $\frac{4}{3}$.
 29. $\frac{53}{60}$. 30. $\frac{52}{105}$. 31. $1\frac{1}{12}$. 32. $\frac{19}{20}$. 33. $\frac{47}{60}$. 34. $\frac{7}{10}$. 35. $2\frac{1}{10}$.
 36. $1\frac{7}{15}$. 37. $1\frac{1}{4}$. 38. $1\frac{23}{40}$. 39. $1\frac{7}{20}$. 40. $1\frac{1}{15}$. 41. $2\frac{23}{32}$. 42. $1\frac{19}{54}$.
 43. $\frac{37}{44}$. 44. $1\frac{15}{78}$. 45. $1\frac{7}{44}$. 46. $\frac{5}{12}$. 47. $\frac{1}{8}$. 48. $\frac{7}{15}$. 49. $\frac{1}{10}$.
 50. $\frac{2}{3}$. 51. $\frac{7}{15}$. 52. $\frac{1}{12}$. 53. $\frac{1}{3}$. 54. $\frac{5}{14}$. 55. $\frac{5}{24}$. 56. $\frac{5}{36}$. 57. $\frac{1}{10}$.
 58. $\frac{1}{3}$. 59. $\frac{1}{2}$. 60. $\frac{3}{20}$. 61. $\frac{35}{40}$. 62. $\frac{4}{70}$. 63. $\frac{1}{30}$. 64. $\frac{2}{3}$. 65. $\frac{7}{12}$.
 66. $\frac{7}{24}$. 67. $\frac{9}{16}$. 68. $\frac{23}{60}$. 69. $\frac{7}{40}$. 70. $\frac{17}{80}$. 71. $\frac{7}{30}$. 72. $\frac{1}{48}$. 73. $\frac{47}{72}$.
 74. $\frac{1}{40}$. 75. $\frac{29}{180}$.

- Page 173.—76. $3\frac{1}{3}$. 77. $4\frac{2}{5}$. 78. $5\frac{1}{6}$. 79. $8\frac{2}{3}$. 80. $9\frac{1}{8}$. 81. $3\frac{1}{2}$.
 82. $3\frac{4}{5}$. 83. $4\frac{7}{20}$. 84. $3\frac{1}{4}$. 85. $4\frac{1}{3}$. 86. $4\frac{3}{10}$. 87. $2\frac{7}{15}$. 88. $3\frac{7}{10}$.
 89. $2\frac{4}{21}$. 90. $3\frac{11}{16}$. 91. $\frac{3}{4}$. 92. $1\frac{2}{3}$. 93. $2\frac{1}{3}$. 94. $1\frac{1}{2}$. 95. $2\frac{1}{4}$.
 96. $1\frac{1}{2}$. 97. $1\frac{1}{3}$. 98. $3\frac{1}{4}$. 99. $4\frac{2}{3}$. 100. $4\frac{1}{3}$. 101. $3\frac{1}{4}$. 102. $3\frac{1}{4}$.
 103. $3\frac{1}{4}$. 104. $5\frac{1}{4}$. 105. $10\frac{1}{4}$. 106. 8. 107. 12. 108. $28\frac{1}{3}$. 109. 40.
 110. $52\frac{1}{2}$. 111. $64\frac{1}{5}$. 112. $73\frac{1}{3}$. 113. $68\frac{1}{4}$. 114. $48\frac{2}{5}$. 115. $63\frac{3}{4}$.

| | | | | | | |
|------------------------|-------------------------|-------------------------|-------------------------|-------------------------|----------------------|----------------------|
| 116. $36\frac{2}{5}$. | 117. 24. | 118. 40. | 119. $48\frac{3}{4}$. | 120. $38\frac{3}{5}$. | 121. $\frac{1}{2}$. | 122. $\frac{1}{4}$. |
| 123. $\frac{2}{3}$. | 124. $\frac{7}{20}$. | 125. $\frac{1}{20}$. | 126. $\frac{2}{3}$. | 127. $\frac{7}{6}$. | 128. $\frac{5}{8}$. | 129. $\frac{4}{5}$. |
| 130. $\frac{7}{10}$. | 131. $\frac{1}{4}$. | 132. $\frac{6}{7}$. | 133. $\frac{5}{8}$. | 134. $\frac{1}{8}$. | 135. $\frac{2}{8}$. | 136. $\frac{3}{4}$. |
| 137. 59. | 138. $74\frac{2}{5}$. | 139. $117\frac{1}{5}$. | 140. $184\frac{1}{2}$. | 141. 519. | 142. 227. | |
| 143. 333. | 144. $152\frac{1}{2}$. | 145. $280\frac{1}{2}$. | 146. $520\frac{1}{2}$. | 147. $811\frac{1}{2}$. | | |

| | | | | | | |
|---|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|------------------------|
| Page 174. — 148. 35. 149. 77. 150. 175. 151. 434. 152. $129\frac{4}{5}$. | | | | | | |
| 153. 70. | 154. 117. | 155. $101\frac{1}{2}$. | 156. 84. | 157. $93\frac{1}{3}$. | 158. $207\frac{1}{2}$. | |
| 159. $333\frac{1}{5}$. | 160. $643\frac{1}{2}$. | 161. 86. | 162. $612\frac{1}{2}$. | 163. $7\frac{1}{2}$. | 164. $\frac{1}{4}$. | |
| 165. $13\frac{3}{5}$. | 166. $171\frac{1}{5}$. | 167. $22\frac{1}{7}$. | 168. $34\frac{1}{2}$. | 169. $62\frac{1}{2}$. | 170. 85. | |
| 171. 182. | 172. $493\frac{2}{5}$. | 173. $23\frac{2}{5}$. | 174. $13\frac{2}{4}$. | 175. 16. | 176. $42\frac{1}{2}$. | |
| 177. $107\frac{1}{2}$. | 178. $1\frac{2}{15}$. | 179. $\frac{3}{15}$. | 180. $\frac{2}{25}$. | 181. $\frac{5}{81}$. | 182. $\frac{1}{15}$. | 183. $\frac{1}{100}$. |
| 184. $1\frac{2}{25}$. | 185. $\frac{4}{35}$. | 186. $1\frac{1}{2}$. | 187. $1\frac{3}{55}$. | 188. $\frac{4}{55}$. | 189. $2\frac{1}{3}$. | 190. $1\frac{9}{11}$. |
| 190. $\frac{2}{7}$. | 191. $\frac{5}{8}$. | 192. $\frac{8}{30}$. | 193. $2\frac{3}{10}$. | 194. $2\frac{5}{22}$. | 195. $1\frac{4}{5}$. | 196. $1\frac{1}{5}$. |
| 196. $1\frac{1}{5}$. | 197. $2\frac{1}{5}$. | 198. $6\frac{1}{8}$. | 199. $6\frac{1}{16}$. | 200. $6\frac{1}{100}$. | 201. $7\frac{2}{3}$. | 202. $8\frac{7}{11}$. |
| 202. $8\frac{7}{11}$. | 203. $6\frac{2}{5}$. | 204. $8\frac{1}{2}$. | 205. $11\frac{1}{2}$. | 206. $13\frac{1}{2}$. | 207. $73\frac{1}{2}$. | 208. $2\frac{1}{2}$. |
| 208. $2\frac{1}{2}$. | 209. $3\frac{7}{15}$. | 210. $4\frac{1}{8}$. | 211. $9\frac{1}{8}$. | 212. $9\frac{3}{8}$. | 213. $1\frac{5}{11}$. | 214. $3\frac{1}{4}$. |
| 214. $3\frac{1}{4}$. | 215. $2\frac{3}{8}$. | 216. $1\frac{1}{2}$. | 217. $4\frac{1}{2}$. | 218. $4\frac{3}{8}$. | 219. $4\frac{1}{2}$. | 220. $4\frac{1}{2}$. |
| 221. $4\frac{3}{8}$. | 222. $3\frac{1}{8}$. | | | | | |

Page 175. — 1. $20\frac{1}{4}$ gal. 2. \$5.02. 3. \$2.85. 4. \$5.14.

Page 176. — 5. \$9.35. 6. \$2.49. 7. \$29.64. 8. \$1.91 $\frac{1}{2}$ a bu., \$14,515.70.
9. At close of deal, \$10,000 Mr. Johnson's share, \$5,000 Mr. Brown's share.
10. \$2410. 11. \$68.75. 12. $7\frac{1}{2}$ A. 13. $17\frac{1}{8}$ bu.

Page 177. — 14. $4\frac{3}{8}$ tons. 15. $97\frac{7}{8}$ hrs. 16. $\frac{7}{300}$. 17. \$24.33. 18. Live weight, \$2.34. 19. \$1548.90.

Page 178. — 20. $6\frac{1}{4}$ bu., $1\frac{1}{9}$. 21. \$4.00. 22. 104 cupsful. 23. $257\frac{1}{4}$ loaves.
24. \$6.12. 25. \$13.20. 26. \$2.00. 27. $\frac{16}{91}$, $\frac{12}{91}$, $\frac{9}{13}$.

Page 179. — 28. $\frac{1}{4}$ rent, $\frac{2}{3}$ service, $\frac{7}{8}$ light, $\frac{3}{4}$ food. 29. \$8.25 saves, \$429, $\frac{11}{10}$. 30. More. 31. \$88. 32. \$94, \$82.

Page 180. — 33. \$5. 34. \$5. 35. \$3.93. 36. \$30.36. 37. \$.83.
38. $264\frac{1}{2}$ A., $\frac{5}{8}$. 39. $11\frac{1}{2}$ bu. 40. \$33. 41. 576 rods. 42. $103\frac{3}{4}$ qts. 43. 39 T., $\frac{4}{5}$ T.

Page 181. — 44. $3\frac{2}{5}$ hrs., 8 gal. 45. \$3750, \$11,250. 46. \$3500.
47. $137\frac{3}{8}$ miles apart. 48. \$6.69. 49. \$125 tuition, \$200 board, \$62.50 clothing, \$112.50 incidentals. 50. \$900,000, A's \$240,000, B's \$90,000 at close.

Page 185. — 1. \$83.40. 2. \$104.50. 3. \$209.50. 4. \$253.20.
5. \$167.50. 6. \$417.30. 7. \$422.06. 8. \$500.

Page 189. — 1. .14, .29, .46, .69, .125. 2. .27, .027, .207, .109, .201. 3. .61, .079, .0011, .0276, .0201. 4. .019, .109, .0109, .0011, .0077. 5. .211, .0211, .2011, .4004. 6. .1137, .3711, .02509. 7. .359. 8. .0108. 9. .00009, 10. 12.12. 11. 351.17. 12. 19.0019. 13. 243.019. 14. 7008.00011, 15. 7000.007. 16. 3500.35. 17. 10,000.010.

Page 190. — 1. 3, .5, .6, .7, .8, .9. 2. .03, .05, .06, .07, .08, .09. 3. .19, .23, .27, .37, .51, .63, .77. 4. .127, .359, .0127, .0579, .1551, 3.1177. 5. 1.3, 1.9, 2.3, 3.1, 4.7, 5.3. 6. 1.21, 1.35, 2.31, 3.51, 4.79, 6.25. 7. 1.225, 2.557, 3.775, 5.7, 7.59. 8. 1.2525, 4.558, 12.5, 37.54.

Page 191.—1. .30, .19. 2. .50, .46. 3. .70, .08. 4. .800, .018.
5. .100, .003. 6. .700, .097. 7. .150, .271. 8. .340, .207. 9. .620, .408.
10. .7500, .0011. 11. .1200, .0020, .0014. 12. .3500, .0710, .0104.
13. .45000, .10070, .05411. 14. .00400, .11000, .00007. 15. .80000,
.00800, .00008.

1. $\frac{10}{20}, \frac{5}{10}, \frac{1}{2}, \frac{3}{6}, \frac{9}{18}, \frac{25}{50}, \frac{25}{50}, \frac{11}{22}, \frac{9}{18}, \frac{2}{5}$. 2. $\frac{7}{10}, \frac{4}{5}, \frac{1}{5}, \frac{4}{5}, \frac{25}{50}, \frac{25}{50}, \frac{50}{100}, \frac{20}{40}, \frac{14}{28}, \frac{17}{34}$.
3. $\frac{3}{20}, \frac{1}{10}, \frac{1}{4}, \frac{1}{20}, \frac{27}{100}, \frac{14}{100}, \frac{125}{1000}, \frac{20}{100}, \frac{9}{200}, \frac{1}{200}$. 4. $\frac{1}{4}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}$.
5. $\frac{1}{10}, \frac{1}{40}, \frac{1}{40}, \frac{1}{8}, \frac{2}{40}, \frac{2}{40}, \frac{1}{5}, \frac{7}{8}$. 6. $25\frac{1}{20}, 30\frac{3}{25}$,
 $36\frac{1}{4}, 45\frac{3}{25}, 55\frac{11}{20}, 75\frac{3}{8}, 80\frac{3}{8}, 90\frac{3}{5}$.

Page 192.—1. 18.71. 2. 19.36. 3. 28.479. 4. 16.3229. 5. 100.

Page 193.—6. 148.131. 7. 105.140. 8. 63.9. 9. 83.46933. 10. 5000.
11. 89.514. 12. 35.69402. 13. 174.6764. 14. 168.96531. 15. 4477.1151.
16. 7070.13205. 17. \$1576.31. 18. \$1738.89. 19. \$1226.56. 20. \$1631.81.
21. \$760.04.

Page 194.—1. 410.4 miles. 2. 6417.86 ft. 3. 589.076 A. 4. \$10,137.65.
5. 754.75 rods. 6. 1565 lbs. 7. \$166.84.

Page 195.—8. \$71.69. 9. \$463.08.
1. 9.19. 2. 8.87. 3. 16.148. 4. 23.907. 5. 19.313. 6. 55.135.
7. 41.915. 8. 109.785. 9. 61.709. 10. 83.03319.

Page 196.—11. 6.88, 41.9284, 41.99284. 12. 54.081, 101.1669, 166.9968.
13. 62.996, 146.06, 121.38981. 14. 61.501, 32.0321, 299.999. 15. 94.901,
32.0001, 1.0001. 16. 149.047885, .9999, 575.0391.
1. \$5.81. 2. 145.75 A. 3. 146.9 A. 4. \$1.55. 5. \$1201.38. 6. \$727.62.

Page 197.—7. \$306.46.

Page 198.—1. 312.5. 2. 31.25. 3. 3.125. 4. 3.125. 5. .03125.
6. .0003125. 7. 5.7558. 8. .4403. 9. .019133. 10. .15105.
11. 84.193009. 12. .071437041. 13. 1.905. 14. 3.264. 15. 4.929.
16. 15.675. 17. 22.165. 18. 37.098. 19. 796.08. 20. 101.436.
21. 196.174. 22. 35.0343. 23. 482.769. 24. 10.7991.

Page 199.—25. 33.12. 26. 35.394. 27. 10.8319. 28. .57612.
29. 1.21676. 30. 1.56288. 31. 24.15644. 32. 1.42434. 33. 346.3684.
34. 1308.104. 35. 95.8568. 36. .67272. 37. 1231.074. 38. .832624.
39. .434418. 40. 1803.88056. 41. 4033.12804. 42. 129.18404.
43. 267.231144. 44. 5093.3393. 45. 1768.35648. 46. 469.422512.
47. 1.8739784. 48. 21629.03028. 49. .02055015. 50. 2417.478625.
1. \$14.53. 2. \$464.46. 3. \$9429.38. 4. 204.1875 tons. 5. \$937.50.
6. \$11739.38.

Page 200.—7. \$46.97 per acre. 8. 106.81406 inches. 9. 441.5125
sq. yds. 10. \$679.38. 11. \$10,568.75. 12. \$4200 eldest, \$2400 second,
\$2700 each remaining. 13. \$637.50 rent, \$510.00 traveling, \$765.00
advertising, \$2337.50 salaries. 14. \$406.25.

Page 204.—1. 32. 2. 58. 3. 2.7. 4. 121. 5. 1210. 6. 121000.
7. 121. 8. 12.1. 9. 121. 10. 12100. 11. 20.3. 12. 20.3. 13. .00203.
14. 2.03. 15. 1690. 16. 1690. 17. 416900000. 18. 1720.3.
19. .17203. 20. .4003. 21. 400.3. 22. .000207. 23. 2500. 24. 1.75.
25. 23. 26. .068574+. 27. 122. 28. 200. 29. 740. 30. 1898.993+.
31. 21,447,400. 32. 1012. 33. 160161.6. 34. .00656. 35. .0000104.

Page 205. — 36. .882+. 37. 882.352+. 38. 4.523+. 39. 4.56.
 40. 308. 41. 953.846+. 42. .207+. 43. 49545.454+. 44. 11428.571+.
 45. 20.930+. 46. 132.743+. 47. 6944.444+. 48. 1.226+. 49. .0000122652+.
 50. 208.067+. 51. \$30. 52. \$26. 53. \$12.50. 54. \$250. 55. \$1.67.
 56. \$600. 57. 400. 58. 2500. 59. 303.0303+. 60. 59. 61. 7.01.
 62. 701. 63. 7010. 64. 70.1. 65. 1.4+. 66. 25.
 1. 600 turns. 2. 68.76. 3. $27\frac{2}{3}$ days. 4. 54 tons.

Page 206. — 5. 20.5486+ pounds. 6. 237.6 ft. 7. 124.24 ft. 8. $144\frac{3}{13}$ cords. 9. \$2.88. 10. 2346.12 gal. 11. \$32.26. 12. \$.44, \$.23 $\frac{1}{2}$. 13. \$24.93.

Page 207. — 1. .6, .8, .5, .625, .875, .8. 2. .5, .5, .25, .25, .8, .2.
 3. .3, .6, .5, .25, .4, $18\frac{1}{8}$. 4. .25, $.53\frac{1}{3}$, .45, .3, .25, $.17\frac{1}{2}$. 5. .03, .05, .05, $.03\frac{1}{2}$, .04, .05.
 6. $.714\frac{2}{3}$, $.857\frac{1}{2}$, $.777\frac{7}{9}$, $.727\frac{3}{11}$, $.937\frac{1}{2}$, $.913\frac{1}{3}$, $.897\frac{7}{9}$.
 7. $.418\frac{2}{3}$, $.777\frac{7}{9}$, $.833\frac{1}{3}$, $.923\frac{1}{13}$, $.882\frac{6}{17}$, $.989\frac{8}{23}$, $.989\frac{8}{23}$. 8. .13, $.471\frac{1}{12}$, $.806\frac{2}{3}$, $.692\frac{3}{11}$, $.981\frac{8}{9}$, .877.

Page 209. — 1. 31,000. 2. 43,000. 3. 70,600. 4. 131,400.
 5. 170,600. 6. 253,700. 7. 261,800. 8. 480,000. 9. 451,400.
 10. 1,031,000. 11. 1,241,000. 12. 1,464,200. 13. 200,000. 14. 229,575.
 15. 733,000. 16. 3,710,000. 17. 3,990,000. 18. 4,270,000.

Page 210. — 1. 9.2. 2. 6. 3. 1.14. 4. 3.49. 5. 11.7. 6. .29.
 7. 17.914. 8. 33. 9. $3.3223-$.

Page 211. — 10. 6.555+. 11. .135. 12. 20.85.

1. 24.328. 2. 76.144. 3. 126.922. 4. 308.66. 5. 1397.78. 6. 2960.

Page 212. — 1. \$808.47. 2. 250 pounds. 3. \$153. 4. \$117.
 5. 95.475 tons. 6. 465 A., 775 A., \$83,700, \$139,500. 7. \$45.80.
 8. \$331.40.

Page 213. — 9. \$62. 10. Contractor owed \$63.88. 11. 140 tons.
 12. \$196.65. 13. \$75.15. 14. \$60.75, \$297.

Page 214. — 15. \$103.85. 16. \$2094.40. 17. 808.1 lb. 18. More, \$743.70. 19. \$84.32. 20. \$308.54.

Page 215. — 21. \$1.40. 22. No. 23. \$52. 24. \$11.20.

Page 219. — 1. 63 in. 2. 88 in. 3. 125 in. 4. 160 in. 5. 289 in.
 6. 460 in. 7. 551 in. 8. 16 ft. 9. 23 ft. 10. 37 ft. 11. 74 ft.
 12. 168 ft. 13. 205 ft. 14. 405 ft. 15. 96 in. 16. 156 in. 17. 240 in.
 18. 384 in. 19. 444 in. 20. 768 in. 21. 996 in. 22. 22 yd. 23. 66 yd.
 24. 57 yd. 25. $86\frac{1}{2}$ yd. 26. 133 yd. 27. $175\frac{1}{2}$ yd. 28. $86\frac{1}{2}$ ft. 29. 73 ft.
 30. 206 ft. 31. $253\frac{1}{2}$ ft. 32. $354\frac{1}{2}$ ft. 33. 540 ft. 34. 3620 yd.
 35. 9000 yd. 36. 7190 yd. 37. $5428\frac{1}{2}$ yd. 38. 9350 yd. 39. 18,128 yd.

Page 220. — 1. 9 in. 2. 1 ft. 6 in. 3. 1 ft. 7.44 in. 4. 2 ft. 3 in.
 5. 2 ft. 9.12 in. 6. 3 ft. 5.4 in. 7. $4\frac{1}{2}$ in. 8. 1 ft. 1.5 in. 9. 1 ft. 10.5 in.
 10. 2 ft. 7.5 in. 11. 8 ft. 3 in. 12. 16 ft. 6 in. 13. 1 yd. 1 ft. $1\frac{1}{2}$ in.
 14. 2 ft. .75 in. 15. 1 yd. 8.55 in. 16. 1 yd. 1 ft. 6.45 in. 17. 2 yd. 2.25 in.
 18. 3 yd. 1 ft. 3.75 in. 19. 480 rd. 20. 400 rd. 21. 40 rd.
 22. 120 rd. 23. 200 rd. 24. 312 rd.

Page 222. — 1. 2 yd. 2 ft. 4 in. 2. 3 yd. 1 ft. 8 in. 3. 3 yd. 2 ft. 8 in.
 4. 4 yd. 1 ft. 1 in. 5. 4 yd. 1 ft. 9 in. 6. 5 yd. 4 in. 7. 5 yd. 1 ft. 1 in.
 8. 2 yd. 1 ft. 9. 4 yd. 1 ft. 10. 1 rd. 1 yd. 6 in. 11. 1 rd. 5 yd. 6 in.

12. 2 rd. 4 yd. 2 ft. 13. 3 rd. 4 yd. $1\frac{1}{2}$ ft. 14. 4 rd. 4 yd. 15. 2 rd. 4 yd.
 16. 3 rd. 2 yd. $1\frac{1}{2}$ ft. 17. 5 rd. 1 yd. $1\frac{1}{2}$ ft. 18. 6 rd. 4 yd. 19. 10 rd.
 20. 11 rd. 3 yd. $1\frac{1}{2}$ ft. 21. 14 rd. 1 yd. 22. 1 mi. 65 rd. 23. 1 mi. 130 rd.
 24. 1 mi. 170 rd. 25. 1 mi. 192 rd. 26. 2 mi. 1 rd. 27. 2 mi. 20 rd.
 28. 2 mi. 95 rd. 29. 19 yd. 2 ft. 30. 24 yd. 1 ft. 31. 133 yd. 1 ft.
 32. 250 yd. 33. 2 yd. 2 ft. 1 in. 34. 3 yd. 11 in. 35. 1 yd. 2 ft. 11 in.
 36. 9 yd. 2 ft. 10 in. 37. 4 rods. 38. 8 rd. 1 yd. 39. 8 rods. 40. 12 rods.
 41. 12 rods. 42. 10 rd. 43. 31 rd. 4 yd. 1 ft. 6 in. 44. 2 rd. 4 yd.

Page 223. — 41. 1 mi. 142 rd. 1 ft. 42. 113 ft. 43. 4 rd. 5 yd.
 2 ft. 4 in. 44. 3294.72 ft. 45. 17,695 ft. 46. 63,560 in. 47. 8.8 yd.
 48. 950 yd. 1 ft. 2.4 in. 49. $\frac{1}{4}$ mi. 50. 3309 in.

- Page 226. — 1. 1440 sq. in. 2. 3456 sq. in. 3. 4320 sq. in. 4. 6480 sq. in.
 5. 8640 sq. in. 6. 90 sq. ft. 7. 135 sq. ft. 8. 216 sq. ft. 9. $45\frac{3}{4}$ sq. yd.
 10. 726 sq. yd. 11. $1058\frac{3}{4}$ sq. yd. 12. 800 sq. rd. 13. 1600 sq. rd.
 14. 2400 sq. rd. 15. 6720 sq. rd. 16. 9600 sq. rd. 17. 1920 A.
 18. 5120 A. 19. 6400 A. 20. 15,360 A. 21. 70,400 A. 22. 1480 sq. in.
 23. 2190 sq. in. 24. 2934 sq. in. 25. 4410 sq. in. 26. 6620 sq. in.
 27. 7803 sq. in. 28. 86 sq. ft. 29. 115 sq. ft. 30. 222 sq. ft.
 31. 316 sq. ft. 32. 436 sq. ft. 33. 588 sq. ft. 34. 360 sq. in.
 35. 705.6 sq. in. 36. 756 sq. in. 37. 388.8 sq. in. 38. 5832 sq. in.
 39. 16588.8 sq. in.

- Page 227. — 40. 305 sq. in., 591 sq. in. 41. 1962 sq. in., 2235 sq. in.
 42. 3000 sq. in., 2724 sq. in. 43. $2\frac{1}{8}$ sq. ft. 44. $3\frac{3}{8}$ sq. ft. 45. $5\frac{1}{2}$ sq. ft.
 46. 108 sq. ft. 47. 135 sq. ft. 48. 274.5 sq. ft. 49. 1089 sq. ft.
 50. 952.875 sq. ft. 51. 3294.225 sq. ft. 52. $423\frac{1}{2}$ sq. yd. 53. $1361\frac{1}{4}$ sq. yd.
 54. 3781 $\frac{1}{2}$ sq. yd. 55. $1\frac{251}{128}$ sq. yd. 56. $60\frac{1}{4}$ sq. yd. 57. $100\frac{1}{2}$ sq. yd.
 58. 19,360 sq. yd. 59. 12,100 sq. yd. 60. 1331 sq. yd. 61. $\frac{400}{1089}$ sq. rd.
 62. $8\frac{288}{121}$ sq. rd. 63. $23\frac{953}{1783}$ sq. rd. 64. $2\frac{58}{121}$ sq. rd. 65. $11\frac{29}{21}$ sq. rd.
 66. $2\frac{39}{121}$ sq. rd. 67. 1280 sq. rd. 68. 360 sq. rd. 69. 580 sq. rd.
 70. $4\frac{1}{2}$ A. 71. $7\frac{1}{2}$ A. 72. $6\frac{3}{121}$ A. 73. 1520 A. 74. $5\frac{1}{2}$ sq. mi.
 75. $8\frac{1}{8}$ sq. mi. 76. 50 sq. mi. 77. 75 sq. mi.

- Page 230. — 1. 150 sq. in. 2. 216 sq. in. 3. 280 sq. in. 4. 540 sq. in.
 5. 1056 sq. in. 6. 20 sq. ft. 7. 30 sq. ft. 8. 247 sq. ft. 9. 180 sq. yd.
 10. 432 sq. yd. 11. 135 sq. ft. 12. 300 sq. ft. 13. 216 sq. ft. 14. 3630 sq. yd.
 15. 6655 sq. yd. 16. $12\frac{1}{2}$ sq. ft. 17. $8\frac{1}{8}$ sq. ft. 18. $141\frac{3}{8}$ sq. ft.
 19. $394\frac{5}{8}$ sq. ft. 20. $584\frac{1}{2}$ sq. ft. 21. $20\frac{7}{8}$ sq. yd. 22. $47\frac{3}{8}$ sq. yd.
 23. 4065.5 sq. yd. 24. 6536 sq. yd. 25. $6515\frac{3}{8}$ sq. rd. 26. 385 sq. ft.
 27. 108 sq. yd. 28. $82\frac{3}{8}$ sq. rd. 29. $1134\frac{3}{8}$ sq. yd. 30. 21 A. 31. 20 ft.
 wide.

Page 231. — 32. 40 ft. 33. 100 ft., 8 lots, 7250 sq. ft. 34. 6750 sq. ft.,
 8316 sq. ft., 15,066 sq. ft. 35. \$94.40. 36. 585 sods.

Page 232. — 37. \$19.01. 38. \$70.40. 39. \$3850.

- Page 236. — 1. 120 sq. in. 2. 252 sq. in. 3. 702 sq. in. 4. 518 sq. in.
 5. 892.5 sq. in. 6. 168 sq. ft. 7. 342 sq. ft. 8. 744 sq. ft. 9. 940.5 sq. ft.
 10. 323 sq. yd. 11. 28.125 sq. ft. 12. 22.5 sq. ft. 13. $2\frac{3}{4}$ sq. rd.
 14. $145\frac{5}{8}$ sq. rd. 15. $555\frac{5}{8}$ sq. yd. 16. $5\frac{1}{8}$ sq. ft. 17. $17\frac{5}{8}$ sq. ft.
 18. $12\frac{3}{8}$ sq. ft. 19. $40\frac{1}{8}$ sq. ft. 20. $38\frac{3}{8}$ sq. ft. 21. $42\frac{1}{8}$ sq. ft.
 22. $19\frac{9}{144}$ sq. ft. 23. $42\frac{5}{144}$ sq. ft. 24. $57\frac{5}{8}$ sq. yd. 25. $177\frac{3}{8}$ sq. yd.
 26. 84 ft. 27. 40 ft. 28. 16 ft. 29. $2343\frac{25}{8}$ sq. ft. 30. 2 ft.
 31. $241\frac{1}{24}$ sq. ft. 32. 28 ft.

Page 237.—33. $3\frac{3}{5}$ sq. in. 34. 9 ft. 35. 6 sq. ft. 36. 1082.5 sq. in.
37. 384 sq. ft. 38. 1400 sq. ft. 39. $5\frac{1}{3}$ sq. yd. 40. 189,000 sq. ft.

Page 238.—41. 360 rd. 42. 89 ft. 43. 6048.144 sq. in., 42.001 sq. ft.
44. 1920 tiles. 45. \$24.18. 46. 300 ft., 44,720 sq. ft.

Page 240.—1. 135 sq. in. 2. 400 sq. in. 3. 864 sq. in. 4. 300 sq. ft.
5. 476 sq. ft. 6. 546 sq. yd. 7. 700 sq. rd. 8. 1260 sq. rd. 9. 1674 sq. rd.
10. $681\frac{3}{11}$ sq. rd. 11. $3\frac{9}{16}$ sq. ft. 12. $43\frac{1}{8}$ sq. ft. 13. $46\frac{1}{8}$ sq. ft.
14. $15\frac{3}{5}$ sq. yd. 15. $19\frac{3}{5}$ sq. yd. 16. $67\frac{3}{11}$ sq. rd.

Page 241.—17. $164\frac{3}{121}$ sq. rd. 18. $36\frac{2}{3}$ sq. rd. 19. $33\frac{905}{1089}$ sq. rd.
20. $269\frac{421}{1089}$ sq. rd. 21. 30 ft. 22. 5 yd. 23. 17,500 sq. ft. 24. No
difference. 25. $3\frac{3}{5}$ rd. 26. 60 ft. 27. 3000 sq. ft., 1400 sq. ft.

Page 243.—1. 5184 cu. in. 2. 20,736 cu. in. 3. 31,104 cu. in.
4. 46,656 cu. in. 5. 51,840 cu. in. 6. 60,480 cu. in. 7. 115,200 cu. in.
8. 73,440 cu. in. 9. 28,800 cu. in. 10. 93,888 cu. in. 11. 540 cu. ft.
12. 729 cu. ft. 13. 945 cu. ft. 14. 3375 cu. ft. 15. 3600 cu. ft.
16. 14,688 cu. in. 17. 5443.2 cu. in. 18. 114.75 cu. ft. 19. 422.55 cu. ft.
20. 1235.25 cu. ft. 21. 1.5 cu. ft. 22. $2\frac{1}{12}$ cu. ft. 23. 4 cu. ft. 24. $1\frac{1}{4}$ cu. ft.
25. $2\frac{1}{2}$ cu. ft. 26. 6 cu. ft. 27. 9 cu. yd. 28. 28 cu. yd. 29. 46 cu. yd.
30. $1.67\frac{1}{3}$ cu. yd. 31. $15.9\frac{2}{3}$ cu. yd. 32. $102.23\frac{4}{7}$ cu. yd.

Page 244.—1. 1800 cu. in. 2. 3200 cu. in. 3. 10,500 cu. in.
4. 1824 cu. in. 5. 8640 cu. in. 6. 16,632 cu. in. 7. 24,948 cu. ft.
8. 34,410 cu. ft. 9. $128\frac{5}{8}$ cu. ft. 10. $7661\frac{1}{2}$ cu. ft. 11. $544\frac{1}{2}$ cu. ft.
12. $9\frac{1}{3}$ cu. yd.

Page 245.—13. 2520 cu. in., $1\frac{1}{4}$ cu. ft. 14. 66 cu. ft. 15. 36 cu. ft.
16. 19,440 cu. in. 17. 6 cu. ft. 18. 3.969+ cu. ft.

Page 246.—1. 96 gi. 2. 152 gi. 3. 216 gi. 4. 60 pt. 5. 90 pt.
6. 140 pt. 7. 109 pt. 8. 165 pt. 9. 200 pt. 10. 216 qt. 11. 248 qt.
12. 290 qt. 13. 10 pt. 14. 20 qt. 15. 28 qt. 16. $14\frac{1}{2}$ pt. 17. 42 qt.
18. $15\frac{1}{2}$ gal. 19. 60 gal. 20. 80 gal. 21. $113\frac{1}{2}$ gal. 22. $31\frac{1}{8}$ gal.
23. $35\frac{5}{8}$ gal. 24. $3\frac{1}{2}$ gal.

Page 247.—25. $14\frac{1}{2}$ pt. 26. $16\frac{1}{2}$ pt. 27. 22 pt. 28. 72 pt.
29. 178 pt. 30. 281 pt. 31. $313\frac{1}{2}$ pt. 32. 228 pt. 33. 516 pt. 34. 840 pt.
35. 39 pt. 36. 60 pt. 37. 648 pt. 38. 1924 pt. 39. 2838 pt. 40. 124 pt.
41. 338 pt. 42. 1250 pt. 43. 1932 pt. 44. 2810 pt. 45. 304 bottles.
46. 34 gal. 47. 935 gal. 48. 2 times. 49. 10,240 bottles. 50. 67.32 gal.
51. 15.147+ gal. 52. \$9.

Page 248.—1. 48 pt. 2. 70 pt. 3. 92 pt. 4. 182 pt. 5. 25 pt.
6. 63 pt. 7. 85 pt. 8. 257 pt. 9. 432 qt. 10. 224 qt. 11. 582 qt.
12. 884 qt. 13. 128 pk. 14. 181 pk. 15. 706 pk. 16. 859 pk. 17. 27 qt.
18. 43 qt. 19. 46 qt. 20. 52 qt. 21. 7 pk. 22. $7\frac{1}{3}$ pk. 23. $9\frac{2}{3}$ pk.
24. $14\frac{3}{4}$ bu. 25. 25 bu. 26. $36\frac{1}{4}$ bu. 27. $60\frac{1}{4}$ bu. 28. $85\frac{1}{2}$ bu. 29. 112 pk.
30. 184 pk. 31. 224 pk. 32. 316 pk. 33. 448 pk. 34. 618 pk. 35. 686 pk.
36. 203 pk. 37. $42\frac{3}{4}$ bu. 38. $58\frac{1}{2}$ bu. 39. 88 bu. 40. 4 bu. 41. $2\frac{7}{15}$ bu.
42. 6 bu. 43. 3.6 bu. 44. $.4031\frac{1}{4}$ bu.

Page 249.—45. 67.2006+ cu. in. 46. \$5.20. 48. 123 boxes. 49. 157.49+ bu.
51. $4\frac{2}{3}$. 52. $3\frac{3}{5}$ bu. 53. \$42.31. 54. \$.20.

Page 250.—1. 240 oz. 2. 384 oz. 3. 560 oz. 4. 170 oz. 5. 570 lb.
 6. 240 lb. 7. 7050 lb. 8. 11,030 lb. 9. 14,065 lb. 10. $12\frac{1}{2}$ lb. 11. $31\frac{1}{4}$ lb.
 12. $112\frac{1}{2}$ lb. 13. 2.95 cwt. 14. 3.67 cwt. 15. 6.74 cwt. 16. 35.4 cwt.
 17. 87.54 cwt. 18. 75.95 cwt. 19. 542.4 cwt. 20. 671.85 cwt.
 21. 876.5 cwt. 22. 240 oz. 23. 512 oz. 24. 864 oz. 25. 1040 oz.
 26. 1296 oz. 27. 1600 oz. 28. $9\frac{3}{8}$ lb. 29. 21 lb. 30. $361\frac{1}{8}$ lb.
 31. 1254 lb. 32. 3570 lb. 33. 2300 lb. 34. 550 lb. 35. 3 T. 7 cwt. 50 lb.
 36. 12 cwt. 50 lb. 37. 32.4375 T.

Page 251.—1. 3000 lb. 2. 5600 lb. 3. 6000 lb. 4. 3840 lb.
 5. 16,500 lb. 6. 12,000 lb. 7. 19,040 lb. 8. 16,000 lb. 9. 21,600 lb.
 10. 22,080 lb. 11. 19,600 lb. 12. 5500 lb. 13. 7680 lb. 14. 210,000 lb.

Page 252.—15. 30 tons. 16. 36 tons. 17. 48 tons. 18. $33\frac{1}{3}$ bu.
 19. $35\frac{1}{2}$ bu. 20. $62\frac{1}{2}$ bu. 21. $33\frac{1}{2}$ bu. 22. $41\frac{1}{2}$ bu. 23. $41\frac{1}{2}$ bu.
 24. $1\frac{1}{2}$ tons. 25. \$7020. 26. \$110.05.
 1. 980 lb. 2. 900 lb. 3. 6000 lb. 4. 4200 lb.

Page 253.—5. 3200 lb. 6. 4800 lb. 7. 17,500 lb. 8. 12,600 lb.
 9. 1875 lb. 10. $4427\frac{1}{2}$ lb. 11. 1462.5 lb. 12. 2756 lb. 13. $10\frac{1}{8}$ tons,
 \$126.56. 14. \$175. 15. $5\frac{2}{3}\frac{1}{2}$ tons. 16. \$190.08, \$.0495.

Page 254.—1. 14,400 sec. 2. 19,200 sec. 3. 6000 sec. 4. 900 min.
 5. 1440 min. 6. 2160 min. 7. 360 hr. 8. 960 hr. 9. 182 days.
 10. 392 days. 11. 108 mo. 12. 324 mo.

Page 255.—13. 4 min. 14. 7 min. 15. 5 hr. 16. 7 hr.
 17. $8\frac{3}{4}$ days. 18. $13\frac{1}{4}$ days. 19. $18\frac{3}{4}$ days. 20. $5\frac{1}{2}$ yr. 21. $18\frac{1}{2}$ wk.
 22. $11\frac{1}{2}$ yr. 23. $108\frac{1}{2}$ wk. 24. $142\frac{1}{2}$ wk. 25. 730 min. 26. 605 hr.
 27. 1899 days. 28. 56 days. 29. 57 days. 30. 104 days. 31. 75 days.
 32. 274 days. 33. 126 days. 34. 102 days. 35. 123 days. 36. 242 days.
 37. 328 days. 38. 224 days. 39. 302 days. 40. 268 days. 41. 232 days.
 42. .125. 43. .0476+. 44. 1440 min., 86,400 sec. 45. Wednesday. 46. $\frac{19221}{17280}$.

Page 257.—1. 1440 min. 2. 2100 min. 3. 3360 min. 4. 4500 min.
 5. 5160 min. 6. 5640 min. 7. 9'. 8. 13'. 9. 14'. 10. 630 min.
 11. 1455 min. 12. 4520 min. 13. 610 min. 14. 1455 min. 15. 4356 min.
 16. 615 sec. 17. 1460 sec. 18. 3251 sec. 19. 920'. 20. 2135'.
 21. 2439'. 22. 36,920''. 23. 163,250''. 24. 244,690''. 25. 127,215''.
 26. 164,745''. 27. 215,999''. 28. 270°. 29. 225°. 30. 450°.

Page 259.—1. 12 %. 2. 31 %. 3. 48 %. 4. 35 %. 5. 87 %.
 6. 99 %. 7. 75 %. 8. $62\frac{1}{2}$ %. 9. $41\frac{3}{4}$ %. 10. $87\frac{1}{2}$ %. 11. 90 %.
 12. $86\frac{3}{4}$ %. 13. $85\frac{1}{4}$ %. 14. $77\frac{1}{2}$ %. 15. 95 %.

Page 261.—1. 12. 2. 18. 3. 56.25. 4. 102. 5. 58.5. 6. 93.78.
 7. 147. 8. 232.2. 9. 400.5. 10. 540. 11. 400.35. 12. 79.8. 13. 40.
 14. 70. 15. 180. 16. 180. 17. 50. 18. 67.2. 19. 210. 20. 250.
 21. 288. 22. 455. 23. 705. 24. 840. 25. 187.5. 26. 690. 27. 900.
 28. 1500. 29. 221. 30. 576. 31. 1202.87. 32. 1800. 33. 720.
 34. 728.5. 35. 3025. 36. 5250.

Page 262.—37. .005, .005 of 150 or .75. 38. .0025, .0025 of 250 or
 .625. 39. .825, 1.575, 3. 40. $1\frac{1}{2}$, $1\frac{1}{2}$, 2.
 1. \$900. 2. Saves \$240, spends \$960.

Page 263.—3. \$7.50. 4. \$240. 5. \$2812.50. 6. \$625. 7. \$550
 (ost. 8. \$87.50; \$437.50. 9. \$2812.50. 10. \$48. 11. 60 bu. 12. \$98.

Page 265.—1. \$6. 2. \$7. 3. \$12. 4. \$13.12½. 5. \$15.75.
 6. \$26.25. 7. \$18. 8. \$30. 9. \$50. 10. \$162.50. 11. \$276.25.
 12. \$281.25. 13. \$20. 14. \$37.50. 15. \$15. 16. \$112.50. 17. \$100.
 18. \$166.25. 19. \$296.25. 20. \$369.37½. 21. \$442.12½. 22. \$586.50.
 23. \$731.25. 24. \$836.35. 25. \$943.31½. 26. \$2932.50. 27. \$2437.50.
 28. \$3120. 29. \$3646.87½. 30. \$4111.87. 31. \$5404. 32. \$7468.50.
 33. \$274.81½. 34. \$577.35. 35. \$781.59. 36. \$999.72. 37. \$1492.66+.
 38. \$2203.42. 39. \$3237.50. 40. \$4005.62.

Page 266.—1. \$50. 2. \$5.00. 3. \$2900. 4. \$141. 5. \$10.50.
 6. \$10.68½. 7. \$470.25. 8. \$1562.50, \$60,937.50. 9. \$1320 cost to buyer;
 \$1243.12½ received by owner.

Page 268.—1. \$48. 2. \$75. 3. \$105. 4. \$96. 5. \$150.
 6. \$180. 7. \$210. 8. \$120. 9. \$252. 10. \$144. 11. \$315.
 12. \$1080. 13. \$756. 14. \$650.

Page 269.—1. \$15. 2. \$17.50. 3. \$22.50. 4. \$30.63. 5. \$42.50.
 6. \$64.17. 7. \$66. 8. \$151.25. 9. \$18.75. 10. \$21.67.
 11. \$49.20. 12. \$71.55.

Page 270.—13. \$77.40. 14. \$114.21. 15. \$238. 16. \$236.61.
 1. \$14.40. 2. \$23.06. 3. \$16.24. 4. \$24.83. 5. \$50.97.
 6. \$41.53. 7. \$85.05. 8. \$31.04. 9. \$119.59. 10. \$131.97.

Page 273.—1. \$1.20. 2. \$3.60. 3. \$6. 4. \$5. 5. \$7.68.
 6. \$12. 7. \$10. 8. \$28.80. 9. \$50. 10. \$1.60.

Page 274.—1. \$4.49+. 2. \$89,756,518.84. 3. Pennsylvania \$2.80+;
 California \$13.04+. 4. 280 cities, \$1,792,640.10. New England States,
 \$2706.16+; Eastern States, \$6559.81+; Southern States, \$2221.90+; Middle
 Western States, \$8096.23+; Western States, \$1865.46+; Pacific States, \$19,668.84+.

Page 276.—1. \$5150. 2. \$300. 3. \$240, \$280. 4. 3 bonds, \$50
 payment on a 4th bond.

Page 277.—1. \$277.20. 2. 384 boxes. 3. \$26.40. 4. 30 A.
 5. 9 cu. ft. 6. 157.5 sq. ft. 7. 66.2 cu. ft. 8. 70 lb. 9. \$20.75.
 10. 6.69+ lb. 11. 45½%.

Page 278.—12. \$60,000. 13. 60%. 14. 11½ ft. 15. \$440.
 16. 336 miles. 17. \$6.08. 18. 19+ sec. 19. The first.

Page 279.—20. 100 A. 21. 4.95, 3.5, 61.2. 22. 72.5 lb. 23. \$20.
 24. \$192. 25. \$.03+. 26. \$8.03.

Page 280.—27. \$3600. 28. 75 lb. timothy, 50 lb. red top, 25 lb. clover.
 29. \$555 first year, \$545 second year. 30. \$34. 31. \$156.25. 32. \$.25.

Page 281.—33. \$4, \$1.25. 34. Tom \$64, Jerry \$60, Dick \$36.
 35. 224 cu. in., 16 sq. in. 36. \$.58. 37. \$2900, 125 doz.

Page 282.—1. 25 ft. 4 in. 2. 30 ft. 6 in. 3. 41 ft. 5 in. 4. 50 ft.
 5. 26 lb. 4 oz. 6. 38 lb. 12 oz. 7. 21 bu. 1 pk. 8. 39 bu. 9. 63 qt. 1 pt.
 10. 49 bu. 11. 64 mi. 125 rd. 12. 33 hr. 20 min. 13. 54 ft. 9 in.
 14. 122 ft. 9 in. 15. 17 T. 1033 lb. 16. 35 wk. 2 da. 17. 60 yd. 2 ft. 3 in.
 18. 22 hr. 27 min. 29 sec. 19. 28 bu. 2 pk. 7 qt.

Page 283.—**20.** 42 sq. yd. 8 sq. ft. 6 sq. in. **21.** 19 A. 1 sq. rd. 11 sq. yd. 8 sq. ft. **22.** 15 yd. 1 ft. 4 in. **23.** 28 hr. 7 min. **24.** 28 T. 4 cwt. 10 lb. **25.** 37 cu. yd. 25 cu. ft. **26.** 16 yr. 10 mo. 15 da. **27.** 30 gal. 3 qt. 1 pt. 1 gi. **28.** 35 hr. 1 min. **29.** 69 cu. yd. 14 cu. ft.

Page 284.—**1.** 8 ft. 2 in. **2.** 22 ft. 11 in. **3.** 24 lb. 11 oz. **4.** 1 T. 17 cwt. 5. 28 lb. 8 oz. **6.** 9 bu. 3 pk. **7.** 14 hr. 30 min. **8.** 4 wk. 3 da. **9.** 5 bu. 3 pk. 6 qt. **10.** 3 yd. 1 ft. 10 in. **11.** 55 min. 49 sec. **12.** 9 T. 14 cwt. 65 lb. **13.** 52 sq. yd. 8 sq. ft. **14.** 10 hr. 56 min. 55 sec. **15.** 2 mo. 25 da. **16.** 1 yr. 3 mo. 25 da. **17.** 1 yr. 5 mo. 24 da. **18.** 17 yr. 3 mo. 7 da.

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